ESE 2021

UPSC ENGINEERING SERVICES EXAMINATION

Main Examination

Mechanical Engineering

Topicwise *Conventional* Solved Papers

Paper-I

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Mechanical Engineering
Topicwise Conventional Solved Questions : Paper-I : (2000-2020)

1st Edition : 2011
2nd Edition : 2012
3rd Edition : 2013
4th Edition : 2014
5th Edition : 2015
6th Edition : 2016
7th Edition : 2017
8th Edition : 2018
9th Edition: 2019

10th Edition: 2020

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Director's Message

In past few years ESE Main exam has evolved as an examination designed to evaluate a candidate's subject knowledge. Studying engineering is one aspect but studying to crack prestigious ESE exam requires altogether different strategy, crystal clear concepts and rigorous practice of previous years' questions. ESE mains being conventional exam has subjective nature of questions, where an aspirant has to write elaborately - step by step with proper and well labeled diagrams and figures. This characteristic of the main exam gave me the aim and purpose to write this book. This book is an effort to cater all the difficulties being faced by students during their preparation right from conceptual clarity to answer writing approach.

MADE EASY Team has put sincere efforts in solving and preparing accurate and detailed explanation for all the previous years' questions in a coherent manner. Due emphasis is made to illustrate the ideal method and procedure of writing subjective answers. All the previous years' questions are segregated subject wise and further they have been categorised topic-wise for easy learning and helping aspirants to solve all previous years' questions of particular area at one place. This feature of the book will also help aspirants to develop understanding of important and frequently asked areas in the exam.

I would like to acknowledge the efforts of entire MADE EASY team who worked hard to solve previous years' questions with accuracy. I hope this book will stand upto the expectations of aspirants and my desire to serve the student community by providing best study material will get accomplished.

B. Singh (Ex. IES)
CMD, MADE EASY Group

ESE 2021 Main Examination

Mechanical Engineering

Conventional **Solved Questions**

Paper-I

Contents

■■■■

1 Thermodynamics

Revised Syllabus of ESE: *Thermodynamic systems and processes; properties of pure substance; Zeroth, First and Second Laws of Thermodynamics; Entropy, Irreversibility and availability; analysis of thermodynamic cycles related to energy conversion, ideal and real gases; compressibility factor; Gas mixtures. (Topic of Power Plant: Rankine) (Topics of IC Engine : Otto, Diesel and Dual Cycles).*

1. Basic Concepts, Work and Heat

1.1 An ideal gas is heated at constant volume until its temperature is 3 times the original temperature, then it is expanded isothermally till it reaches its original pressure. The gas is then cooled at constant pressure till it is restored to the original state. Determine the net work done per kg of gas if the initial temperature is 350 K.

[10 marks : 2003]

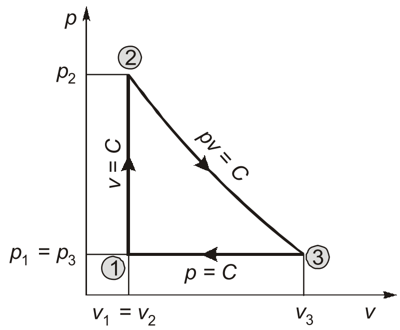
Solution:

The following three processes that form the cycle are shown in p-v diagram.

(i) Process 1-2: Heating at $v = C$

(ii) Process 2-3: Expansion at $T = C$

(iii) Process 3-1: Cooling at $p = C$



Given data:

$$T_2 = 3T_1$$

$$T_1 = 350 \text{ K}$$

$$\therefore \quad T_2 = 3 \times 350 = 1050 \text{ K}$$

For process 1-2,

$$\frac{p_2}{p_1} = \frac{T_2}{T_1} = 3$$

Work done per kg of gas: $w_{1\text{-}2} = 0$ $\quad [\because v_1 = v_2]$

For process 2-3,

$$p_2 v_2 = p_3 v_3$$

or

$$\frac{v_2}{v_3} = \frac{p_3}{p_2} = \frac{p_1}{p_2} = \frac{1}{3} \quad [\because p_3 = p_1]$$

Work done per kg of gas:

$$w_{2\text{-}3} = RT_2 \log_e \frac{v_3}{v_2} = 0.287 \times 1050 \log_e 3 = 331.06 \text{ kJ/kg}$$

For process 3-1,

Work done per kg of gas:

$$w_{3\text{-}1} = p_3(v_1 - v_3) = R(T_1 - T_3)$$

$$= 0.287\,(350 - 1050) = -200.9 \text{ kJ/kg}$$

Net work done per kg of gas: $w_{net} = w_{1\text{-}2} + w_{2\text{-}3} + w_{3\text{-}1} = 0 + 331.06 - 200.9 =$ **130.16 kJ/kg**

1.2 The heat capacity at constant pressure of a certain system is a function of temperature only and may be expressed as

$$C_p = 2.093 + \frac{41.87}{t+100} \text{ J/°C}$$

where t is the temperature in °C. The system is heated while it is maintained at a pressure of 1 atmosphere until its volume increases from 2000 cm^3 to 2400 cm^3 and its temperature increases from 0°C to 100°C.

(i) Find the magnitude of heat interaction.

