

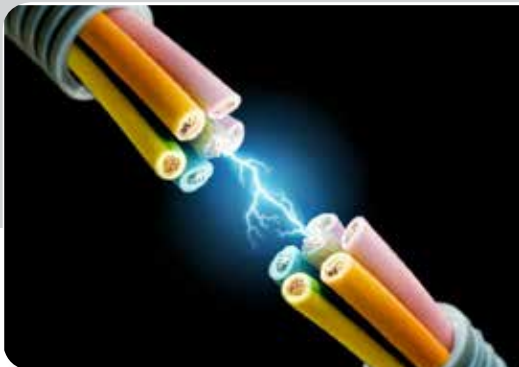
Fully Solved
Multiple Choice Questions
for

ESE

GATE

PSUs

**ELECTRICAL
ENGINEERING**



3500

Fully Solved
Multiple Choice Questions



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3500 Multiple Choice Questions for ESE, GATE, PSUs : Electrical Engineering

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First Edition: 2016

PREFACE



It gives me great happiness to introduce the revised edition on Electrical Engineering containing nearly 3500 MCQs which focuses in-depth understanding of subjects at basic and advanced level which has been segregated topic-wise to disseminate all kind of exposure to students in terms of quick learning and deep apt. The chapter wise segregation has been done to align with contemporary competitive examination pattern. Attempt has been made to bring out all kind of probable competitive questions for the aspirants preparing for ESE, GATE & PSUs. The content of this book ensures threshold level of learning and wide range of practice questions which is very much essential to boost the exam time confidence level and ultimately to succeed in all prestigious engineer's examinations. It has been ensured from made easy team to have broad coverage of subjects at chapter level.

Year by year number of competitors are increasing and the variety of questions asked in examination is widening, under such scenario this book will definitely help students to enhance their skills required to succeed in competitive exams like ESE, GATE, PSUs, State Engineering Services etc.

While preparing this book utmost care has been taken to cover all the chapters and variety of concepts which may be asked in the exams. The solutions and answers provided are upto the closest possible accuracy. The full efforts have been made by MADE EASY Team to provide error free solutions and explanations.

I have true desire to serve student community by way of providing good sources of study and quality guidance. I hope this book will be proved an important tool to succeed in competitive examinations. Any suggestions from the readers for the improvement of this book are most welcome.

B. Singh (Ex. IES)

Chairman and Managing Director
MADE EASY Group

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I

Electromagnetic Theory

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Vector Analysis

- Q.1** If $\vec{P} = x^2y^2\vec{i} + (x-y)\vec{k}$, $\vec{Q} = zx\vec{i}$ and $\phi = xy^2z^3$, then match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

List-I **List-II**

A. Div. \vec{Q} 1. $y^2z^3\vec{i} + 2yxz^3\vec{j} + 3z^2y^2x\vec{k}$

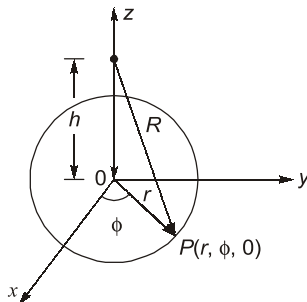
B. Grad ϕ 2. $-\vec{i} + \vec{k}x^2$

C. Curl \vec{P} 3. z

Codes:

	A	B	C
(a)	1	2	3
(b)	2	1	3
(c)	3	1	2
(d)	3	2	1

- Q.2** The unit vector \vec{a}_R which points from $z = h$ on the z -axis towards $(r, \phi, 0)$ in cylindrical co-ordinates as shown below is given by



(a) $\frac{h\vec{a}_r - r\vec{a}_z}{\sqrt{r^2 + h^2}}$	(b) $\frac{r\vec{a}_r - h\vec{a}_z}{\sqrt{r^2 + h^2}}$
(c) $\frac{h\vec{a}_\phi - r\vec{a}_z}{\sqrt{r^2 + h^2}}$	(d) $\frac{r\vec{a}_z - h\vec{a}_\phi}{\sqrt{r^2 + h^2}}$

