

Electrical Engineering

Signals and Systems

Comprehensive Theory

with Solved Examples and Practice Questions



MADE EASY
Publications

**MADE EASY Publications Pvt. Ltd.**

Corporate Office: 44-A/4, Kalu Sarai (Near Hauz Khas Metro Station), New Delhi-110016

E-mail: infomep@madeeasy.in

Contact: 011-45124612, 0-9958995830, 8860378007

Visit us at: www.madeeasypublications.org

Signals and Systems

© Copyright, by MADE EASY Publications Pvt. Ltd.

All rights are reserved. No part of this publication may be reproduced, stored in or introduced into a retrieval system, or transmitted in any form or by any means (electronic, mechanical, photo-copying, recording or otherwise), without the prior written permission of the above mentioned publisher of this book.

First Edition : 2015

Second Edition : 2016

Third Edition : 2017

Fourth Edition : 2018

Fifth Edition : 2019

Sixth Edition : 2020

Seventh Edition : 2021

Contents

Signals and Systems

Chapter 1

Introduction to Signals 2

- 1.1 Elementary Signals 2
- 1.2 Classification of Signals..... 22
- 1.3 Basic Operations on Signals 48
 - Student Assignments-1* 69
 - Student Assignments-2* 70

Chapter 2

Introduction to Systems..... 71

- 2.1 Continuous-time and discrete-time systems..... 72
- 2.2 Classification of Systems..... 72
- 2.3 Linear Time-Invariant (LTI) Systems 82
- 2.4 Continuous time LTI systems 82
- 2.5 Discrete-time LTI Systems 96
- 2.6 LTI System Properties and the Impulse Response..... 100
- 2.7 Step Response of an LTI System..... 104
 - Student Assignments-1* 107
 - Student Assignments-2* 109

Chapter 3

Continuous-time Fourier Series 110

- 3.1 Different Forms of Fourier Series 110
- 3.2 Symmetry Conditions in Fourier Series..... 113
- 3.3 Dirichlet Conditions 117
- 3.4 Properties of Fourier Series 119
- 3.5 Systems with Periodic Inputs..... 127
- 3.6 Limitations of Fourier Series..... 127
 - Student Assignments-1* 127
 - Student Assignments-2* 128

Chapter 4

Continuous Time Fourier Transform..... 129

- 4.1 The Definition 129
- 4.2 Fourier Transform of Some Basic Signals 130
- 4.3 Inverse Fourier Transform of Some Basic Functions 135
- 4.4 Properties of Fourier Transform 137
- 4.5 Fourier Transform of Periodic Signal 155
- 4.6 Application of Fourier Transform..... 158
- 4.7 Ideal and Practical Filters..... 159
- 4.8 Energy Spectral Density (ESD)..... 160
- 4.9 Power Spectral Density (PSD) 161
- 4.10 Correlation 162
- 4.11 Limitation of Fourier Transform and its Solution 164
 - Student Assignments-1* 165
 - Student Assignments-2* 166

Chapter 5

Laplace Transform 167

- 5.1 The Definition 167
- 5.2 Relationship between Laplace Transform and Fourier Transform 168
- 5.3 Eigen Value and Eigen Function 168
- 5.4 Region of Convergence (ROC) for Laplace Transform..... 169
- 5.5 Laplace Transforms to Some Basic Signals..... 170
- 5.6 Properties of Laplace Transform 177
- 5.7 Inverse Laplace Transform 185
- 5.8 LTI System and Laplace Transform 190
- 5.9 Interconnection of LTI Systems (Block Diagrams) 194
- 5.10 Laplace Transform of Causal Periodic Signals 195
- 5.11 Unilateral Laplace Transform 197
- 5.12 Properties of Unilateral Laplace Transform (ULT) 199

5.13 Application of Laplace Transform in	
Solving Differential Equations.....	204
Student Assignments-1	208
Student Assignments-2	210

Chapter 6

Sampling211

6.1 The Sampling Theorem.....	211
6.2 Sampling Techniques.....	216
6.3 Sampling Theorem for Band Pass Signals	218
6.4 Reconstruction of Signal	219
Student Assignments-1	224
Student Assignments-2	224

Chapter 7

z-Transform225

7.1 The Definition.....	225
7.2 Region of Convergence for z-transform.....	226
7.3 z-Transform of Some Basic Signals.....	229
7.4 Properties of z-Transform.....	235
7.5 Inverse z-Transform	243
7.6 Discrete-time LTI Systems and z-Transform.....	250
7.7 z-Transform of Causal Periodic Signals.....	256
7.8 Relation Between Laplace Transform and z-Transform...	257
7.9 Unilateral z-Transform	259
7.10 Properties of Unilateral z-transform (UZT).....	260
7.11 z-Transform Solution of Linear Difference Equations.....	263
Student Assignments-1	268
Student Assignments-2	270

