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(2001-2023)

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Civil Services Main Examination Previous Solved Papers : Electrical Engineering Paper-I

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Preface

B. Singh (Ex. IES)

Civil Service is considered as the most prestigious job in India and it has become a preferred destination by all engineers. In order to reach this estimable position every aspirant has to take arduous journey of Civil Services Examination (CSE). Focused approach and strong determination are the pre-requisites for this journey. Besides this, a good book also comes in the list of essential commodity of this odyssey.

I feel extremely glad to launch the revised edition of such a book which will not only make CSE plain sailing, but also with 100% clarity in concepts.

MADE EASY team has prepared this book with utmost care and thorough study of all previous years papers of CSE. The book aims to provide complete solution to all previous years questions with accuracy.

On doing a detailed analysis of previous years CSE question papers, it came to light that a good percentage of questions have been asked in Engineering Services, Indian Forest Services and State Services exams. Hence, this book is a one stop shop for all CSE, ESE and other competitive exam aspirants.

I would like to acknowledge efforts of entire MADE EASY team who worked day and night to solve previous years papers in a limited time frame and I hope this book will prove to be an essential tool to succeed in competitive exams and my desire to serve student fraternity by providing best study material and quality guidance will get accomplished.

With Best Wishes

B. Singh (Ex. IES)

CMD, MADE EASY Group

Previous Years Solved Papers of

Civil Services Main Examination

Electrical Engineering : Paper-I

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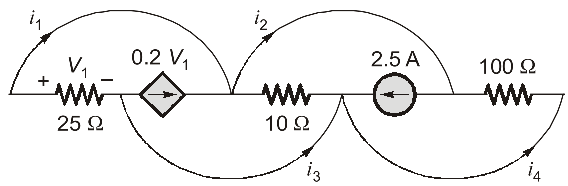
1 Circuit Theory

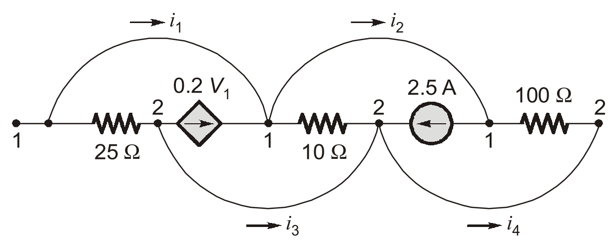
1. Circuit Element, Nodal and Mesh Analysis

1.1 For the circuit shown below, find i_1, i_2, i_3 and i_4.

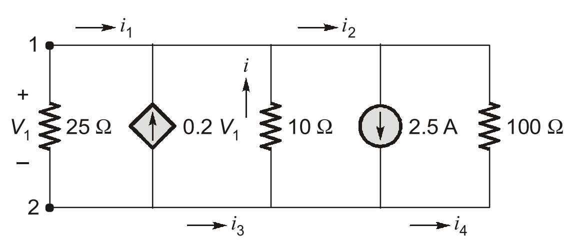


[CSE-2005 : 5 marks]

Solution:



Circuit is redrawn as:



By Nodal equation,

$$-\frac{V_1}{100} - 2.5 - \frac{V_1}{10} + 0.2V_1 - \frac{V_1}{25} = 0$$

$$\Rightarrow \qquad V_1 = \frac{250}{5} = 50 \text{ V}$$

$$i_4 = -\frac{V_1}{100} = -0.5 \text{ A}$$

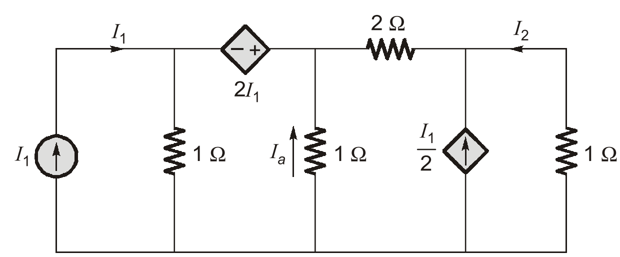
$$i_1 = -\frac{V_1}{25} = \frac{-50}{25} = -2 \text{ A}$$

$$i_2 = i_1 + 0.2\,V_1 + i = -2 + 10 - 5 = 3 \text{ A}$$

$$i_3 = i - 2.5 + i_4 = -5 - 2.5 - 0.5 = -8 \text{ A}$$

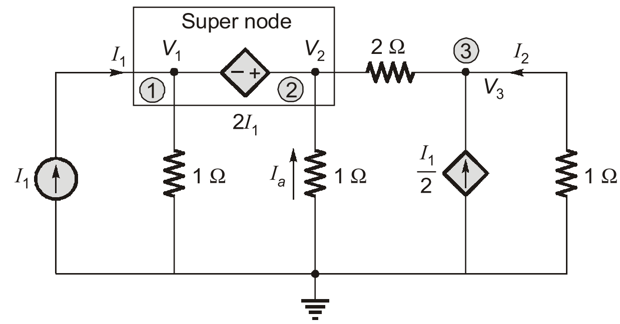
$$i_1 = -2 \text{ A} \; ; \; i_2 = 3 \text{ A} \; ; \; i_3 = -8 \text{ A} \; ; \; i_4 = -0.5 \text{ A}$$

1.2 For the network shown below, find the current ratio transfer function given by $\alpha = I_2/I_1$.



[CSE-2005 : 10 marks]

Solution:



Consider nodes (1) and (2). These two nodes constitutes a super node.