- Q.3** If the vector V given below is irrotational, then the values of a , b and c will be respectively
- $$V = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$$

- (a) $a = 4$, $b = 2$ and $c = -1$
 (b) $a = 2$, $b = -1$ and $c = 4$
 (c) $a = 4$, $b = -1$ and $c = 2$
 (d) $a = 2$, $b = 4$ and $c = -1$

- Q.4** Match **List-I (Vector Identities)** with **List-II (Equivalent expression)** and select the correct answer using the codes given below the lists:

List-I

A. $(\vec{A} \times \vec{B}) \cdot (\vec{C} \times \vec{D})$

B. $\vec{A} \times (\vec{B} \times \vec{C})$

C. $(\vec{A} \times \vec{B}) \times (\vec{C} \times \vec{D})$

List-II

1. $(\vec{A} \cdot \vec{C} \cdot \vec{D})\vec{B} - (\vec{B} \cdot \vec{C} \cdot \vec{D})\vec{A}$

2. $[(\vec{A} \cdot \vec{C})(\vec{B} \cdot \vec{D}) - (\vec{A} \cdot \vec{D})(\vec{B} \cdot \vec{C})]$

3. $(\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$

Codes:

	A	B	C
(a)	1	3	2
(b)	3	1	2
(c)	2	1	3
(d)	2	3	1

- Q.5** The vector differential operator, Del(∇) in spherical co-ordinate system is given by

(a) $\nabla = \vec{e}_r \frac{\partial}{\partial r} + \vec{e}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} + \vec{e}_\phi \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi}$

(b) $\nabla = \vec{e}_r \frac{1}{r} \frac{\partial}{\partial r} + \vec{e}_\theta \frac{1}{r \sin \theta} + \vec{e}_\phi \frac{\partial}{\partial \phi}$

(c) $\nabla = \vec{e}_r \frac{\partial}{\partial r} + \vec{e}_\theta \frac{\partial}{\partial \theta} + \vec{e}_\phi \frac{1}{r \cos \theta} \frac{\partial}{\partial \phi}$

(d) $\nabla = \vec{e}_r \frac{1}{r} \frac{\partial}{\partial r} + \vec{e}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} + \vec{e}_\phi \frac{1}{r \cos \theta} \frac{\partial}{\partial \phi}$

- Q.6 Assertion (A):** Divergence of a vector function \vec{A} at each point gives the rate per unit volume at which the physical entity is issuing from that point.

Reason (R): If some physical entity is generated or absorbed within a certain region of the field, then that region is known as source or sink respectively and if there are no sources or sinks in the field, the net outflow of the incompressible physical entity over any part of the region is zero. However, the net outflow is said to be positive, if the total strength of the sources are greater than the total strength of sink and vice-versa.

- (a) Both A and R are true and R is a correct explanation of A.
 (b) Both A and R are true but R is not a correct explanation of A.
 (c) A is true but R is false.
 (d) A is false but R is true.

Q.7 Which of the following identity is not true?

- (a) $\vec{A}(\vec{B} \cdot \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$
 (b) $\nabla \cdot (\nabla \times \vec{A}) = 0$
 (c) $\nabla \times \nabla \phi \neq 0$
 (d) None of the above

Q.8 The vector \vec{A} directed from $(2, -4, 1)$ to $(0, -2, 0)$ in Cartesian coordinates is given by

- (a) $-2\vec{a}_x + 2\vec{a}_y + \vec{a}_z$ (b) $-2\vec{a}_x + 2\vec{a}_y - \vec{a}_z$
 (c) $-\vec{a}_x - 2\vec{a}_y + 2\vec{a}_z$ (d) $\vec{a}_x - 2\vec{a}_y - \vec{a}_z$

Q.9 What is the value of $\iint_S \vec{F} \cdot d\vec{s}$, where

$$\vec{F} = 4xz\vec{i}_1 - y^2\vec{i}_2 + yz\vec{i}_3 ?$$

Here, s is the surface bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$ and $\vec{i}_1, \vec{i}_2, \vec{i}_3$ are unit vectors along x , y and z axes respectively.