Chapter 8

Discrete Fourier Transform (DFT)271

8.1 The Definition.....	272
8.2 Properties of DFT.....	276
8.3 Introduction to FFT (Fast Fourier Transform).....	281
Student Assignment.....	281

Chapter 9

Digital Filters.....282

9.1 Introduction.....	282
9.2 Filter Basics.....	282
9.3 Butterworth Filters	283
9.4 Digital Filters.....	285
9.5 Basics Structures for IIR Systems	286
9.6 Basic Structures for FIR Systems.....	298
9.7 IIR Filter Design from Continuous-Time Filters.....	302
9.8 Impulse Invariant Method.....	302
9.9 Design of IIR Filter by Approximation of Derivatives.....	308
9.10 IIR Filter design by the bilinear transformation	311
9.11 Design of FIR filters	315
9.12 Design of linear phase FIR filters using frequency sampling method	326
9.13 Comparison of Designing methods.....	328

Chapter 10

Fourier Analysis of Discrete Time Signals329

10.1 The Definition.....	329
10.2 Properties of DTFS	331
10.3 The Definition : DTFT	331
10.4 DTFT of Some Basic Signals.....	333
10.5 Properties of DTFT	338
10.6 Fourier Transform Pairs Using Inverse DTFT	347
10.7 Fourier Transform of Periodic Signals	349
10.8 LTI System Analysis and DTFT	350
10.9 Application of DTFT.....	351
10.10 Ideal and Practical Filters.....	353
10.11 Relationship between CTFT and DTFT	357
10.12 Energy Spectral Density	358
10.13 Power Spectral Density (PSD)	358
10.14 Correlation.....	358
Student Assignments-1	359
Student Assignments-2	360



Signals and Systems

Introduction to Signals and Systems

This book starts with basic and extensive chapter on signals in which continuous and discrete-time case are discussed in parallel. A variety of basic signals, functions with their mathematical description, representation and properties are incorporated. A substantial amount of examples are given for quick sketching of functions. A chapter on systems is discussed separately which deals with classification of systems, both in continuous and discrete domain and more emphasize is given to LTI systems and analytical as well as graphical approach is used to understand convolution operation. These two chapters makes backbone of the subject.

Further we shall proceed to transform calculus which is important tool of signal processing. A logical and comprehensive approach is used in sequence of chapters. The continuous time Fourier series which is base to the Fourier transform, deals with periodic signal representation in terms of linear complex exponential, is discussed.

The Fourier transform is discussed before Laplace transform. The sampling, a bridge between continuous-time and discrete-time, is discussed to understand discrete-time domain.

A major emphasis is given on proof of the properties so that students can understand and analyzes fundamental easily.

A point wise recapitulation of all the important points and results in every chapter proves helpfull to students in summing up essential developments in the chapter which is an integral part of any competitive examination.

Introduction to Signals

Introduction

A signal is any quantity having information associated with it. It may also be defined as a function of one or more independent variables which contain some information. A function defines a relationship between two sets i.e. one is domain and another is range.

It means function defines mapping from one set to another and similarly a signal may also be defined as mapping from one set (domain) to another (range). e.g.

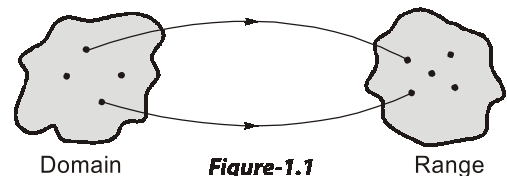


Figure-1.1

- A speech signal would be represented by acoustic pressure as a function of time.
- A monochromatic picture would be represented by brightness as a function of two spatial variable.
- A voltage signal is defined by a voltage across two points varying as function of time.
- A video signal, in which color and intensity as a function of 2-dimensional space (2D) and 1-dimensional time (i.e. hybrid variables).

NOTE: In this course of “signals and systems”, we shall focus on signals having only one variable and will consider ‘time’ as independent variable.

1.1 Elementary Signals

These signals serve as basic building blocks for construction of somewhat more complex signals. The list of elementary signals mainly contains singularity functions and exponential functions.

These elementary signals are also known as basic signals/standard signals.

Let us discuss these basic signals one-by-one.

1.1.1 Unit Impulse Function

A continuous-time unit impulse function $\delta(t)$, also called as dirac delta function is defined as

$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad \int_{-\infty}^{\infty} \delta(t) dt = 1$$

The unit-impulse function is represented by an arrow with strength of ‘1’ which represents its ‘area’ or ‘weight’.