$$V_2 - V_1 = 2I_1 \quad ...(1)$$

Super node equation

$$\frac{V_1}{1} - I_1 + \frac{V_2}{1} + \frac{V_2 - V_3}{2} = 0 \quad ...(2)$$

$$V_1 + V_2 + 0.5V_2 - 0.5V_3 = I_1$$

$$V_1 + 1.5V_2 - 0.5V_3 = I_1$$

Node (3):

$$\frac{V_3 - V_2}{2} - I_2 - \frac{I_1}{2} = 0 \quad ...(3)$$

Put

$$\frac{V_3 - V_2}{2} = 0.5I_1 + I_2$$

$$V_1 - I_1 + V_2 - 0.5I_1 - I_2 = 0$$

$$V_1 + V_2 = 1.5I_1 + I_2 \quad ...(4)$$

From (1) and (4)

$$V_2 = 1.75I_1 + 0.5I_2$$

Also

$$V_3 - V_2 = I_1 + 2I_2$$

$$V_3 = V_2 + I_1 + 2I_2$$

$$V_3 = 2.75I_1 + 2.5I_2 \quad ...(5)$$

Also

$$I_2 = -V_3$$

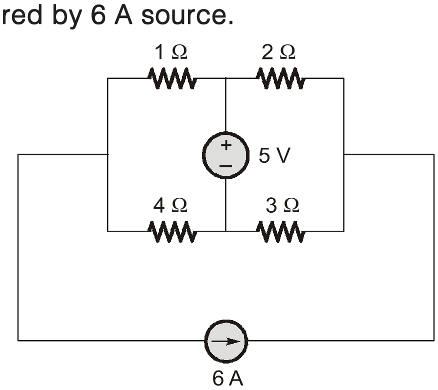
From (5),

$$-I_2 = 2.75I_1 + 2.5I_2$$

$$-3.5I_2 = 2.75I_1$$

$$\alpha = \frac{I_2}{I_1} = \frac{-2.75}{3.5} = -0.786$$

1.3 Determine the power delivered by 6 A source.



[CSE-2007 : 10 marks]

2 Signals and Systems

1. Basics and LTI Systems

1.1 Use the properties of the unit impulse function $\delta(t)$ to evaluate the following integrals:

(i) $\int_{-\infty}^{\infty} [t^2 + \cos \pi t]\, \delta[(t-5)]\, dt$ (ii) $\int_{-\infty}^{\infty} [e^{-\pi t} + \sin(10\pi t)]\, \delta(2t+1)\, dt$

[CSE-2002 : 10 + 10 marks]

Solution:

For a unit impulse function,

$$\delta(t - t_0) = 1 \text{ for } t = t_0$$
$$= 0 \text{ for } t \neq t_0$$

(i)
$$y(t) = \int_{-\infty}^{\infty} \left(t^2 + \cos \pi t\right) \delta(t-5) dt$$

This integral contains $\delta(t-5)$ which is zero for all values of 't' except for $t = 5$.

$\Rightarrow$
$$y(5) = (5^2 + \cos 5\pi) = 25 - 1 = 24$$

$\therefore$
$$y(t) = \begin{cases} 24; & \text{for } t = 5 \\ 0; & \text{otherwise} \end{cases}$$

(ii)
$$y(t) = \int_{-\infty}^{\infty} \left[e^{-\pi t} + \sin(10\pi t)\right] \delta(2t+1)\, dt$$

As we know, for unit impulse function

$$\delta(bt) = \frac{1}{|b|} \delta(t)$$

Here,
$$\delta(2t+1) = \delta\left[2\left(t + \frac{1}{2}\right)\right] = \frac{1}{2} \delta\left(t + \frac{1}{2}\right)$$

Also,
$$\delta\left(t + \frac{1}{2}\right) = \begin{cases} \infty & t = -\frac{1}{2} \\ 0 & t \neq -\frac{1}{2} \end{cases}$$

$\therefore$
$$y(t) = \frac{1}{2} \int_{-\infty}^{\infty} \left[e^{-\pi t} + \sin(10\pi t)\right] \delta\left(t + \frac{1}{2}\right) dt$$

$\Rightarrow$
$$y\left(-\frac{1}{2}\right) = \frac{1}{2}\left[e^{\pi/2} + \sin(-5\pi)\right] = \frac{1}{2} e^{\pi/2}$$

$\therefore$
$$y(t) = \begin{cases} \frac{1}{2} e^{\pi/2}, & t = -\frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

4 Analog Electronics

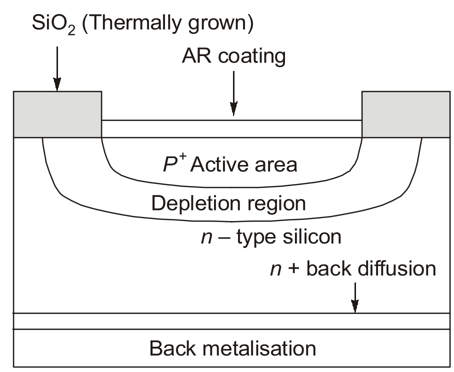
1. Diode Circuits

1.1 Explain clearly the construction of a p-n junction and its use to convert sun-light directly into electricity. What distinguishes a solar cell from a conventional *p-n* junction diode?

[CSE-2002 : 20 marks]

Solution:

Theory of operation: When photons having sufficient energy strike the device, they are absorbed and electron hole pairs are created. The depths at which photons are absorbed depends on their energy; the lower the energy of photons, the deeper they are absorbed. The electron hole pairs drift apart and when the minority carries reach the junction, they are swept across by the electric field. If the two sides are electrically connected, an electric current flows through the region.