- (a) $\frac{1}{2}$ (b) $\frac{5}{2}$
 (c) 2 (d) $\frac{3}{2}$

Q.10 The vector field given by

$$\vec{A} = yz\vec{a}_x + xz\vec{a}_y + xy\vec{a}_z \text{ is}$$

- (a) rotational and solenoidal
 (b) rotational but not solenoidal
 (c) irrotational and solenoidal
 (d) irrotational but not solenoidal

Q.11 If $\vec{A} = \frac{\vec{a}_x}{\sqrt{x^2 + y^2}}$, then the value of $\nabla \cdot \vec{A}$ at

$(2, 2, 0)$ will be

- (a) -0.0884 (b) 0.0264
 (c) -0.0356 (d) 0.0542

Q.12 If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, then the value of $\vec{i} \times (\vec{r} \times \vec{i}) + \vec{j} \times (\vec{r} \times \vec{j}) + \vec{k} \times (\vec{r} \times \vec{k})$ is

- (a) \vec{r} (b) $2\vec{r}$
 (c) $3\vec{r}$ (d) $6\vec{r}$

Q.13 What is the value of constant b so that the vector

$$\vec{V} = (x + 3y)\vec{i} + (y - 2x)\vec{j} + (x + bz)\vec{k}$$

is solenoidal?

- (a) 2 (b) -1
 (c) 3 (d) -2

Q.14 Match List-I with List-II and select the correct answer using the codes given below the lists:

List-I

- A. Gauss's divergence theorem
 B. Stoke's theorem
 C. The divergence
 D. The curl

List-II

1. $\nabla \cdot \vec{A}$
 2. $\oint_L \vec{A} \cdot d\vec{l} = \iiint_S (\nabla \times \vec{A}) \cdot d\vec{s}$
 3. $\iiint_S \vec{A} \cdot d\vec{s} = \iiint_S \vec{A} \cdot \vec{e} d\vec{s}$ (\vec{e} - An unit vector)
 4. $\nabla \times \vec{A}$

Codes:

	A	B	C	D
(a)	3	2	4	1
(b)	2	3	1	4
(c)	3	2	1	4
(d)	2	3	4	1

Q.15 Assertion (A): Vector differential operator is a vector quantity and it signifies that certain operations of a differentiation are to be carried out on the scalar function following it.

Reason (R): Vector differential operator possesses properties similar to ordinary vectors.

- (a) Both A and R are true and R is a correct explanation of A.
 (b) Both A and R are true but R is not a correct explanation of A.
 (c) A is true but R is false.
 (d) A is false but R is true.

Q.16 Consider the following statements:

1. Divergence of a vector function \vec{A} at each point gives the rate per unit volume at which the physical entity is issuing from that point.
2. If a vector function ϕ represents temperature, then $\text{grad. } \phi$ or $\nabla\phi$ will represent rate of change of temperature with distance.
3. The curl of a vector function A gives the measure of the angular velocity at every point of the vector field.

Which of the above statements is/are correct?

- (a) 2 and 3 only (b) 1, 2 and 3
 (c) 1 and 3 only (d) 2 only

Q.17 Assertion (A): The Gauss's divergence theorem permits us to express certain integrals by means of surface integrals.

Reason (R): Gauss's divergence theorem states that "the surface integral of the curl of a vector field taken over any surface s is equal to the line integral of the vector field around the closed periphery (contour) of the surface."

- (a) Both A and R are true and R is a correct explanation of A.
 (b) Both A and R are true but R is not a correct explanation of A.
 (c) A is true but R is false.
 (d) A is false but R is true.