(ii) How much does the internal energy of the system increase? [5 + 5 = 10 marks : 2009]

Solution:

Given data: V_1 = 2000 cm^3; V_2 = 2400 cm^3; T_1 = 0°C = 273 K; T_2 = 100°C = 373 K

(i) Given $C_p = 2.093 + \frac{41.87}{t+100}$ J/°C

For constant pressure process,

$$\int_1^2 \delta Q = \int_{t_1}^{t_2} C_p dt$$

$$Q_{1-2} = \int_{t_1}^{t_2}\left(2.093 + \frac{41.87}{t+100}\right)dt = 2.0936\,(t_2 - t_1) + 41.87 \ln\left(\frac{t_2+100}{t_1+100}\right)$$

$$= 209.3 + 29.02 = \mathbf{238.32\ J}$$

(ii) Work done in the process,

$$W_{1-2} = \int_1^2 pdV = p\int_1^2 dV = p(V_2 - V_1) = 1.01325 \times 10^5 \times (2400 - 2000) \times 10^{-6}$$

$$= 40.53 \text{ J}$$

Change in internal energy,

$$\Delta U = U_2 - U_1 = \Delta Q_{1-2} - \Delta W_{1-2} = 238.32 - 40.53 = \mathbf{197.79\ J}$$

1.3 Obtain an expression for the specific work output of a gas turbine unit in terms of pressure ratio, isentropic efficiencies of the compressor and turbine, and the maximum and minimum temperature, T_3 and T_1. Hence show that the pressure ratio r_p for maximum power is given by

$$r_p = \left[\eta_T \eta_C \frac{T_3}{T_1}\right]^{\gamma/2(\gamma-1)}$$

[10 marks : 2011]

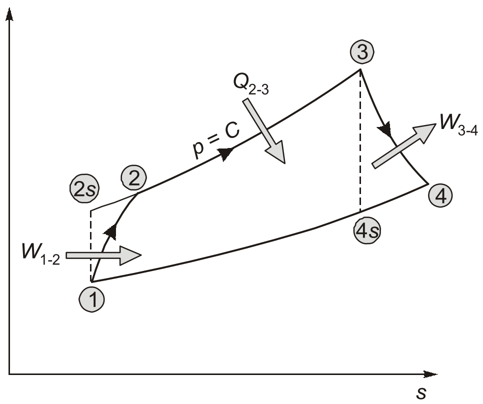
Solution:

In an actual gas turbine, the compressor and the turbine are not isentropic, some losses occur due to internal friction.

For actual compression process 1-2 in the compressor:

Compressor efficiency: η_c. It is defined as the ratio of isentropic increase in temperature to the actual increase in temperature in the compressor.

Mathematically,



Actual cycle without pressure losses

Compressor efficiency: $$\eta_C = \frac{(\Delta T)_{\text{isentropic}}}{(\Delta T)_{\text{actual}}}$$

$$\eta_C = \frac{T_{2s} - T_1}{T_2 - T_1} \quad \text{...(i)}$$

The compressor efficiency is also called the isentropic efficiency of the compressor.
For isentropic process 1-2s,

$$\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

or
$$T_{2s} = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{...(ii)}$$

For given values of T_1, p_1 and p_2, we can determine the value of T_{2s}. By substituting the value of T_{2s} in Eq. (i), we can find out the value of T_2 at given value of η_C.

For actual expansion process 3-4 in the turbine:

Turbine efficiency: η_T. It is defined as the ratio of the actual decrease in temperature to the isentropic decrease in temperature in the turbine.

Mathematically,

Turbine efficiency:
$$\eta_T = \frac{(\Delta T)_{\text{actual}}}{(\Delta T)_{\text{isentropic}}}$$

$$\eta_T = \frac{T_3 - T_4}{T_3 - T_{4s}} \quad \text{...(iii)}$$

The turbine efficiency is also called the isentropic efficiency of the turbine.
For isentropic process 3-4s,

$$\frac{T_3}{T_{4s}} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

or
$$\mathrm{T}_{4s} = \frac{T_3}{\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}} \quad \text{...(iv)}$$

For given values of T_3, p_1 and p_2, we can determine the value of T_{4s}. By substituting the value of T_{4s} in Eq. (iii), we can find out the value of T_4 at given value of η_T.

Process 2-3: Heat supplied at $p = C$

Heat supplied: $Q_{2\text{-}3} = mc_p(T_3 - T_2)$

For unit mass flow rate,

Heat supplied: $q_{2\text{-}3} = c_p(T_3 - T_2)$

Actual turbine work output: $W_{3\text{-}4} = W_T = mc_p(T_3 - T_4)$

For unit mass flow rate, $w_{3\text{-}4} = w_T = c_p(T_3 - T_4)$

Actual compression work required: $W_{1\text{-}2} = W_C = mc_p(T_2 - T_1)$

For unit mass flow rate, $w_{1\text{-}2} = w_C = c_p(T_2 - T_1)$

Net work output: w_{net} = Turbine work – Compressor work

$$w_{\text{net}} = w_T - w_C$$
$$= c_p(T_3 - T_4) - c_p(T_2 - T_1)$$

$$\eta_T = \frac{T_3 - T_4}{T_3 - T_{4s}} \qquad\qquad \eta_C = \frac{T_{2s} - T_1}{T_2 - T_1}$$

or
$$T_3 - T_4 = \eta_T(T_3 - T_{4s})$$

or
$$T_2 - T_1 = \frac{T_{2s} - T_1}{\eta_c}$$

$\therefore \quad w_{net} = c_p \eta_T (T_3 - T_{4s}) - \frac{c_p}{\eta_C}(T_{2s} - T_1)$