Differences between a solar cell and a normal *p-n* junction diode:

1. The back side of the solar cell is completely metallized while the front side electrode is partially metallized in the form of metal fingers and bus bars to allow light to pass to the active solar material and get absorbed in the material to generate the photo current. The normal diode is fully metallized from both sides and so is opaque to light.
2. The doping of the substrate material in a solar cell is selected to optimize the open circuit voltage while collecting an appreciable part of the incident radiation by adjusting its thickness.

1.2 Describe the factors on which the conductivity of an intrinsic semiconductor depends. A semiconductor is made of *n*-type germanium. From the following data, calculate the conductivity of *n*-type germanium:

(i) Density of Ge = 5.32×10^3 kg/m^3

(ii) At weight of Ge = 72.6 kg/K-mol.

(iii) Charge of electron = 1.6×10^{-9} C

(iv) Mobility of holes = 0.18 m^2/V-s

(v) Mobility of electrons = 3.8 m^2/V-s

(Assume one donor atom in each 10^8 atoms)

[CSE-2003 : 20 marks]

Solution:

Factors on which the conductivity of intrinsic semiconductor depends:

1. Mobility of electrons and holes. Higher the mobility, higher the conductivity.
2. Concentration of electrons and holes. Higher the concentration, higher the conductivity.
3. Charge on electrons and holes.

The mobility and concentration are highly dependent on temperature. So conductivity also depends on temperature.

Now, $\sigma = n e \mu_n$ [For n-type semiconductor]

n = Density of electrons

$e = 1.6 \times 10^{-19}$ C

$\mu_n = 3.8$ m²/V-s [Given]

Now, given 1 donor atom for each 10^8 atoms

$\Rightarrow$ 10^{15} donor atoms for each 10^{23} atoms

$\Rightarrow$ 10^{15} donor atoms for each 72.6 gm Ge

$\Rightarrow$ 10^{15} donor atoms for $\dfrac{72.6 \times 10^{-3}}{5.32 \times 10^{3}}$ m³ Ge

$\Rightarrow$ 10^{15} donor atoms for 13.65×10^{-6} m³ Ge

$\Rightarrow$ 7.32×10^{19} donor atoms for 1 m³ of Ge

$\therefore$ $n = 7.32 \times 10^{19}$ / m³

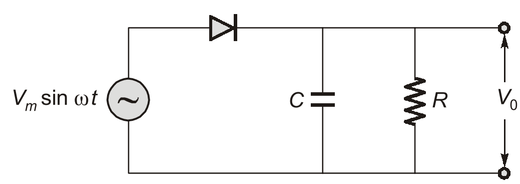
$$\sigma = n e \mu_n$$

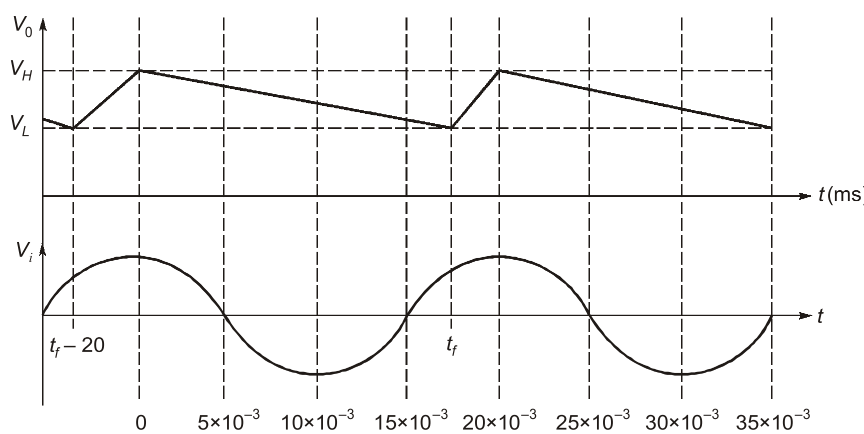
$$= 7.32 \times 10^{19} \times 1.6 \times 10^{-19} \times 3.8 = 44.50\ (\Omega\text{-m})^{-1}$$

1.3 **Draw the circuit diagram of a half wave rectifier with a reservoir capacitor that smooths the output. Draw its output waveform. Determine the reservoir capacitor value for a HWR and smoothing circuit to supply 20 V to a load of 500 Ω. Maximum ripple amplitude is to be 10% of the average output voltage, and the input signal frequency is 50 Hz.**

[CSE-2004 : 20 marks]

Solution:





Given: $R_L = 500\ \Omega$

$V_{dc} = 20$ V

Maximum ripple amplitude = 10% of V_{dc}

$\therefore$ $\dfrac{V_r}{2} = 10\%\ V_{dc}$

$\therefore$ $V_r = \dfrac{2}{10} \times 20 = 4$ V

5 Digital Electronics

1. Number System and Logic Gates

1.1 A Farmer (F) has two sheds (N_1 and S_1) in his farm to keep his farm produce and belongings. He keeps Vegetables (V) as farm produce, a Goat (G) which can eat the vegetables and a Dog (D) that can bite the goat in the absence of the farmer. The farmer alone has to manage both sheds without loosing either vegetables or the goat. Suggest a logic circuit that can alarm him under any catastrophic condition by means of a buzzer.

[CSE-2001 : 20 marks]

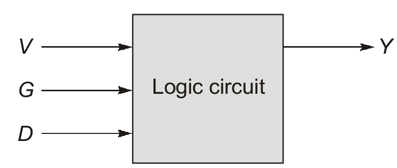
Solution:

We have three inputs : V, G, D

Each of these inputs can be either 0 or 1. An input is zero when the corresponding variable is in N_1 and it is 1 when the corresponding variable is in shed S_1.