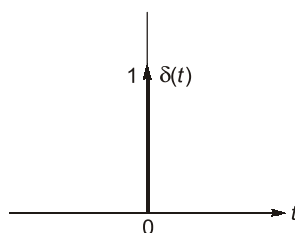


Figure-1.2

The above definition of an impulse function is more generalised and can be represented as limiting process without any regard to shape of a pulse. For example, one may define impulse function as a limiting case of rectangular pulse, triangular pulse Gaussian pulse, exponential pulse and sampling pulse as shown below:

(i) Rectangular Pulse

$$\delta(t) = \lim_{\varepsilon \rightarrow 0} p(t)$$

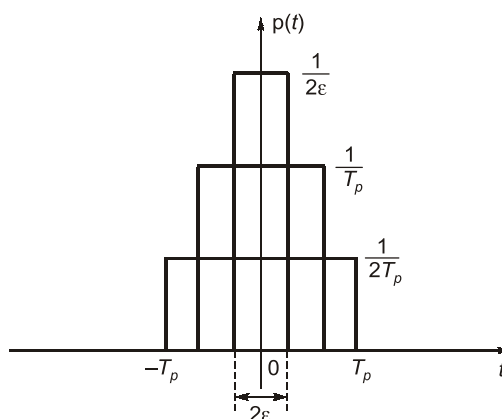


Figure-1.3

(ii) Triangular Pulse

$$\delta(t) = \begin{cases} \lim_{\tau \rightarrow 0} \frac{1}{\tau} \left[1 - \frac{|t|}{\tau} \right] & ; |t| < \tau \\ 0 & ; |t| > \tau \end{cases}$$

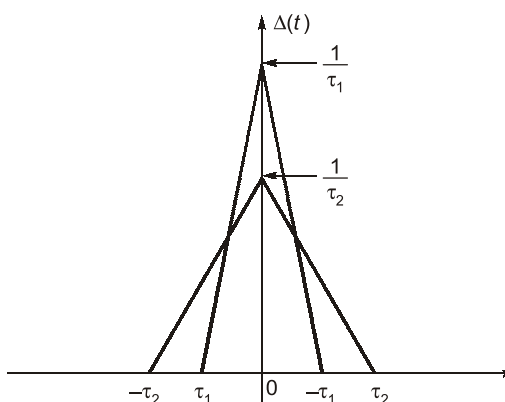


Figure-1.4

(iii) Gaussian Pulse

$$\delta(t) = \lim_{\tau \rightarrow 0} \frac{1}{\tau} \left[e^{-\pi t^2 / \tau^2} \right]$$

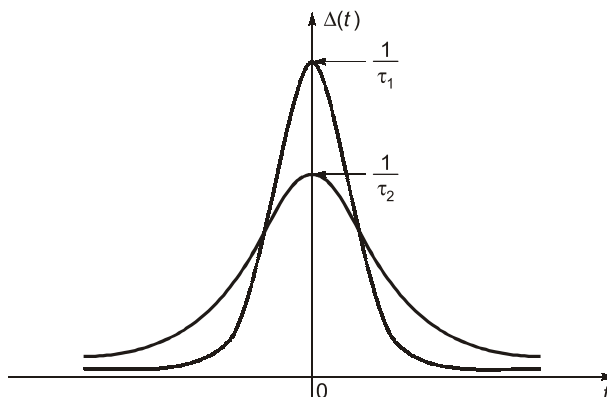


Figure-1.5

(iv) Exponential Pulse

$$\delta(t) = \lim_{\tau \rightarrow 0} \frac{1}{2\tau} \left[e^{-|t|/\tau} \right]$$

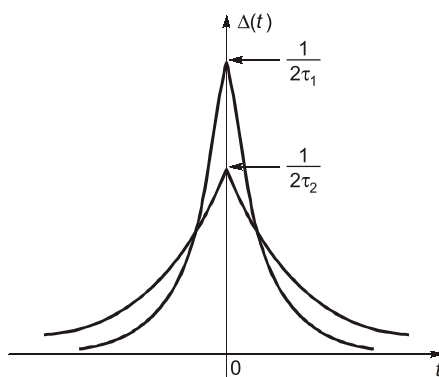


Figure-1.6

(v) Sampling Function

$$\int_{-\infty}^{\infty} \frac{k}{\pi} \text{Sa}(kt) dt = 1$$

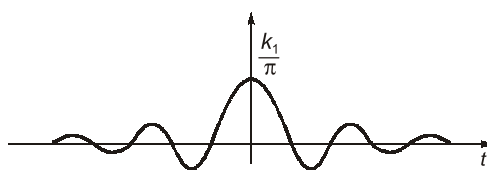


Fig. (a)