For ideal process 3-4s,

$$\frac{T_3}{T_{4s}} = r_p^{\frac{\gamma-1}{\gamma}} = r_p^z \qquad \left(\because \frac{\gamma-1}{\gamma} = z\right)$$

or $T_{4s} = \frac{T_3}{r_p^z}$

$$\mathbf{T_{4s} = T_3 \cdot r_p^{-z}}$$

For ideal process 1-2s,

$$\frac{T_{2s}}{T_1} = r_p^{\frac{\gamma-1}{\gamma}} = r_p^z$$

or $\mathbf{T_{2s} = T_1 r_p^z}$

$\therefore \quad w_{net} = c_p \eta_T \left(T_3 - T_3 r_p^{-z}\right) - \frac{c_p}{\eta_C}\left(T_1 r_p^z - T_1\right)$

For maximum work output condition for the actual cycle,

$$\frac{dw_{net}}{dr_p} = 0$$

$$c_p \eta_T \left[0 - T_3(-z) r_p^{-z-1}\right] - \frac{c_p}{\eta_C}\left(T_1 z r_p^{z-1} - 0\right) = 0$$

or $\eta_T T_3 z r_p^{-z-1} - \frac{T_1 z r_p^{z-1}}{\eta_C} = 0$

or $\eta_T T_3 r_p^{-z-1} = \frac{T_1 r_p^{z-1}}{\eta_C}$

$$\eta_T \eta_C \frac{T_3}{T_1} = \frac{r_p^{z-1}}{r_p^{-z-1}} = r_p^{z-1+z+1} = r_p^{2z}$$

or $\eta_T \eta_C \frac{T_3}{T_1} = r_p^{2z}$

or $\sqrt{\eta_T \eta_C \frac{T_3}{T_1}} = r_p^z$

or $\sqrt{\eta_T \eta_C \frac{T_3}{T_1}} = r_p^{\frac{\gamma-1}{\gamma}} \qquad \left(\because z = \frac{\gamma-1}{\gamma}\right)$

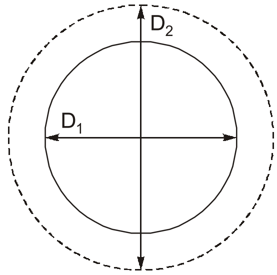
or $\mathbf{r_p} = \left[\eta_T \eta_C \frac{T_3}{T_1}\right]^{\frac{\gamma}{2(\gamma-1)}}$

1.4 A spherical balloon of 1 m diameter contains a gas at 200 kPa and 300 K. The gas inside the balloon is heated until the pressure reaches 500 kPa. During the process of heating, the pressure is proportional to the diameter of the balloon. Determine the work done by the gas inside the balloon.

[10 marks : 2013]

Solution:

Given data: $D_1 = 1$ m; $p_1 = 200$ kPa; $T_1 = 300$ K; $p_2 = 500$ kPa



As $p \propto D$

$$\therefore \quad \frac{p_1}{p_2} = \frac{D_1}{D_2}$$

$$\frac{200}{500} = \frac{D_1}{D_2}$$

or $D_2 = D_1 \times 2.5 = 1 \times 2.5 = 2.5$ m

$$\frac{p}{D} = \frac{p_1}{D_1} = \frac{p_2}{D_2} = \text{Constant(C)} \qquad \left[C = \frac{p_1}{D_1} = 200 \text{ kPa/m}\right]$$

$$W = \int_1^2 pdV$$

$$V = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 = \frac{4}{8\times 3}\times\pi\left(D^3\right) = \frac{\pi}{6}D^3$$

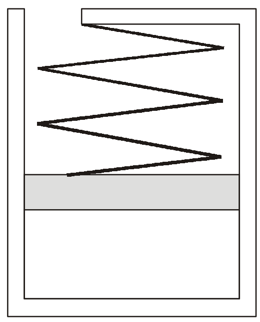
$$\therefore \quad dV = \frac{3\pi}{6}D^2 dD = \frac{\pi}{2}D^2 dD$$

$$W = \int_1^{2.5}(CD)\times\frac{\pi}{2}D^2 dD = \frac{\pi C}{2}\int_1^{2.5}D^3 dD = \frac{\pi}{2}C\left(\frac{D^4}{4}\right)_1^{2.5}$$

$$W = \frac{3.14}{2}\times\frac{200\times 10^3}{1}\times\frac{\left(2.5^4 - 1^4\right)}{4}$$

$W = 2988 \times 10^3$ J = **2988 kJ**

1.5 A cylinder having a piston restrained by a linear spring (of spring constant 15 kN/m) contains 0.5 kg of saturated vapour water at 120°C, as shown in the figure. Heat is transferred to the water, causing the piston to rise. If the piston cross-sectional area is 0.05 m^2, and the pressure varies linearly with volume until a final pressure of 500 kPa is reached. Find the final temperature in the cylinder and the heat transfer for the process.