$Y = 0 \Rightarrow$ Buzzer is off

$Y = 1 \Rightarrow$ Buzzer is on



Now,

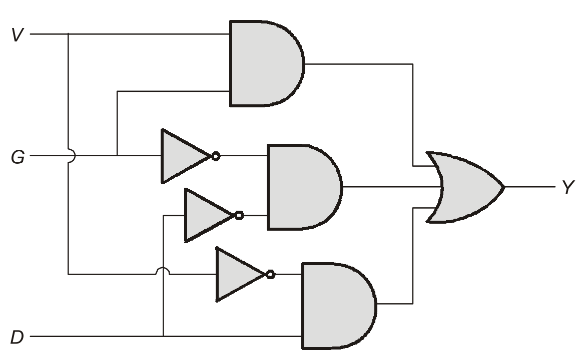
V	G	D	Y
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

The Karnaugh map for above truth table is as follows:

$Y =$

V \ GD	00	01	11	10
0	1	1	1	0
1	1	0	1	1

$$Y = \bar{G}\bar{D} + \bar{V}D + VG$$



1.2 Minimize and implement the following multiple-output functions:

$$f_1(A, B, C, D) = \Sigma m(1, 2, 3, 6, 8, 12, 14, 15)$$
$$f_2(A, B, C, D) = \Sigma m(1, 2, 3, 5, 6, 7, 8, 12, 13)$$

[CSE-2006 : 10 marks]

Solution:

$$f_1(A, B, C, D) = \Sigma m(1, 2, 3, 6, 8, 12, 14, 15)$$
$$f_2(A, B, C, D) = \Sigma m(1, 2, 3, 5, 6, 7, 8, 12, 13)$$

Karnaugh maps:

$f_1 =$

AB \ CD	00	01	11	10
00	0	1	1	1
01	0	0	0	1
11	1	0	1	1
10	1	0	0	0

$f_2 =$

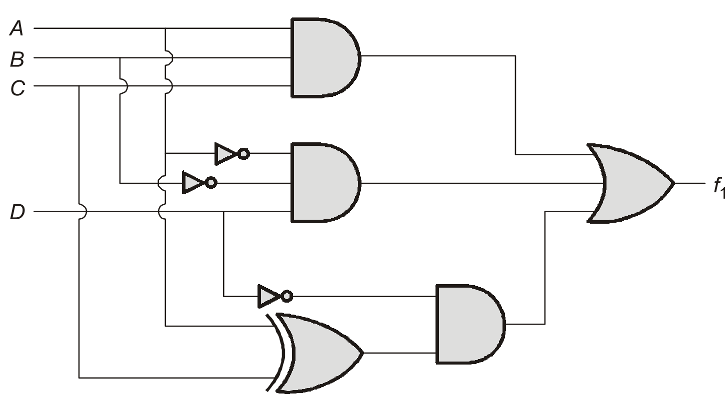
AB \ CD	00	01	11	10
00	0	1	1	1
01	0	1	1	1
11	1	1	0	0
10	1	0	0	0

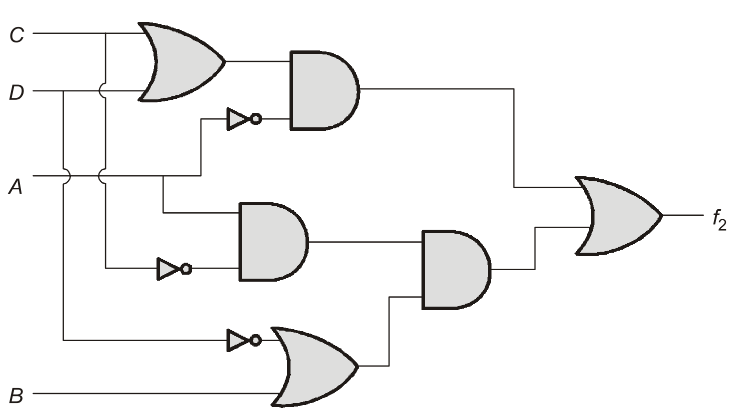
$$f_1 = \bar{A}\bar{B}D + \bar{A}C\bar{D} + ABC + A\bar{C}\bar{D}$$

$$f_2 = \bar{A}C + \bar{A}D + AB\bar{C} + A\bar{C}\bar{D}$$

$$f_1 = \bar{A}(C + D) + A\bar{C}(B + \bar{D})$$

$$f_2 = \bar{A}\bar{B}D + ABC + (A \oplus C)\bar{D}$$

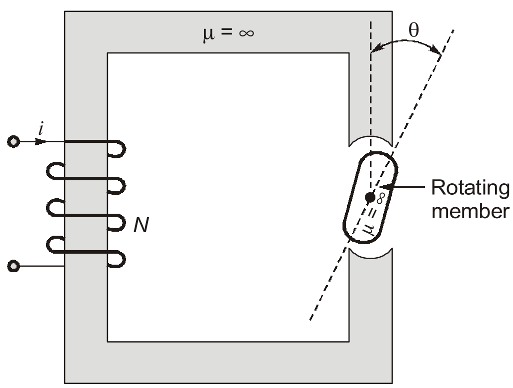




6 Energy Conversion

1. Electro Mechanical Energy Conversion

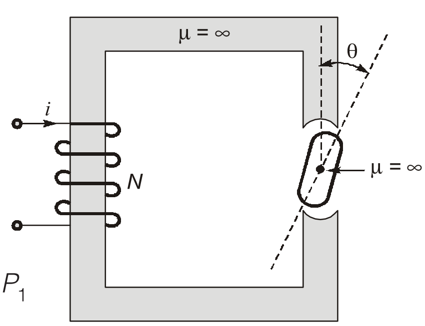
1.1 Figure below shows a singly excited electromechanical system with a stationary member and a rotating member. The stationary member is an electromagnet excited by a coil of N turns carrying a current i wound on a ferromagnetic core of infinite permeability. The rotating member is an unwound ferromagnetic core of infinite permeability causing variable permeance at changing rotor position θ from a maximum permeance of P_1 at $\theta = 0$ to a minimum permeance of P_2 at $\theta = 90°$. Assuming this variation of permeance to be sinusoidal develop an expression for the developed torque on the rotating member as a function of θ.