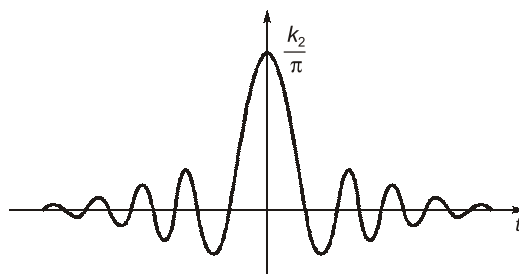


Fig. (b)

Figure-1.7

Properties of Continuous Time Unit Impulse Function**(i) Scaling property:**

$$\delta(at) = \frac{1}{|a|} \delta(t) \quad ; \quad 'a' \text{ is a constant, positive or negative}$$

Proof:

$$\delta(at) = \frac{1}{|a|} \delta(t)$$

Integrating above equation on both the sides with respect to 't'.

$$\int_{-\infty}^{+\infty} \delta(at) dt = \int_{-\infty}^{+\infty} \frac{1}{|a|} \delta(t) dt$$

Let

$$at = \tau$$

$$a \cdot dt = d\tau \quad ; \quad 'a' \text{ is a constant, positive or negative}$$

or

$$|a| \cdot dt = d\tau$$

Now,

$$\int_{-\infty}^{+\infty} \delta(at) dt = \int_{-\infty}^{+\infty} \delta(\tau) \cdot \frac{d\tau}{|a|} = \int_{-\infty}^{+\infty} \frac{1}{|a|} \delta(t) \cdot dt$$

By definition,

$$\int_{-\infty}^{+\infty} \delta(t) dt = \int_{-\infty}^{+\infty} \delta(\tau) d\tau = 1$$

**Important Expressions**

- $\delta(at \pm b) = \frac{1}{|a|} \delta\left(t \pm \frac{b}{a}\right)$
- $\delta(-t) = \delta(t) \quad \dots \delta(t) \text{ is an even function of time.}$

(ii) Product property/multiplication property:

$$x(t)\delta(t - t_o) = x(t_o)\delta(t - t_o)$$

Proof:The function $\delta(t - t_o)$ exists only at $t = t_o$. Let the signal $x(t)$ be continuous at $t = t_o$.

$$\begin{aligned} \text{Therefore,} \quad x(t) \delta(t - t_o) &= x(t)|_{t=t_o} \cdot \delta(t - t_o) \\ &= x(t_o) \delta(t - t_o) \end{aligned}$$

**Important Expressions**

- $x(t) \delta(t) = x(0) \delta(t)$

(iii) Sampling property:

$$\int_{-\infty}^{+\infty} x(t) \delta(t - t_o) dt = x(t_o)$$

Proof :

Using product property of impulse function

$$x(t) \delta(t - t_0) = x(t_0) \delta(t - t_0)$$

Integrating above equation on both the sides with respect to 't'.

$$\begin{aligned} \int_{-\infty}^{+\infty} x(t) \delta(t - t_0) dt &= \int_{-\infty}^{+\infty} x(t_0) \delta(t - t_0) dt \\ &= x(t_0) \int_{-\infty}^{+\infty} \delta(t - t_0) dt = x(t_0) \end{aligned}$$

**Important Expressions**

- $\int_{-\infty}^{+\infty} x(t) \delta(t) dt = x(0)$

(iv) The first derivative of unit step function results in unit impulse function.

$$\delta(t) = \frac{d}{dt} u(t)$$

Proof :

Let the signal $x(t)$ be continuous at $t = 0$.

$$\begin{aligned} \text{Consider the integral } \int_{-\infty}^{+\infty} \frac{d}{dt} [u(t)] x(t) dt &= [u(t) x(t)]_{-\infty}^{+\infty} - \int_{-\infty}^{+\infty} x'(t) u(t) dt \\ &= x(\infty) - \int_0^{\infty} x'(t) dt \\ &= x(\infty) - [x(t)]_0^{\infty} \\ &= x(0) \end{aligned} \quad \dots(i)$$

$$\text{We know from sampling property } x(0) = \int_{-\infty}^{+\infty} x(t) \delta(t) dt \quad \dots(ii)$$

From equations (i) and (ii), we get

$$\int_{-\infty}^{+\infty} \frac{d}{dt} [u(t)] x(t) dt = \int_{-\infty}^{+\infty} x(t) \delta(t) dt$$

On comparing, we get $\delta(t) = \frac{d}{dt} u(t)$

(v) Derivative property:

$$\int_{t_1}^{t_2} x(t) \delta^n(t - t_0) dt = (-1)^n x^n(t) \Big|_{t=t_0} ; t_1 < t_0 < t_2 \text{ and suffix } n \text{ means } n^{\text{th}} \text{ derivative}$$

Proof:

Let the signal $x(t)$ be continuous at $t = t_0$ where $t_1 < t_0 < t_2$.