The properties of water are given in the Table below:

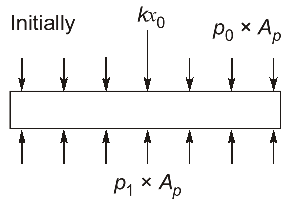
t (°C)	p (kPa)	v_g (m^3/kg)	u_g (kJ/kg)	h_g (kJ/kg)
120	198.50 (p_{sat})	0.89186	2529.2	2705.9
151.83	500.00 (p_{sat})	0.37477	2559.5	2746.6
801	500.00	0.99055	3664.2	4159.2
802	500.00	0.99147	3666.1	4161.6
803	500.00	0.99240	3668.0	4163.9
804	500.00	0.99333	3669.9	4166.3
805	500.00	0.99425	3671.8	4168.6

[10 marks : 2014]

Solution:

Let the initial compression in the spring be x_0

Equilibrium equation at state 1



$$p_1 \times A_p = kx_0 + p_0 \times A_p \quad ...(1)$$

where, p_1 = Saturation pressure at 120°C i.e., 198.50 kPa.

A_p = Area of piston, 0.05 m^2

x_0 = Initial compression

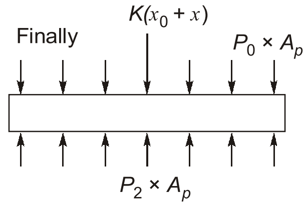
p_0 = Atmospheric pressure

After the heat is transferred in the cylinder, let the piston has moved by a distance x.

Equilibrium equation at state 2

$$p_2 \times A_p = k(x_0 + x) + p_0 \times A_p \quad ...(2)$$

where, p_2 = Final pressure inside the cylinder, 500 kPa



Subtracting equation (1) from (2)

$$(p_2 - p_1) \times A_p = K \cdot x$$

$$(500 - 198.50) \times 0.05 = 15 \times x$$

$$x = 1.005 \text{ m}$$

At state 1, $m = 0.5$ kg, $u_1 = u_{g@120°C} = 2529.2$ kJ/kg

$V_1 = v_1 \times \text{m}$

$\because$ $v_1 = v_{g@120°C} = 0.89186$ m^3/kg

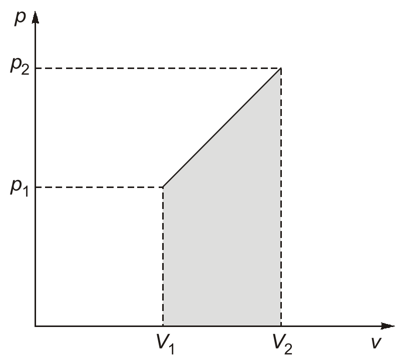
$\therefore$ $V_1 = 0.89186 \times 0.5 = 0.44593$ m^3

At state 2, $V_2 = V_1 + x \times A_p$

$= 0.44593 + 1.005 \times 0.05$

$= 0.49618$ m^3

Specific volume, $v_2 = \dfrac{V_2}{m} = 0.99236$ m^3/kg



From given table, at $v = 0.99236$ m^3/kg

$T_2 = 803$°C and $u_2 = 3668.0$ kJ/kg

$$W_{1-2} = \int pdV$$

$$W_{1-2} = \frac{1}{2}(p_1 + p_2)(V_2 - V_1)$$

$$W_{1-2} = \frac{1}{2}(198.5 + 500)(0.49618 - 0.44593) = 17.5498 \text{ kJ}$$

According to 1st law of thermodynamics

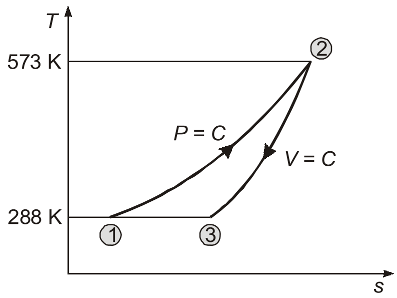
$$Q_{1-2} = U_{1-2} + W_{1-2} = m(u_2 - u_1) + W_{1-2}$$

$$Q_{1-2} = 0.5(3668.0 - 2529.2) + 17.5498$$

$$= 586.9498 \text{ kJ}$$

1.6 1 m^3 of air is heated at constant pressure from 15°C to 300°C and then cooled at constant volume back to its initial temperature. If the initial pressure is 1.03 bar calculate the net heat flow and overall change in entropy. Show the process on a *T-s* diagram. [5 marks : 2014]

Solution:



Given data: $V_1 = 1\ \text{m}^3$; $p_1 = p_2$; $T_1 = 15°\text{C} = (15 + 273)\ \text{K} = 288\ \text{K}$

$T_2 = 300°\text{C} = (300 + 273)\ \text{K} = 573\ \text{K}$; $p_1 = 1.03\ \text{bar} = 1.03 \times 10^5\ \text{Pa}$

$c_p = 1.005\ \text{kJ/kgK}$; $c_v = 0.718\ \text{kJ/kgK}$; $R = 0.287\ \text{kJ/kgK}$

Mass of air: $$m = \frac{p_1V_1}{RT_1} = \frac{1.03 \times 10^5}{287 \times 288} = 1.246\ \text{kg}$$

$$Q = Q_{1\text{-}2} + Q_{2\text{-}3} = mc_p(T_2 - T_1) + mc_v(T_3 - T_2)$$
$$= mc_p(T_2 - T_1) + mc_v(T_1 - T_2) \qquad (\because\ T_1 = T_3)$$
$$= m(T_2 - T_1)(c_p - c_v) = 1.246\,(573 - 288)\,(1.005 - 0.718)$$
$$= 102\ \text{kJ}$$