[CSE-2001 : 20 marks]

Solution:

Symbols used,



Ψ = Magnetic flux linkage

ϕ = Magnetic flux

τ = Torque induced

L = Inductance = N^2P

R = Reluctance

$$P = \frac{1}{R} = \text{Permeance} = P_0 \cos\theta + P_1$$

[This is because the permeance is provided as sinusoidal]

Case-1: When the rotor is moved slowly. In this case, current i will not change much. So, by energy conservation,

$$\tau(\Delta\theta) = \text{change in energy stored}$$

$$= \frac{1}{2} i^2 [L_{\theta_1} - L_{\theta_2}]$$

where, L_θ = inductance at θ angle

$$\Rightarrow \quad \tau = \frac{1}{2} i^2 \left(\frac{\Delta L}{\Delta\theta}\right) = \frac{1}{2} i^2 \frac{dL}{d\theta}$$

$$\Rightarrow \qquad \tau = \frac{1}{2} i^2 \frac{d}{d\theta} [N^2 (P_0 \cos\theta + P_1)]$$

$$\tau = -\frac{1}{2} i^2 N^2 P_0 \sin\theta$$

Case-2 : When the rotor is moved quickly. In this case, the flux linkage Ψ will not change instantaneously. Also, energy stored is given by the expression

$$= \frac{\Psi^2}{2L}$$

So, by applying energy conservation,

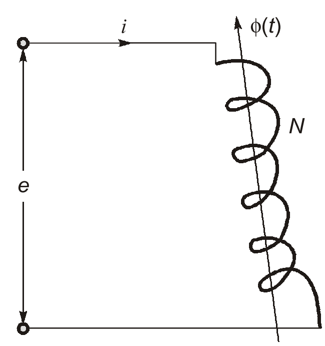
$$\tau(\Delta\theta) = -\Delta\left(\frac{\Psi^2}{2L}\right) = -\frac{\Psi^2}{2}\Delta\left(\frac{1}{L}\right)$$

$$\Rightarrow \qquad \tau = -\frac{\Psi^2}{2}\frac{\Delta\left(\frac{1}{L}\right)}{\Delta(\theta)} = -\frac{\Psi^2}{2}\frac{d}{d\theta}\left(\frac{1}{L}\right) = \frac{\Psi^2}{2L^2}\frac{dL}{d\theta}$$

$$= \frac{i^2}{2}\frac{dL}{d\theta} \qquad [\text{as } \Psi = Li]$$

$$\Rightarrow \qquad \tau = -\frac{1}{2} i^2 N^2 P_0 \sin\theta$$

1.2 Figure below shows a coil of N turns placed in a time varying flux $\phi(t)$. The coil orientation is such that for the direction of current i (as shown) in the flux direction is as indicated in the figure. Assume infinite conductivity of the conductor of the coil. Using relevant Maxwell's equation obtain an expression for e. Indicate how this expression will be in conformity with Lenz's law.

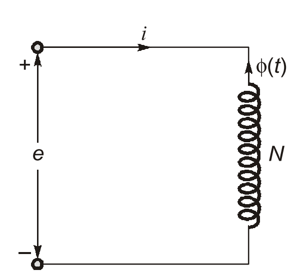


[CSE-2001 : 20 marks]

Solution:

We will use Maxwell's third equation,

$$\nabla \times \bar{E} = -\frac{\partial B}{\partial t}$$



Integrate this equation over a surface area $d\bar{S}$

$$\Rightarrow \qquad \iint (\nabla \times \bar{E}) \cdot d\bar{S} = -\iint \frac{\partial \bar{B}}{\partial t} \cdot d\bar{S}$$

Now, using Stoke's theorem,

$$\Rightarrow \qquad \oint E \cdot d\bar{l} = -\frac{\partial}{\partial t}\iint \bar{B} \cdot d\bar{S}$$

where $d\bar{l}$ is an element of the perimeter of area $d\bar{S}$.

$$E_f \cos(\theta_s - \delta) = 56.57 \quad \text{...(i)}$$

$$Q_{in} = -100 \times 10^3 = \frac{V^2}{Z_s}\sin\theta_s - \frac{VE_f}{Z_s}\sin(\theta_s - \delta)$$

$$\frac{100000 + \dfrac{V^2}{Z_s}\sin\theta_s}{\dfrac{V}{Z_s}} = E_f \sin(\theta_s - \delta)$$

$$E_f \sin(\theta_s - \delta) = 1280.98 \quad \text{...(ii)}$$

$$E_f = \sqrt{1280.98^2 + 56.57^2} = 1282.23$$

$$\sin(\theta_s - \delta) = \frac{1280.98}{1282.23} = 0.999$$

$$\delta = -5.567°$$

$$I_m = \frac{\dfrac{400}{\sqrt{3}}\angle 0° - \dfrac{1282.23}{\sqrt{3}}\angle -5.567°}{(0.5 + j3.5)}$$

$$I_m = 144.52\angle 90°\ \text{A}$$

■■■■

7 Power Electronics and Electric Drives

1. Power Semiconductor Devices

1.1 While the thyristors is used in power electronics converters it is being replaced by other devices recently. Mention any two of these devices and explain how they differ from the thyristor. Name typical convertors in which the thyristors are being replaced.