Consider the derivative $\frac{d}{dt}[x(t) \delta(t - t_0)] = x(t) \delta'(t - t_0) + x'(t) \delta(t - t_0)$

Integrating above equation on both the sides with respect to 't'.

$$\begin{aligned} \int_{t_1}^{t_2} \frac{d}{dt}[x(t) \delta(t - t_0)] dt &= \int_{t_1}^{t_2} x(t) \delta'(t - t_0) dt + \int_{t_1}^{t_2} x'(t) \delta(t - t_0) dt \\ [x(t) \delta(t - t_0)]_{t_1}^{t_2} &= \int_{t_1}^{t_2} x(t) \delta'(t - t_0) dt + \int_{t_1}^{t_2} x'(t) \delta(t - t_0) dt \\ [x(t_2) \delta(t_2 - t_0) - x(t_1) \delta(t_1 - t_0)] &= \int_{t_1}^{t_2} x(t) \delta'(t - t_0) dt + \int_{t_1}^{t_2} x'(t) \delta(t - t_0) dt \end{aligned}$$

Here, $\delta(t_1 - t_0) = 0$ and $\delta(t_2 - t_0) = 0$ because $t_0 \neq t_1$ or $t_0 \neq t_2$

$$\text{So,} \quad 0 = \int_{t_1}^{t_2} x(t) \delta'(t - t_0) dt + \int_{t_1}^{t_2} x'(t) \delta(t - t_0) dt$$

$$\begin{aligned} \int_{t_1}^{t_2} x(t) \delta'(t - t_0) dt &= (-1) \int_{t_1}^{t_2} x'(t) \delta(t - t_0) dt \quad (\because \text{using sampling property}) \\ \Rightarrow &= (-1) x'(t_0) \end{aligned}$$

$$\text{Hence,} \quad \int_{t_1}^{t_2} x(t) \delta'(t - t_0) dt = (-1)^1 x'(t_0)$$

If same procedure is repeated for second derivative, we get

$$\int_{t_1}^{t_2} x(t) \delta''(t - t_0) dt = (-1)^2 x''(t_0)$$

On generalising aforementioned results, we get

$$\int_{t_1}^{t_2} x(t) \delta^n(t - t_0) dt = (-1)^n x^n(t_0)$$

(vi) Shifting Property:

According to shifting property, any signal can be produced as combination of weighted and shifted impulses.

$$x(t) = \int_{-\infty}^{+\infty} x(\tau) \delta(t - \tau) d\tau$$

Proof:

Using product property

$$x(t) \delta(t - t_0) = x(t_0) \delta(t - t_0)$$

Replacing t_0 by τ

$$x(t) \delta(t - \tau) = x(\tau) \delta(t - \tau)$$

Integrating above equation on both the sides with respect to 'τ'.

$$\int_{-\infty}^{+\infty} x(t) \delta(t - \tau) d\tau = \int_{-\infty}^{+\infty} x(\tau) \delta(t - \tau) d\tau$$

$$x(t) \int_{-\infty}^{+\infty} \delta(t - \tau) d\tau = \int_{-\infty}^{+\infty} x(\tau) \delta(t - \tau) d\tau$$

$$x(t) \cdot 1 = \int_{-\infty}^{+\infty} x(\tau) \delta(t - \tau) d\tau$$

$$x(t) = \int_{-\infty}^{+\infty} x(\tau) \delta(t - \tau) d\tau$$

(vii) The derivative of impulse function is known as **doublet** function.

$$\delta'(t) = \frac{d}{dt} \delta(t)$$

Graphically,

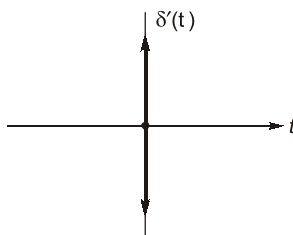


Figure-1.8

Area under the **doublet** function is always zero.

Discrete-Time Case

The discrete time unit impulse function $\delta[n]$, also called unit sample sequence or delta sequence is defined as

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$

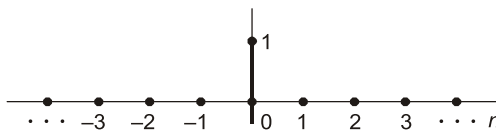


Figure-1.9

It is also known as **Kronecker delta**.