Q.18 Match **List-I (Physical quantities)** with **List-II (Dimensions)** and select the correct answer using the codes given below the lists:

List-I	List-II
A. Electric potential	1. $MT^{-2}I^{-1}$
B. Magnetic flux	2. $ML^2T^{-3}I^{-1}$
C. Magnetic field intensity	3. IL^{-1}
D. Magnetic flux density	4. $ML^2T^{-2}I^{-1}$

Codes:

A	B	C	D
(a) 2	4	3	1
(b) 4	2	3	1
(c) 1	2	1	3
(d) 4	2	1	3

Q.19 Which of the following statements is not true regarding vector algebra?

- (a) Dot product of like unit vector is unity.
 (b) Dot product of unlike unit vector is zero.
 (c) Cross product of two like unit vectors is a third unit vector having positive sign for normal rotation and negative for reverse rotation.
 (d) All the above statements are true.

Q.20 A rigid body is rotating with an angular velocity of ω where, $\vec{\omega} = \omega_x\vec{i} + \omega_y\vec{j} + \omega_z\vec{k}$ and v is the line velocity.

if \vec{r} is the position vector given by $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, then the value of $\text{curl } \vec{v}$ will be equal to

- (a) $\frac{1}{2}\omega$ (b) ω
 (c) $\frac{1}{3}\omega$ (d) 2ω

Q.21 If $\vec{r} = x\vec{i}_x + y\vec{i}_y + z\vec{i}_z$, then which of the following relation will hold true?

- (a) $\nabla\vec{r} = 3$ (b) $\nabla \times \vec{r} = 0$
 (c) Both (a) and (b) (d) Neither (a) nor (b)

Q.22 If \vec{E} is any vector field in cartesian co-ordinates system, then

- (a) $\nabla \cdot (\nabla \times \vec{E}) = \nabla \times \nabla \times \vec{E} - \nabla^2 \vec{E}$
 (b) $\text{Div. curl } \vec{E} = 0$
 (c) $\nabla \cdot (\nabla \times \vec{E}) = \nabla^2 \vec{E} - \nabla \times \nabla \times \vec{E}$
 (d) $\text{Div. curl } \vec{E} \neq 0$

Q.23 If $\vec{c} = \vec{a} \times \vec{b}$ and $\vec{b} = \vec{a} \times \vec{c}$, then

- (a) $\vec{b} = 0$ and $\vec{c} = 0$ (b) Only $\vec{b} = 0$
 (c) Only $\vec{c} = 0$ (d) $\vec{b} \neq 0$ and $\vec{c} \neq 0$

Q.24 If S is any closed surface enclosing a volume V and $\vec{A} = ax\vec{i} + by\vec{j} + cz\vec{k}$, then the value of

$\oint_S \vec{A} \cdot \hat{n} d\vec{s}$ (\hat{n} is a unit vector) will be equal to

- (a) $\frac{1}{3}(a+b+c)V$ (b) $(a+b+c)V$
 (c) $\frac{1}{2}(a+b+c)V$ (d) $(a+b+c)V$

Q.25 Assertion (A): The laplacian operator of a scalar function ϕ can be defined as "Gradient of the divergence of the scalar ϕ ".

Reason (R): Laplacian operator may be a "scalar laplacian" or a "vector laplacian" depending upon whether it is operated with a scalar function or a vector, respectively.

- (a) Both A and R are true and R is a correct explanation of A.
 (b) Both A and R are true but R is not a correct explanation of A.
 (c) A is true but R is false.
 (d) A is false but R is true.

Q.26 Match **List-I (Terms)** with **List-II (Type)** and select the correct answer using the codes given below the lists:

List-I**List-II**

- | | |
|---------------------------------|----------------------------|
| A. Curl $(\vec{F}) = 0$ | 1. Laplace equation |
| B. Div $(\vec{F}) = 0$ | 2. Irrotational |
| C. Div grad $(\phi) = 0$ | 3. Solenoidal |
| D. Div div $(\phi) = 0$ | 4. Not defined |

Codes:

- | | A | B | C | D |
|-----|----------|----------|----------|----------|
| (a) | 2 | 3 | 1 | 4 |
| (b) | 4 | 1 | 3 | 2 |
| (c) | 2 | 1 | 3 | 4 |
| (d) | 4 | 3 | 1 | 2 |

Q.27 Which of the following relations are not correct?