For constant p process,

$$S_2 - S_1 = mc_p \log_e\left(\frac{T_2}{T_1}\right) = 1.246 \times 1.005 \times \log_e\left(\frac{573}{288}\right) = 0.8614\ \text{kJ/K}$$

For constant volume process,

$$S_3 - S_2 = mc_v \log_e\left(\frac{T_3}{T_2}\right) = mc_v \log_e\left(\frac{T_1}{T_2}\right)$$
$$= 1.246 \times 0.718 \times \log_e\left(\frac{288}{573}\right) = -0.6154\ \text{kJ/K}$$

∴ Overall change of entropy,

$$\Delta S = (S_2 - S_1) + (S_3 - S_2) = 0.8614 - 0.6154 = \mathbf{0.246\ kJ/K}$$

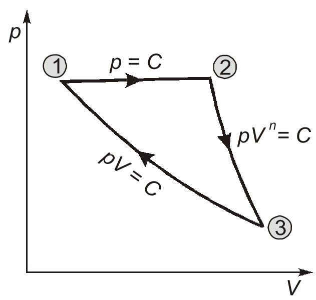
1.7 A certain mass of air is initially at 260°C and 700 kPa and occupies 0.028 m^3. The air is expanded at constant pressure to 0.084 m^3. A polytropic process with n = 1.50 is then carried out, followed by a constant temperature process which completes the cycle. All the processes are reversible processes.

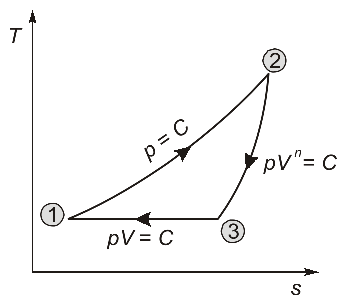
(i) Sketch the cycle on *p-v* and *T-s* coordinates and

(ii) Find the efficiency of the cycle. [10 marks : 2015]

Solution:

Given data: $T_1 = 260°\text{C} = (260 + 273)\ \text{K} = 533\ \text{K}$, $p_1 = 700\ \text{kPa}$, $V_1 = 0.028\ \text{m}^3$, $V_2 = 0.084\ \text{m}^3$





Applying equation of state at state 1,

$$p_1V_1 = mRT_1$$
$$700 \times 0.028 = m \times 0.287 \times 533$$

or $$m = 0.1281\ \text{kg}$$

For isobaric process 1-2,

$$\frac{T_2}{V_2} = \frac{T_1}{V_1} \quad \text{(Charles law)}$$

$$\frac{T_2}{0.084} = \frac{533}{0.028}$$

or $T_2 = 1599$ K

For polytropic process 2-3,

$$\frac{T_3}{T_2} = \left(\frac{V_2}{V_3}\right)^{n-1}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_3}\right)^{n-1} \quad (\because T_3 = T_1)$$

$$\frac{533}{1599} = \left(\frac{0.084}{V_3}\right)^{1.5-1} = \left(\frac{0.084}{V_3}\right)^{0.5}$$

or
$$\left(\frac{533}{1599}\right)^{\frac{1}{0.5}} = \frac{0.084}{V_3}$$

$$\left(\frac{533}{1599}\right)^{2} = \frac{0.084}{V_3}$$

or $V_3 = 0.756$ m^3

For process 1-2, Work done: $W_{1\text{-}2} = p_1 (V_2 - V_1)$

$= 700 (0.084 - 0.028) = 39.2$ kJ

Heat transfer: $Q_{1\text{-}2} = m\, c_p (T_2 - T_1)$

$= 0.1281 \times 1.005 (1599 - 533) = 137.23$ kJ

For process 2-3, Work done:
$$W_{2\text{-}3} = \frac{mR(T_3 - T_2)}{1-n}$$

$$= \frac{0.1281 \times 0.287(533 - 1599)}{1-1.5} = 78.38 \text{ kJ}$$

Heat transfer:
$$Q_{2\text{-}3} = \left(\frac{\gamma - n}{\gamma - 1}\right) \times W_{2-3} = \left(\frac{1.4 - 1.5}{1.4 - 1}\right) \times 78.38 = -19.59 \text{ kJ}$$

For process 3-1, Work done:
$$W_{3\text{-}1} = Q_{3-1} = mRT_1 \log_e \frac{V_1}{V_3}$$

$$= 0.1284 \times 0.287 \times 533 \log_e \frac{0.028}{0.756} = -64.73 \text{ kJ}$$

Net work done: $W_{net} = W_{1\text{-}2} + W_{2\text{-}3} + W_{3\text{-}1}$

$= 39.2 + 78.38 - 64.73 = 52.85$ kJ

Heat supplied: $Q_s = Q_{1\text{-}2} = 137.23$ kJ

The efficiency of the cycle,

$$\eta = \frac{\text{Net work done}}{\text{Heat supplied}} = \frac{W_{net}}{Q_s} = \frac{52.85}{137.23} = 0.3851 = \mathbf{38.51\%}$$