[CSE-2001 : 20 marks]

Solution:

Transistors used in power electronic converters are being replaced by more advanced power devices like power MOSFET, IGBT, GTO etc.

Comparison between thyristors and MOSFET/IGBT

- Higher input impedance than SCR.
- Low on state power losses than SCR.
- Faster switching characteristics than SCR.
- Higher power ratings than SCR.
- These devices are more popular where high switching speed and high power application is desirable for example: SMPS, choppers.

1.2 How are the power semiconductor devices specified? What do these specification signify?

[CSE-2001 : 20 marks]

Solution:

The following ratings are used to specify power semiconductor devices”

1. **Voltage rating:** It signifies the maximum allowable voltage that can be applied to the terminals of a device. Values can be different for reverse and forward biased connections. These ratings also include the maximum allowable rate of change of voltage of the terminals.
2. **Forward current rating:** It is usually specified after consideration of the following factors:
   (i) Current at which the junction temperature does not exceed a rated value.
   (ii) Current at which internal leads and contacts are not evaporated.
   (iii) External connector current handling capabilities.
3. **Temperature rating:** This specification signifies the maximum allowable temperature that the device can reach safely. It is determined by the quality of materials used. It also signifies the conditions under which the device can be reliably used.
4. **Power rating:** This signifies the maximum amount of power that can be safely dissipated by the device. It is directly related to the maximum allowable temperature of the device.

1.3 Germanium and Silicon are both semiconducting materials and we have both diodes and triodes with germanium as well as silicon. However, in respect of controlled rectifiers, we have only silicon controlled rectifiers but no germanium controlled rectifier. Explain why?

[CSE-2002 : 10 marks]

Solution:

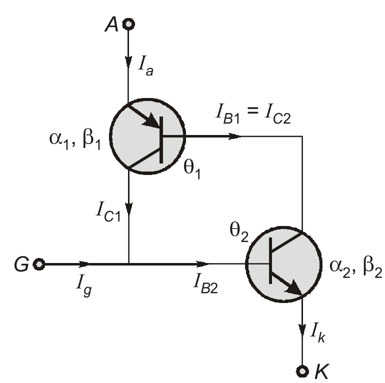
We do not have Germanium controlled rectifier because:

1. Germanium has a lower band gap than silicon. So, silicon controlled rectifiers can with stand higher temperatures than Germanium controlled rectifiers without risking thermal runaway.
2. The silicon controlled rectifiers can have higher peak inverse voltage than Germanium controlled rectifiers.
3. Leakage current of Germanium devices is high.
4. Silicon is cheaper and easily available.

1.4 Use the two transistor equivalent circuit of the SCR and show that when reverse biased, the SCR cannot conduct even if it is triggered.

[CSE-2002 : 10 marks]

Solution:



When the device is reverse biased, $V_A < V_K$

The device will not conduct when it is triggered because when it is triggered,

$$V_G > V_K$$

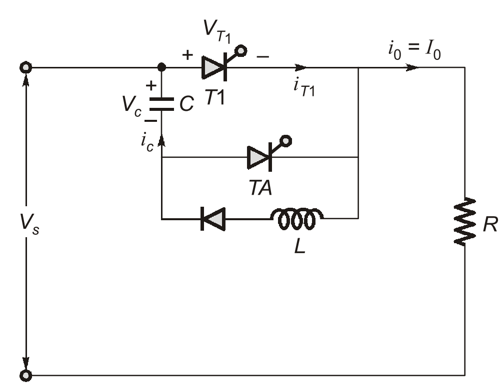
$\Rightarrow$ $V_G > V_A$ $[V_K > V_A]$

$\Rightarrow$ Q_1 will not conduct.

1.5 Consider the circuit shown below. Main thyristor $T1$ and auxiliary thyristor TA are off and capacitor is charged to voltage V_s with upper plate positive. Sketch the waveforms for i_o, i_c, i_{T1}, v_c and v_{T1} (assuming constant load current), after

(i) thyristor $T1$ is turned-on

(ii) steady-state is reached in part (a) and then thyristor TA is turned-on.

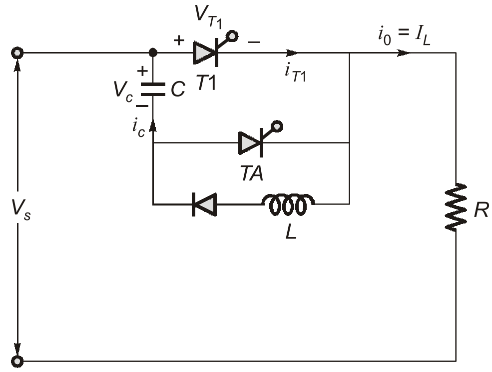


From these waveforms, obtain the expressions for circuit turn-off time for both SCRs.

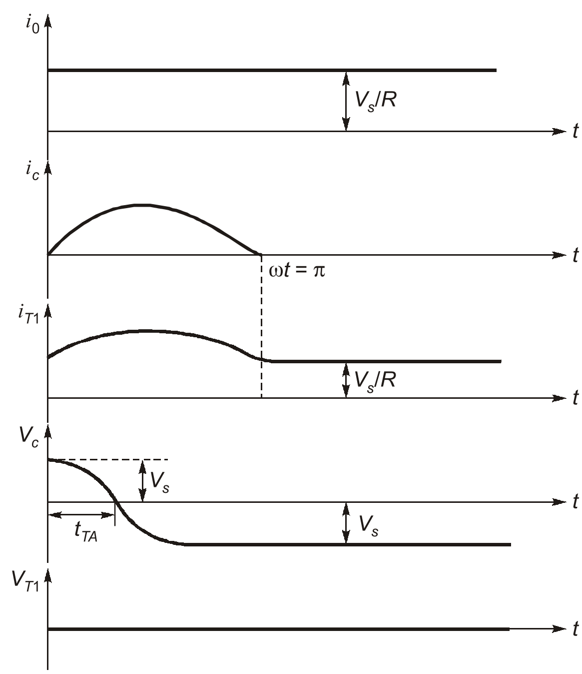
In case V_s = 220 V, R = 10 Ω and maximum value of current through main SCR is 3 times the load current, calculate the value of commutating inductance L for C = 20 μF and the circuit turn-off time for the auxiliary SCR.