- (a) $[B \times C, C \times A, A \times B] = [ABC]^2$
 (b) $A \times [B \times (C \times D)] = B \cdot D (A \times C) - B \cdot C (A \times D)$
 (c) $(B \times C) \cdot (A \times D) + (C \times A) \cdot (B \times D) + (A \times B) \cdot (C \times D) = 0$
 (d) $(A \times B)^2 = A^2 B^2 - (A \cdot B)^2$

Q.28 If $uF = \nabla v$, where u and v are scalar fields and F is a vector field, then $F \cdot \text{curl } F$ is equal to

- (a) zero
 (b) $\frac{\nabla^2 v}{u^2}$
 (c) $\frac{(\nabla v \cdot \nabla) v}{u^2}$
 (d) not defined

Q.29 Which of the following option is not correct?

- (a) A vector field \vec{A} is solenoid, if $\nabla \cdot \vec{A} = 0$
 (b) A vector field \vec{A} is irrotational, if $\nabla \times \vec{A} = 0$
 (c) A vector field V is harmonics, if $\nabla^2 V \neq 0$
 (d) All options are correct

Q.30 Which of the following statements is not true of a phasor?

- (a) It may be a scalar or a vector.
 (b) It is a time dependent quantity.
 (c) It is a complex quantity.
 (d) All are true

Q.31 A scalar function, V is given by $V = xyz^2$. The gradient of V is given by

- (a) $xz^2 \hat{a}_x + 2xyz \hat{a}_y + xz^2 \hat{a}_z$
 (b) $yz^2 \hat{a}_x + xz^2 \hat{a}_y + xyz \hat{a}_z$
 (c) $2xyz \hat{a}_z + yz^2 \hat{a}_y + xz^2 \hat{a}_z$
 (d) $yz^2 \hat{a}_x + xz^2 \hat{a}_y + 2xyz \hat{a}_z$

Q.32 The scalar potential is given by
 $V = (x^2 - y^2 - z^2)$ Volts

The laplacian of V is

- (a) 0
 (b) -2
 (c) 1
 (d) -1



1

Answers & Explanations | Vector Analysis

• AnswerKey •

- | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (c) | 2. (b) | 3. (a) | 4. (d) | 5. (a) | 6. (a) | 7. (c) | 8. (b) | 9. (d) |
| 10. (c) | 11. (a) | 12. (b) | 13. (d) | 14. (c) | 15. (d) | 16. (b) | 17. (c) | 18. (a) |
| 19. (c) | 20. (d) | 21. (c) | 22. (b) | 23. (a) | 24. (d) | 25. (d) | 26. (a) | 27. (d) |
| 28. (a) | 29. (c) | 30. (a) | 31. (d) | 32. (b) | | | | |

• Explanations •

1. (c)

Here,

$$\begin{aligned}\text{Div. } \vec{Q} &= \nabla \cdot \vec{Q} = \left(\vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \cdot (\vec{i}zx) \\ &= (\vec{i} \cdot \vec{i}) \left(\frac{\partial}{\partial x} \cdot zx \right) = 1 \cdot z = z\end{aligned}$$

Also,

$$\text{Grad } \phi = \nabla \phi = \left(\frac{\partial}{\partial x} \vec{i} + \frac{\partial}{\partial y} \vec{j} + \frac{\partial}{\partial z} \vec{k} \right) (xy^2z^3) = (y^2z^3 \vec{i} + 2yxz^3 \vec{j} + 3z^2y^2x \vec{k})$$