2 Refrigeration and Air-conditioning

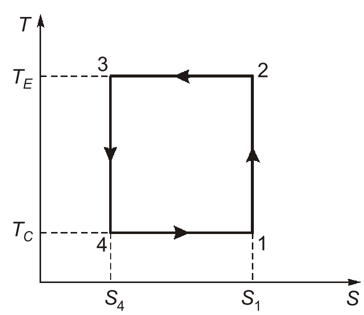
Revised Syllabus of ESE: *Vapour compression refrigeration, Refrigerants and Working cycles, (Topic of Power Plant : Compressors), Condensers, Evaporators and Expansion devices, Other types of refrigeration systems like Vapour Absorption, Vapour jet, thermo electric and Vortex tube refrigeration. Psychometric properties and processes, Comfort chart, Comfort and industrial air conditioning, Load calculations and Heat pumps.*

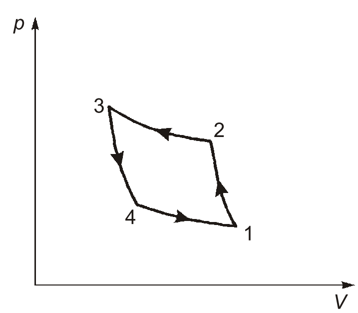
1. Air-Refrigerating Cycle

1.1 If gas is used as refrigerant in Reverse Carnot refrigeration cycle, draw *T-s* and *P-v* diagrams with heat rejection and heat absorption temperature of T_E and T_C respectively. Show that this cycle requires two compressors and two expanders. Find their work requirement and the *COP* of the cycle.

[10 marks : 2002]

Solution:





1 – 2 ⟶ Isentropic compression
2 – 3 ⟶ Isothermal compression
3 – 4 ⟶ Isentropic expansion
4 – 1 ⟶ Isothermal expansion

As we can see that in p-V diagram, both in process 1-2, and 2-3 the pressure is increasing so two compressors are required.

Similarly in process 3-4 and 4-1 the pressure is decreasing and volume is increasing so two expanders are required.

Work transfer in compressors (per kg) = $w_{12} + w_{23}$

$$= \frac{\gamma}{\gamma-1}R(T_1 - T_2) + RT_E \log_e\left(\frac{V_3}{V_2}\right) = c_p(T_C - T_E) + RT_E \log_e\left(\frac{V_3}{V_2}\right)$$

Work transfer in expanders = $c_p(T_3 - T_4) + RT_C \log_e\left(\frac{V_1}{V_4}\right) = c_p(T_E - T_C) + RT_C \log_e\left(\frac{V_1}{V_4}\right)$

Alternatively Net work required = Heat rejected – Heat absorbed

$$= T_E(S_1 - S_4) - T_C(S_1 - S_4) = (T_E - T_C)(S_1 - S_4)$$

$$(COP)_R = \frac{\text{Heat absorbed}}{\text{Work done}} = \frac{T_C(S_1 - S_4)}{(T_E - T_C)(S_1 - S_4)} = \frac{T_C}{T_E - T_C}$$

Net work required, $W = W_C + W_T = RT_E \log_e\left(\frac{V_3}{V_2}\right) + RT_C \log_e\left(\frac{V_1}{V_4}\right)$

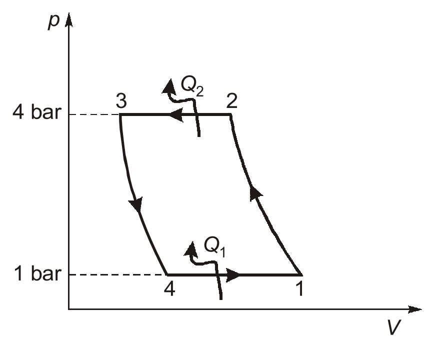
$\because \quad \frac{V_3}{V_2} = \frac{V_4}{V_1}$

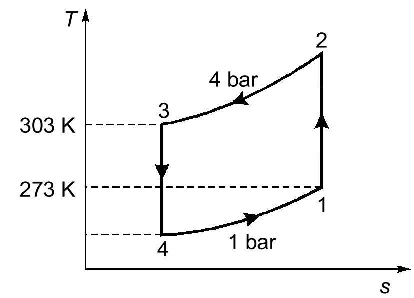
So, $W = R(T_E - T_C)\log_e\left(\frac{V_3}{V_2}\right) = (T_E - T_C)(S_2 - S_3) = (T_E - T_C)(S_1 - S_4)$

1.2 Dense air is used as refrigerant in reverse Brayton or Bell coleman or Joule cycle. Draw *p-V* and *T-s* diagram for the cycle. Derive the expression for *COP* in terms of pressure ratio. If the temperatures at the end of heat absorption and heat rejection are 0°C and 30°C respectively. The pressure ratio is 4 and pressure in the cooler is 4 bar, determine the temperature at all state points and volume flow rates at inlet to compressor and at exit of turbine for 1 TR cooling capacity.