[CSE-2003 : 20 marks]

Solution:



(i)



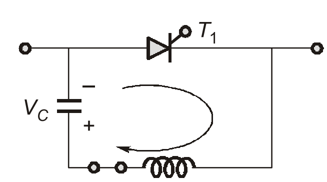
At, $t = 0$

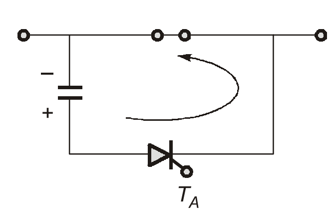
T_1 is ON

$$i_{T1(\max)} = i_0 + i_P$$

$\therefore$ $i_C = i_P \sin \omega t,\quad i_P = V_S\sqrt{\frac{C}{L}}$

$$V_C = +V_S$$





At, $t = T_1$

$$i_C = 0$$

$$\omega_0 t_1 = \pi$$

$$i_{T1} = i_0$$

$$V_C = -V_S$$

8 Analog Communication

1. Signals, Random Variable and Noise Basics

1.1 Show that the multiplication of a sinusoidal signal by a signal $f(t)$ translates the whole frequency spectrum. Can we retranslate the spectrum to its original position? If yes, how?

[CSE-2001 : 20 marks]

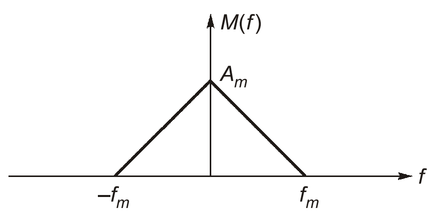
Solution:

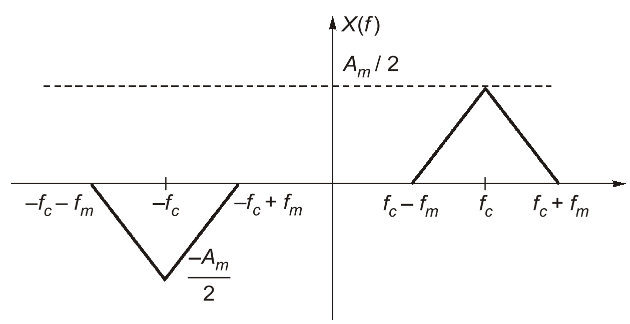
Let the signal being multiplied is $m(t)$

$$\Rightarrow \quad x(t) = m(t)\sin 2\pi f_c t$$

Taking Fourier transform on both sides

$$\Rightarrow \quad X(f) = M(f) * \frac{[\delta(f-f_c)-\delta(f+f_c)]}{2} = M(f) * \frac{\delta(f-f_c)}{2} - M(f) * \frac{\delta(f+f_c)}{2}$$





Yes, we can retranslate the spectrum to its original position by multiplying it with the same sinusoid and then passing it through a low pass filter of gain 2.

$$x(t)\sin 2\pi f_c t = m(t)\sin^2 2\pi f_c t$$

$$= \frac{m(t)}{2}[1-\cos 4\pi f_c t] = \frac{m(t)}{2} - \frac{m(t)}{2}\cos 4\pi f_c t$$

The low pass filter with gain = 2, will reject the 2 f_c component and we will get $m(t)$.

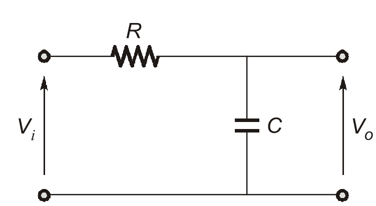
$$\therefore \quad y(t) = m(t)$$

1.2 White Gaussian noise of two sided spectral density 10^{-12} V^2/Hz is applied to a low pass (RC) filter having a 3 dB cut-off frequency of 1 kHz. Find the output noise power.

[CSE-2003 : 5 marks]

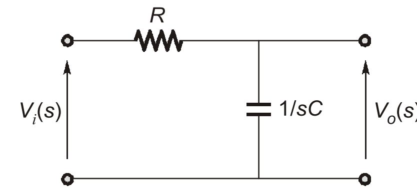
Solution:

Taking the Laplace transform we get,



$$\frac{V_o(s)}{V_i(s)} = H(s) = \frac{\frac{1}{SC}}{R+\frac{1}{SC}}$$

$$\Rightarrow \quad H(s) = \frac{1}{1+SCR}$$



$$\Rightarrow \quad H(f) = \frac{1}{1+j2\pi fCR}$$

$$\Rightarrow \quad |H(f)| = \frac{1}{\sqrt{1+4\pi^2 f^2 C^2 R^2}}$$

$$\text{3 dB frequency} = \frac{1}{2\pi RC}$$

$$\Rightarrow \quad 1000 = \frac{1}{2\pi RC}$$

$$\Rightarrow \quad 2\pi RC = \frac{1}{1000}$$

So,
$$H(f) = \frac{1}{1+j\dfrac{f}{1000}} = \frac{1000}{jf+1000}$$

$$\Rightarrow \quad |H(f)|^2 = \frac{10^6}{f^2+10^6}$$

Now, output noise power,
$$P_0 = \int_{-\infty}^{\infty} S_{ni}(f)|H(f)|^2\,df = \int_{-\infty}^{\infty} 10^{-12} \times \frac{10^6}{f^2+10^6}\,df$$

$$= \int_0^{\infty} \frac{2\times 10^{-6}}{f^2+10^6}\,df = \int_0^{\infty} \frac{2\times 10^{-12}}{1+10^{-6}f^2}\,df$$