Properties of Discrete Time Unit Impulse Sequence

(i) **Scaling property:**

$$\delta[kn] = \delta[n]; k \text{ is an integer}$$

Proof:

By definition of unit impulse sequence

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

Similarly,

$$\begin{aligned} \delta[kn] &= \begin{cases} 1, & kn = 0 \\ 0, & kn \neq 0 \end{cases} \\ &= \begin{cases} 1, & n = \frac{0}{k} = 0 \\ 0, & n \neq \frac{0}{k} \neq 0 \end{cases} \\ &= \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases} = \delta[n] \end{aligned}$$

(ii) Product property:

$$x[n] \delta[n - n_0] = x[n_0] \delta[n - n_0]$$

From definition,

$$\delta[n - n_0] = \begin{cases} 1, & n = n_0 \\ 0, & n \neq n_0 \end{cases}$$

We see that impulse has a non zero value only at $n = n_0$

Therefore, $x[n] \delta[n - n_0] = x[n]_{n=n_0} \delta[n - n_0]$

$$x[n] \delta[n - n_0] = x[n_0] \delta[n - n_0]$$

(iii) Shifting property:

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k] \delta[n - k]$$

Proof:

From product property

$$x[n] \delta[n - n_0] = x[n_0] \delta[n - n_0]$$

Replacing n_0 by 'k'

$$x[n] \delta[n - k] = x[k] \delta[n - k]$$

$$\Rightarrow \sum_{k=-\infty}^{+\infty} x[n] \delta[n - k] = \sum_{k=-\infty}^{+\infty} x[k] \delta[n - k]$$

$$\Rightarrow x[n] \sum_{k=-\infty}^{+\infty} \delta[n - k] = \sum_{k=-\infty}^{+\infty} x[k] \delta[n - k]$$

$$\Rightarrow x[n] \cdot 1 = \sum_{k=-\infty}^{+\infty} x[k] \delta[n - k]$$

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k] \delta[n - k]$$

(iv) The first difference of unit step sequence results in unit impulse sequence.

$$\delta[n] = u[n] - u[n-1]$$

Proof:

By definition of unit step sequence

$$\begin{aligned} u[n] &= \sum_{k=0}^{\infty} \delta[n-k] \\ &= \delta[n] + \sum_{k=1}^{\infty} \delta[n-k] \end{aligned} \quad \dots(i)$$

But,

$$u[n-1] = \sum_{k=1}^{\infty} \delta[n-k]$$

$$u[n] = \delta[n] + u[n-1]$$

$$\delta[n] = u[n] - u[n-1]$$

We get,

Graphically we can see,

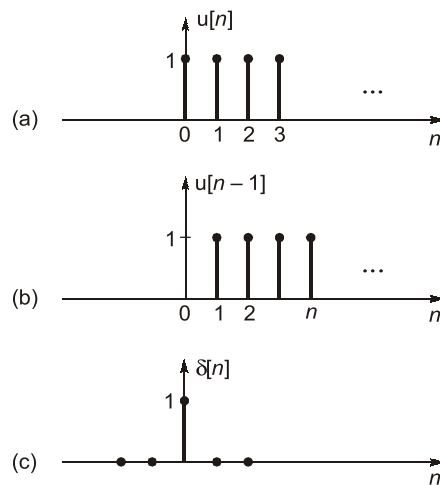


Figure-1.10

Summary Table:

S.No.	Properties of CT unit Impulse Function	Properties of DT unit impulse sequence
1.	$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty} \delta(t) dt = 1$	$\delta[n] = \begin{cases} 1, & \text{if } n = 0; \\ 0, & \text{otherwise} \end{cases}$
2.	$x(t) \delta(t - t_0) = x(t_0) \delta(t - t_0)$	$x[n] \delta[n - k] = x[k] \delta[n - k]$
3.	$\delta(t) = \frac{d}{dt} u(t)$	$\delta[n] = u[n] - u[n - 1]$
4.	$\int_0^{\infty} \delta(t - \tau) d\tau = u(t)$	$\sum_{k=0}^{\infty} \delta[n - k] = u[n]$
5.	$x(t) = \int_{-\infty}^{\infty} x(\tau) \delta(t - \tau) d\tau$	$x[n] = \sum_{k=-\infty}^{\infty} x[k] \delta[n - k]$
6.	$\int_{-\infty}^{\infty} x(t) \delta(t - t_0) dt = x(t_0)$	$\sum_{n=-\infty}^{\infty} x[n] \delta[n - n_0] = x[n_0]$
7.	$\delta(at) = \frac{1}{ a } \delta(t)$ $\delta(at \pm b) = \frac{1}{ a } \delta\left(t \pm \frac{b}{a}\right)$ $\delta(-t) = \delta(t)$	$\delta[kn] = \delta[n]$ $\delta[-n] = \delta[n]$
8.	$\int_{t_1}^{t_2} x(t) \delta(t) dt = \begin{cases} x(0), & t_1 < t < t_2 \\ 0, & \text{otherwise} \end{cases}$	
9.	$\int_{t_1}^{t_2} x(t) \delta^n(t - t_0) dt = (-1)^n x^n(t_0), t_1 < t_0 < t_2$ where suffix n mean n^{th} derivative	
10.	$\delta'(t) = \frac{d}{dt} \delta(t)$	