[15 marks : 2002]

Solution:





Pressure ratio: $r_p = \frac{p_2}{p_1} = \frac{p_3}{p_4}$

Assuming 1-2 and 3-4 as isentropic

$$\therefore \quad T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = T_1 \cdot r_p^{\left(\frac{\gamma-1}{\gamma}\right)}$$

$$\therefore \quad T_3 = T_4 \cdot r_p^{\left(\frac{\gamma-1}{\gamma}\right)}$$

$$\therefore \quad T_2 = 273(4)^{\frac{0.4}{1.4}} = 405.676 \text{ K}$$

$$T_4 = \frac{303}{4^{0.4/1.4}} = 203.9 \text{ K}$$

Coefficient of performance,

$$COP = \frac{\text{Refrigerating effect}}{\text{Work input}} = \frac{Q_1}{W_{Net}} = \frac{Q_1}{W_c - W_e} = \frac{c_p(T_1 - T_4)}{c_p(T_2 - T_1) - c_p(T_3 - T_4)}$$

$$= \frac{(T_1 - T_4)}{(T_2 - T_1) - (T_3 - T_4)} \quad \text{...(i)}$$

$$= \frac{\left(T_1 - \frac{T_3}{r_p^{\frac{\gamma-1}{\gamma}}}\right)}{T_1\left(r_p^{\frac{\gamma-1}{\gamma}} - 1\right) - T_3\left(1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}}\right)} = \frac{\left[T_1 - \frac{T_3}{r_p^{\frac{\gamma-1}{\gamma}}}\right]}{\left[T_1 - \frac{T_3}{r_p^{\frac{\gamma-1}{\gamma}}}\right]\left[r_p^{\frac{\gamma-1}{\gamma}} - 1\right]} = \frac{1}{\left(r_p^{\frac{\gamma-1}{\gamma}} - 1\right)}$$

Volume flow rate of air for 1 TR capacity

$$\therefore \quad \dot{m}_a c_p(T_1 - T_4) = 1 \text{ TR} = 3.5 \text{ kW}$$

$$\dot{m}_a = \frac{3.5}{1.005 \times (273 - 203.9)} = 0.0504 \text{ kg/s}$$

Density of air at inlet to compressor,

$$\rho_1 = \frac{p_1}{RT_1} = \frac{10^5}{287 \times 273} = 1.276 \text{ kg/m}^3$$

Volume flow rate at inlet to compressor,

$$V_1 = \frac{\dot{m}}{\rho} = \frac{0.0504}{1.276} = 0.0395 \text{ m}^3\text{/s} = 2.37 \text{ m}^3\text{/min}$$

Volume flow rate at exit to turbine,

$$V_4 = \frac{\dot{m}_a R T_4}{p_4} = \frac{0.0504 \times 287 \times 203.9}{10^5}$$

$$= 0.0295 \text{ m}^3\text{/s} = \mathbf{1.77 \text{ m}^3\text{/min}}$$

1.3 The compressed air from main compressor of an air-craft cooling system is bled off at 4.5 bar and 200°C. It is then passed through heat exchanger in which the ram air is forced through for cooling purposes by the fan driven by cooling turbine. The condition of the inlet to cooling turbine is 4 bar and 30°C. The air expands in cooling turbine upto 0.7 bar, the isentropic efficiency of cooling turbine is 80% and flow rate through cooling turbine is 30 kg/min. By drawing the system and showing the process on *T-s* diagram. Find

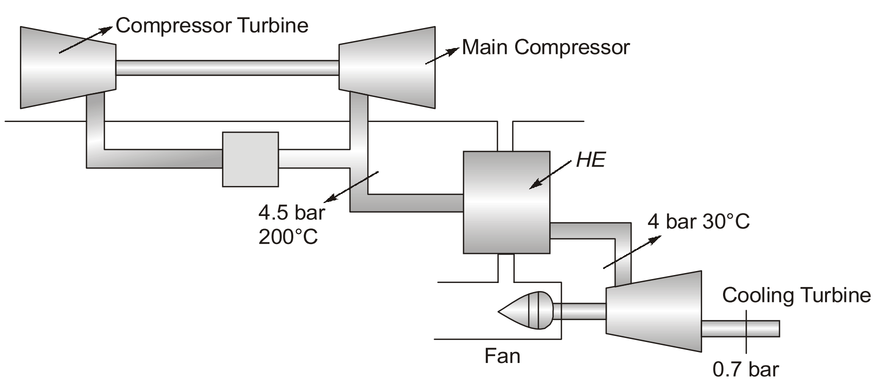
(i) The actual exit temperature from the cooling turbine.

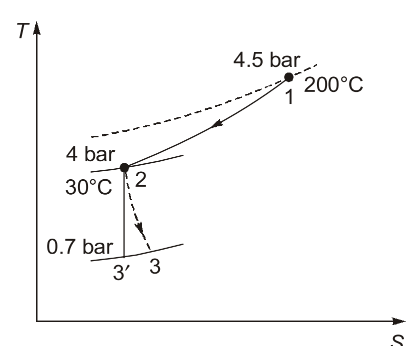
(ii) The power delivered to the ram air blow fan.

(iii) The tons of refrigeration, if the cold air is tempered by mixing with by passed warm air and delivered to the cabin cockpit area where it warms upto 25°C before exhausting out to waste.