And,

$$\begin{aligned}\text{Curl } \vec{P} = \nabla \times \vec{P} &= \begin{vmatrix} \vec{i} & 0 & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & 0 & (x-y) \end{vmatrix} \\ &= \vec{i} \left[\frac{\partial}{\partial y} (x-y) - 0 \right] + \vec{k} \left[0 - \frac{\partial}{\partial y} (x^2y) \right] \\ &= \vec{i} [-1] + \vec{k} [x^2] = -\vec{i} + x^2 \vec{k}\end{aligned}$$

2. (b)

Let the unit vector be given by \vec{a}_R .

Now,

$$\begin{aligned}\vec{R} &= \text{Difference of two vectors} \\ &= r\vec{a}_r - h\vec{a}_z\end{aligned}$$

\therefore Unit vector,

$$\vec{a}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{r\vec{a}_r - h\vec{a}_z}{\sqrt{r^2 + h^2}}$$

3. (a)

Since the given vector V is irrotational, therefore $\text{curl } V = 0$ or, $\nabla \times V = 0$.

Now,

$$\nabla \times V = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (x+2y+az) & (bx-3y-z) & (4x+cy+2z) \end{vmatrix}$$

$$\begin{aligned}
&= \left\{ \frac{\partial}{\partial y} (4x + cy + 2z) - \frac{\partial}{\partial z} (bx - 3y - z) \right\} \vec{i} + \left\{ \frac{\partial}{\partial z} (x + 2y + az) - \frac{\partial}{\partial x} (4x + cy + 2z) \right\} \vec{j} \\
&\quad + \left\{ \frac{\partial}{\partial x} (bx - 3y - z) - \frac{\partial}{\partial y} (x + 2y + az) \right\} \vec{k} \\
&= (c+1)\vec{i} + (a-4)\vec{j} + (b-2)\vec{k}
\end{aligned}$$

Since, $\nabla \times V = 0$, therefore, $a = 4$, $b = 2$, and $c = -1$

4. (d)

- $(\vec{A} \times \vec{B}) \cdot (\vec{C} \times \vec{D})$ is called "product of four vectors".
- $\vec{A} \times (\vec{B} \times \vec{C})$ is called "vector triple product".
- $(\vec{A} \times \vec{B}) \cdot (\vec{C} \times \vec{D})$ is called "vector product of four vectors".

6. (a)

Both assertion and reason are true and reason is the correct explanation of assertion. Reason is the physical interpretation of divergence.

7. (c)

- $\vec{A} \times (\vec{B} \times \vec{C})$ is called "vector triple product" which is a correct expression.

$$\nabla = \left(\vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right)$$

and let $\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}$

Then, $\nabla \times \vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$

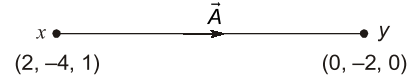
$$\begin{aligned}
\therefore \nabla(\nabla \times \vec{A}) &= \left(\vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \times \left[\vec{i} \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \vec{j} \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \vec{k} \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \right] \\
&= \frac{\partial}{\partial x} \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \frac{\partial}{\partial y} \left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z} \right) + \frac{\partial}{\partial z} \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \\
&= \frac{\partial^2 A_z}{\partial x \partial y} - \frac{\partial^2 A_y}{\partial x \partial z} + \frac{\partial^2 A_z}{\partial y \partial x} - \frac{\partial^2 A_x}{\partial y \partial z} + \frac{\partial^2 A_y}{\partial z \partial x} - \frac{\partial^2 A_x}{\partial z \partial y} = 0
\end{aligned}$$