Example - 1.1

The Dirac delta function $\delta(t)$ is defined as

(a) $\delta(t) = \begin{cases} 1 & ; \quad t = 0 \\ 0 & ; \quad \text{otherwise} \end{cases}$

(b) $\delta(t) = \begin{cases} \infty & ; \quad t = 0 \\ 0 & ; \quad \text{otherwise} \end{cases}$

(c) $\delta(t) = \begin{cases} 1 & ; \quad t = 0 \\ 0 & ; \quad \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty} \delta(t) dt = 1$

(d) $\delta(t) = \begin{cases} \infty & ; \quad t = 0 \\ 0 & ; \quad \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty} \delta(t) dt = 1$

Solution: (d)

Example - 1.2

The integral $\int_{-\infty}^{\infty} \delta\left(t - \frac{\pi}{6}\right) 6 \sin(t) dt$ evaluate to

- (a) 6 (b) 3
(c) 1.5 (d) 0

Solution: (b)

Given signal is

$$x(t) = \int_{-\infty}^{\infty} \delta\left(t - \frac{\pi}{6}\right) 6 \sin t dt$$

By shifting property of unit impulse function

$$\int_{t_1}^{t_2} x(t) \delta(t - t_0) dt = \begin{cases} x(t_0); & t_1 < t_0 < t_2 \\ 0; & \text{elsewhere} \end{cases}$$

$$\begin{aligned} \int_{-\infty}^{\infty} \delta\left(t - \frac{\pi}{6}\right) 6 \sin(t) dt &= 6 \cdot \sin \frac{\pi}{6} \\ &= 6 \times \frac{1}{2} = 3 \end{aligned}$$

Example - 1.3

If $y(t) + \int_0^{\infty} y(\tau) x(t - \tau) d\tau = \delta(t) + x(t)$, then $y(t)$ is

- (a) $u(t)$ (b) $\delta(t)$
(c) $r(t)$ (d) 1

Solution: (b)

Let

$$y(t) = \delta(t)$$

$$\begin{aligned} y(t) + \int_0^{\infty} y(\tau) x(t - \tau) d\tau &= \delta(t) + \int_0^{\infty} \delta(\tau) x(t - \tau) d\tau \\ &= \delta(t) + x(t) \end{aligned}$$

So, $y(t) = \delta(t)$ satisfies the given equation.

Example - 1.4

Which of the following is NOT a property of impulse function?

- (a) $x(t) \delta(t - t_0) = x(t_0) \delta(t - t_0)$ (b) $x(t) * \delta(t - t_0) = x(t - t_0)$
(c) $\int_{t_1}^{t_2} x(t) \delta(t - t_0) dt = x(t_0); t_1 < t < t_2$ (d) $\int_{-\infty}^{+\infty} x(t) \delta^n(t - t_0) dt = (-1)^n \frac{d^n}{dt^n} x(t) \Big|_{t=t_0}$

Solution: (d)

By derivative property

$$\int_{-\infty}^{+\infty} x(t) \delta^n(t - t_0) dt = (-1)^n \frac{d^n}{dt^n} x(t) \Big|_{t=t_0}$$



**Student's
Assignments**

1

Objective Questions

Q.1 Which one of the following relations is **not** correct?

- (a) $f(t)\delta(t) = f(0)\delta(t)$ (b) $\int_{-\infty}^{\infty} f(t)\delta(\tau) d\tau = 1$
(c) $\int_{-\infty}^{\infty} \delta(\tau) d(\tau) = 1$ (d) $f(t)\delta(t-\tau) = f(\tau)\delta(t-\tau)$

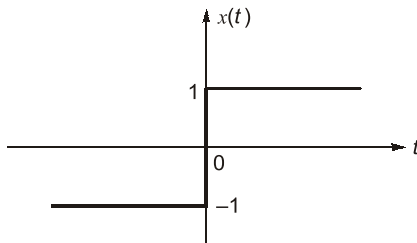
Q.2 The odd component of the signal $x(t) = e^{-2t} \cos t$ is

- (a) $\cosh(2t) \cos t$ (b) $-\sinh(2t) \cos t$
(c) $-\cosh(2t) \cos t$ (d) $\sinh(2t) \cos t$

Q.3 The value of

- $\int_{-2}^2 [(t-3)\delta(2t+2) + 8\cos\pi t \delta'(t-0.5)] dt$ is
(a) 23.13 (b) 13.56
(c) 6.39 (d) 7.85

Q.4 Function $x(t)$ is shown in the figure.