Let $10^{-3} f = x$

$\Rightarrow \quad df = 1000\,dx$

So,
$$P_0 = \int_0^{\infty} \frac{2\times 10^{-12}\times 10^3\,dx}{1+x^2} = \frac{10^{-9}}{\pi} W$$

1.3 The received signal is $r(t) = s(t) + n(t)$ where $s(t)$ is the signal and $n(t)$ is the noise signal. The signal process is random with an auto-correlation $R_s(\tau) = 2\,e^{-|t|}$. The noise is a sample function of a random process with auto-correlation $R_n(\tau) = e^{-2|t|}$. If the two processes are having zero mean value and are independent of each other determine the auto-correlation and total power of $r(t)$.

[CSE-2003 : 5 marks]

Solution:

Using principle of super position,

$$S_R(\omega) = S_s(\omega) + S_n(\omega)$$

Now,
$$R_s(\tau) = 2\,e^{-|t|}$$

$$S_s(\omega) = \text{Fourier transform of } R_s(\tau)$$

$$= 2\int_{-\infty}^{\infty} e^{-|t|} e^{-j\omega t}\,dt = 2\left[\int_{-\infty}^{0} e^{t}\cdot e^{-j\omega t}\,dt + \int_0^{\infty} e^{-t}\cdot e^{-j\omega t}\,dt\right]$$

$$= 2\left[\int_{-\infty}^{0} e^{t|1-j\omega|}\,dt + \int_0^{\infty} e^{-(1+j\omega)t}\,dt\right] = 2\left[\frac{1}{1-j\omega} + \frac{1}{1+j\omega}\right] = \frac{4}{1+\omega^2}$$

$$R_n(\tau) = e^{-2|t|}$$

$$S_n(\omega) = \int_{-\infty}^{\infty} e^{-2|t|}\cdot e^{-j\omega t}\,dt$$

$$\Rightarrow \quad S_n(\omega) = \int_{-\infty}^{0} e^{(2-j\omega)t}\,dt + \int_0^{\infty} e^{-(2+j\omega)t}\,dt = \frac{1}{2-j\omega} + \frac{1}{2+j\omega} = \frac{4}{4+\omega^2}$$

1.5 How much noise voltage is generated in a 1 MΩ resistance at 27°C over a 10 kHz bandwidth? List the sources of noise in an amplifier.

[CSE-2005 : 2 + 3 marks]

Solution:

$$\text{Noise voltage generated} = \sqrt{4KTR(\Delta f)}$$

$$= \sqrt{4 \times 1.38 \times 10^{-23} \times 300 \times 10^{6} \times 10 \times 10^{3}}$$

$$= \sqrt{4 \times 1.38 \times 300 \times 10^{-13}} = \sqrt{4 \times 1.38 \times 3 \times 10^{-11}}$$

$$= 12.86\,\mu\text{V}$$

Source of noise in an amplifier:

1. Shot noise
2. Partition noise
3. Thermal noise

1.6 Three regulator dice are thrown. Assign probabilities to the following events: The sum of the points appearing on three dice is 4, 9 and 15.

[CSE-2005 : 10 marks]

Solution:

Let the three dice be D_1, D_2 and D_3. The matrices below indicate their sum.

$D_1 = 1$ (rows D_2, columns D_3)

D_2 \ D_3	1	2	3	4	5	6
1	3	4	5	6	7	8
2	4	5	6	7	8	9
3	5	6	7	8	9	10
4	6	7	8	9	10	11
5	7	8	9	10	11	12
6	8	9	10	11	12	13

$D_1 = 2$

D_2 \ D_3	1	2	3	4	5	6
1	4	5	6	7	8	9
2	5	6	7	8	9	10
3	6	7	8	9	10	11
4	7	8	9	10	11	12
5	8	9	10	11	12	13
6	9	10	11	12	13	14

$D_1 = 3$

D_2 \ D_3	1	2	3	4	5	6
1	5	6	7	8	9	10
2	6	7	8	9	10	11
3	7	8	9	10	11	12
4	8	9	10	11	12	13
5	9	10	11	12	13	14
6	10	11	12	13	14	15

$D_1 = 4$

D_2 \ D_3	1	2	3	4	5	6
1	6	7	8	9	10	11
2	7	8	9	10	11	12
3	8	9	10	11	12	13
4	9	10	11	12	13	14
5	10	11	12	13	14	15
6	11	12	13	14	15	16

$D_1 = 5$

D_2 \ D_3	1	2	3	4	5	6
1	7	8	9	10	11	12
2	8	9	10	11	12	13
3	9	10	11	12	13	14
4	10	11	12	13	14	15
5	11	12	13	14	15	16
6	12	13	14	15	16	17

$D_1 = 6$

D_2 \ D_3	1	2	3	4	5	6
1	8	9	10	11	12	13
2	9	10	11	12	13	14
3	10	11	12	13	14	15
4	11	12	13	14	15	16
5	12	13	14	15	16	17
6	13	14	15	16	17	18

Total possibilities = 216

Cases when sum is 4 = 3

Cases when sum is 9 = 25

Cases when sum is 15 = 10