$$\begin{aligned}
\bullet \quad \nabla \times \nabla \phi &= \left(\frac{\partial}{\partial x} \vec{i} + \frac{\partial}{\partial y} \vec{j} + \frac{\partial}{\partial z} \vec{k} \right) \times \left(\frac{\partial \phi}{\partial x} \vec{i} + \frac{\partial \phi}{\partial y} \vec{j} + \frac{\partial \phi}{\partial z} \vec{k} \right) \\
&= \left\{ (\vec{i} \times \vec{j}) \frac{\partial^2 \phi}{\partial x \partial y} + (\vec{i} \times \vec{k}) \frac{\partial^2 \phi}{\partial x \partial z} \right\} + \left\{ (\vec{j} \times \vec{i}) \frac{\partial^2 \phi}{\partial x \partial y} + (\vec{j} \times \vec{k}) \frac{\partial^2 \phi}{\partial y \partial z} \right\} + \left\{ (\vec{k} \times \vec{i}) \frac{\partial^2 \phi}{\partial z \partial x} + (\vec{k} \times \vec{j}) \frac{\partial^2 \phi}{\partial y \partial z} \right\} \\
&= \vec{k} \frac{\partial^2 \phi}{\partial x \partial y} - \vec{j} \frac{\partial^2 \phi}{\partial x \partial z} - \vec{k} \frac{\partial^2 \phi}{\partial y \partial x} + \vec{i} \frac{\partial^2 \phi}{\partial y \partial z} + \vec{j} \frac{\partial^2 \phi}{\partial z \partial x} - \vec{i} \frac{\partial^2 \phi}{\partial z \partial y} = 0
\end{aligned}$$

8. (b)

The vector \vec{A} is given as

$$\begin{aligned}\vec{A} &= (0-2)\vec{a}_x + [-2-(-4)]\vec{a}_y + (0-1)\vec{a}_z \\ &= -2\vec{a}_x + 2\vec{a}_y - \vec{a}_z\end{aligned}$$



9. (d)

By divergence theorem,

$$\begin{aligned}\iint_s \vec{F} \cdot d\vec{s} &= \iiint_v \nabla \cdot \vec{F} dv \\ &= \iiint_v \left[\vec{i}_1 \frac{\partial}{\partial x} + \vec{i}_2 \frac{\partial}{\partial y} + \vec{i}_3 \frac{\partial}{\partial z} \right] \times (4xz\vec{i}_1 - y^2\vec{i}_2 + yz\vec{i}_3) dv \\ &= \iiint_v \left[\frac{\partial}{\partial x}(4xz) - \frac{\partial}{\partial y}(y^2) + \frac{\partial}{\partial z}(yz) \right] dv \\ &= \iiint_v [4z - 2y + y] dv = \iiint_v [4z - y] dv\end{aligned}$$

Since, the surface s is bounded by $x = 0, 1$; $y = 0, 1$ and $z = 0, 1$ so, putting the limits, we have:

$$\begin{aligned}\iint_s \vec{F} \cdot d\vec{s} &= \int_0^1 \int_0^1 \int_0^1 (4z - y) dx dy dz = \int_0^1 \int_0^1 \left(\frac{4z^2}{2} - yz \right)_0^1 dx dy \\ &= \int_0^1 \int_0^1 (2 - y) dx dy = \int_0^1 \left(2y - \frac{y^2}{2} \right)_0^1 dy = \int_0^1 \frac{3}{2} dy = \frac{3}{2}\end{aligned}$$

10. (c)

The vector field \vec{A} will be irrotational, if $\nabla \times \vec{A} = 0$.

Now,

$$\begin{aligned}\nabla \times \vec{A} &= \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz & xy \end{vmatrix} \\ &= \left[\frac{\partial}{\partial y}(xy) - \frac{\partial}{\partial z}(xz) \right] \vec{a}_x + \left[\frac{\partial}{\partial x}(xy) - \frac{\partial}{\partial z}(yz) \right] \vec{a}_y + \left[\frac{\partial}{\partial x}(xz) - \frac{\partial}{\partial y}(yz) \right] \vec{a}_z \\ &= [x - x] \vec{a}_x + [y - y] \vec{a}_y + [z - z] \vec{a}_z = 0\end{aligned}$$