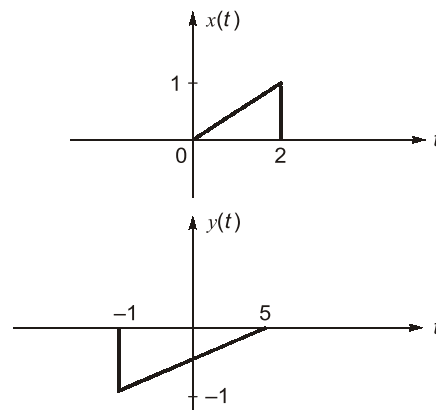
The $x(t)$ in terms of unit step function is _____ and the odd part of unit step function is _____ respectively.

- (a) $2u(t) + 1; \frac{1}{2}\text{sgn}(t)$
(b) $2u(t) - 1; \frac{1}{2}\text{sgn}(t)$
(c) $2u(t) - 1; \frac{1}{2}$
(d) $2u(-t) - 1; \frac{1}{2}$

Q.5 An LTI system has the input signal $x[n]$. The correct sequence of operation to get output $y[n] = x[n - M/L]$; $M > 1, L > 1$ is

- (a) Interpolation by L , Delay by M , Decimation by L
(b) Delay by M , Interpolation by L , Decimation by M
(c) Decimation by L , Delay by M , Interpolation by L
(d) Interpolation by L , Decimation by L , Delay by M

Q.6 Two signals $x(t)$ and $y(t)$ are shown below.



then $x(t)$ in terms of $y(t)$ can be written as

- (a) $-x\left(\frac{t-5}{3}\right)$ (b) $-x\left(\frac{t+5}{3}\right)$
(c) $-x\left(\frac{-(t+5)}{3}\right)$ (d) $-x\left(\frac{-(t-5)}{3}\right)$

Q.7 Fundamental frequency of periodic signal $e^{j\omega_0 n}$ is given as

(where m is integer and N is the period of the signal)

- (a) $m\left(\frac{N}{2\pi}\right)$ (b) $N\left(\frac{2\pi}{m}\right)$
(c) $m\left(\frac{2\pi}{N}\right)$ (d) None of these

Q.8 A discrete time system is given as:

$$x[n] = \cos\left(\frac{n}{4}\right) \cdot \sin\left(\frac{\pi n}{4}\right)$$

The signal is

- (a) periodic with 8 (b) periodic with $8(\pi + 1)$
(c) periodic with 4 (d) non-periodic

Numerical Questions

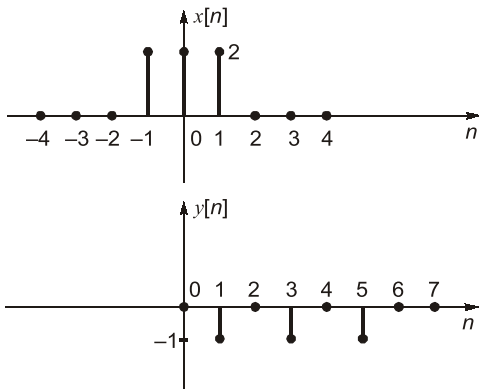
Q.9 The power of signal $x[n] = (-1)^n u[n]$ is ____ W.

Q.10 A discrete time signal is given as

$$x[n] = \cos\left(\frac{\pi n}{3}\right) \cdot (u[n] - u[n-6])$$

The energy of the signal is ____ J.

Q.11 Two functions $x[n]$ and $y[n]$ are shown in following figures.



If $y[n] = \alpha x\left[\frac{n-n_0}{k}\right]$ then value of $n_0 + \alpha + k$ is

_____.

Answers :

1. (b) 2. (b) 3. (a) 4. (b)
5. (a) 6. (d) 7. (c) 8. (d)
9. (0.5) 10. (3) 11. (4.5)

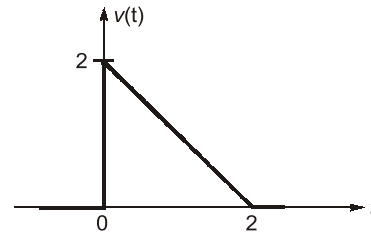


Student's Assignments

2

Q.1 With sketches of waveforms, explain the four class of signals.

Q.2 For the non-recurring waveform shown below, express $v(t)$ in terms of steps, ramps and related functions as needed.



Ans. $[v(t) = 2u(t) - r(t) + r(t-2)]$

Q.3 Show that,

(i) $\int_{-\infty}^{+\infty} \delta(t) e^{-j\omega t} dt = 1$

(ii) $\int_{-\infty}^{+\infty} \delta(t-2) \cos\left(\frac{\pi t}{4}\right) dt = 0$

(iii) $\int_{-\infty}^{+\infty} e^{-2(x-t)} \delta(2-t) dt = e^{-2(x-2)}$

Q.4 Calculate the energy of following signal

$$y(t) = \int_{-\infty}^t [\delta(\tau+2) - \delta(\tau-2)] d\tau$$

Q.5 Prove that shifting a signal does not affect its energy.

■■■■