Hence, \vec{A} is irrotational.The vector field \vec{A} will be solenoidal, if $\nabla \cdot \vec{A} = 0$

Here,

$$\begin{aligned}\nabla \cdot \vec{A} &= \left(\vec{a}_x \frac{\partial}{\partial x} + \vec{a}_y \frac{\partial}{\partial y} + \vec{a}_z \frac{\partial}{\partial z} \right) \cdot (yz\vec{a}_x + xz\vec{a}_y + xy\vec{a}_z) \\ &= \vec{a}_x \cdot \vec{a}_x \frac{\partial}{\partial x}(yz) + \vec{a}_y \cdot \vec{a}_y \frac{\partial}{\partial y}(xz) + \vec{a}_z \cdot \vec{a}_z \frac{\partial}{\partial z}(xy) \\ &= 0 + 0 + 0 = 0\end{aligned}$$

Hence, \vec{A} is solenoidal.

11. (a)

Given,
$$\vec{A} = \frac{1}{\sqrt{x^2 + y^2}} \vec{a}_x$$

$$\begin{aligned} \therefore \nabla \cdot \vec{A} &= \frac{\partial}{\partial x}(A_x) + \frac{\partial}{\partial y}(A_y) + \frac{\partial}{\partial z}(A_z) \\ &= \frac{\partial}{\partial x} \left(\frac{1}{\sqrt{x^2 + y^2}} \right) + 0 + 0 = \frac{\partial}{\partial x} (x^2 + y^2)^{-1/2} = -\frac{1}{2} (x^2 + y^2)^{-3/2} \cdot 2x \\ &= \nabla \cdot \vec{A} = -\frac{x}{\sqrt{(x^2 + y^2)(x^2 + y^2)}} \end{aligned}$$

Now,
$$(\nabla \cdot \vec{A})_{2,2,0} = -\frac{2}{\sqrt{(2^2 + 2^2) \cdot (2^2 + 2^2)}} = -\frac{2}{\sqrt{8 \cdot 8}} = -0.0884$$

12. (b)

Given,
$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\therefore \vec{r} \times \vec{i} = (x\vec{i} + y\vec{j} + z\vec{k}) \times \vec{i} = -y\vec{k} + z\vec{j}$$

Also,
$$\vec{i} \times (\vec{r} \times \vec{i}) = \vec{i} \times (-y\vec{k} + z\vec{j}) = \vec{j}y + z\vec{k}$$

Similarly,
$$\vec{j} \times (\vec{r} \times \vec{j}) = \vec{i}x + \vec{k}z$$

and
$$\vec{k} \times (\vec{r} \times \vec{k}) = \vec{i}x + \vec{j}y$$

Thus,
$$\vec{i} \times (\vec{r} \times \vec{i}) + \vec{j} \times (\vec{r} \times \vec{j}) + \vec{k} \times (\vec{r} \times \vec{k}) = 2(x\vec{i} + y\vec{j} + z\vec{k}) = 2\vec{r}$$

13. (d)

Since vector \vec{V} is solenoidal, therefore

$$\nabla \cdot \vec{V} = 0$$

$$\therefore \left[\vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right] \cdot \left[\vec{i}(x+3y) + \vec{j}(y-2x) + \vec{k}(x+bz) \right] = 0$$

or,
$$[1 + 1 + b] = 0 \text{ or } b = -2$$

15. (d)

Vector differential operator (' ∇ ') is not a vector quantity. Hence, assertion is a false statement.

16. (b)

All the given statements are correct.

17. (c)

Reason is a statement of stroke's theorem not that of Gauss's divergence theorem.

18. (a)

• Electric potential,
$$V = \frac{\text{Work done}}{\text{Test charge}} = \frac{\text{Force} \times \text{Displacement}}{\text{Current} \times \text{Time}}$$

$$\therefore [V] = \frac{[MLT^{-2}][L]}{[I][T]} = [ML^2T^{-3}I^{-1}]$$