[20 marks : 2005]

Solution:





(i) Exit temperature of turbine consider process 2-3′

$$\frac{T^{\gamma}}{p^{\gamma-1}} = C$$

$$\frac{T_2^{1.4}}{p_2^{1.4-1}} = \frac{T_{3'}^{1.4}}{p_{3'}^{1.4-1}}$$

$$\frac{303^{1.4}}{4^{0.4}} = \frac{T_{3'}^{\ 1.4}}{0.7^{0.4}}$$

$$T_{3'} = 303\left(\frac{0.7}{4}\right)^{\frac{0.4}{1.4}} = 184.15 \text{ K}$$

Efficiency of turbine,

$$0.8 = \frac{T_2 - T_3}{(T_2 - T_3')}$$

$$T_3 = T_2 - (T_2 - T_3') \times 0.8 = 303 - (303 - 184.15) \times 0.8 = \mathbf{207.92\ K}$$

(ii) Power delivered to fan = Work output from turbine

$$= \frac{30}{60} \times 1.005\,(T_2 - T_3) = 0.5 \times 1.005 \times (303 - 207.92) = \mathbf{47.78\ kW}$$

(iii) Refrigeration effect:

$$\text{RE} = \dot{m}_a\, c_p\, (T_{\text{Cabin}} - T_3) = 0.5 \times 1.005 \times (298 - 207.92)$$

$$= 45.2652 \text{ kW} = \frac{45.2652}{3.5} \text{ TR} = \mathbf{12.933\ TR}$$

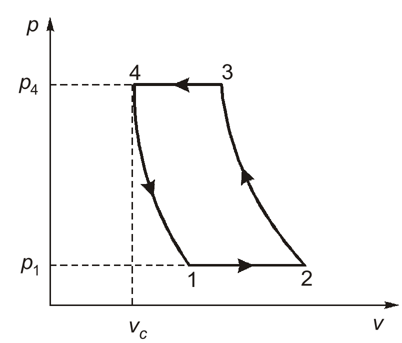
1.4 **Obtain an expression for the capacity of a refrigeration system (tonnage) in terms of the rate of upper and lower pressure limits, expansion index (n), v_c/v_s (v_c = clearance volume and v_s = swept volume), N (rpm), v_c, v_1 (specific volume at the inlet to compressor) and the refrigeration effect ($h_1 - h_4$).**
If the pressure ratio is 6.5, n = 1.1, v_c/v_s = 0.025, N = 900 rpm, v_s = 600 cc, v_1 = 0.078 m^3/kg and refrigeration effect = 150 kJ/kg, calculate the capacity of the system.

[15 marks : 2006]

Solution:

Volumetric efficiency of reciprocating compressor is defined as the ratio of actual volume of refrigerant taken to the swept volume

$$v_s = \text{Swept volume} = \left(\frac{\pi}{4}D^2L\right)$$



2 - 3 – Isentropic compression
4 - 1 – Isentropic expansion
Assuming that expansion and compression follows $pv^n = C$

Actual volume = $(v_2 - v_1)$

$$\therefore \quad \eta_v = \frac{(v_2 - v_1)}{v_{swept}} = \frac{v_2 - v_1}{v_2 - v_C}$$

$$\eta_v = \frac{(v_2 - v_1 - v_C + v_C)}{(v_2 - v_C)} = \frac{(v_2 - v_C) - (v_1 - v_C)}{(v_2 - v_C)}$$

$$\eta_v = 1 - \frac{(v_1 - v_C)}{(v_2 - v_C)} = 1 - \frac{v_C\left(\frac{v_1}{v_C} - 1\right)}{v_{Swept}} = 1 - \frac{v_C\left(\frac{v_1}{v_C} - 1\right)}{v_{Swept}} = 1 - \frac{v_C}{v_S}\left(\frac{v_1}{v_C} - 1\right)$$

$$\eta_v = 1 + \frac{v_C}{v_S} - \frac{v_C}{v_S}\left(\frac{v_1}{v_C}\right) \qquad \text{...(i)}$$

For process 4 – 1:

$$p_4 V_4^n = p_1 v_1^n ; \qquad \frac{p_4}{p_1} = \left(\frac{v_1}{v_C}\right)^n \qquad [\because V_4 = V_c]$$

Put this value in equation (i) we get

$$\eta_v = 1 + \frac{v_C}{v_S} - \frac{v_C}{v_S}\left(\frac{p_4}{p_1}\right)^{\frac{1}{n}}$$

Where p_4 = Higher pressure

p_1 = Lower pressure

$\frac{v_C}{v_S}$ = Clearance ratio = c

Also,

$$\eta_v = \frac{\dot{m} \times v_1}{\left(\frac{\pi}{4} D^2 \times L\right) \times N/60}$$ (v_1 is the specific volume at entry to the compressor)

$$\Rightarrow \qquad \dot{m} = \frac{\left[1 + \left(\frac{v_C}{v_S}\right) - \left(\frac{v_C}{v_S}\right)\left(\frac{p_4}{p_1}\right)^{\frac{1}{n}}\right] \times \frac{v_S}{v_1} \times N}{60}$$

$$\text{Refrigeration capacity (in ton)} = \frac{\dot{m}(h_1 - h_4)}{3.5}$$

$$= \frac{\left[1 + \left(\frac{v_C}{v_S}\right) - \left(\frac{v_C}{v_S}\right)\left(\frac{p_4}{p_1}\right)^{\frac{1}{n}}\right] \times v_S \times N \times (h_1 - h_4)}{v_1 \times 60 \times 3.5}$$

Now, $\frac{p_D}{p_S} = 6.5, n = 1.1, \frac{v_C}{v_S} = 0.025, N = 900 \text{ rpm}, v_S = 600 \text{ cc}$

v_1 = 0.078 m^3/kg, $(h_1 - h_4)$ = 150 kJ/kg

$$\text{Capacity} = \frac{\left[1 + 0.025 - 0.025 \times (6.5)^{\frac{1}{1.1}}\right] \times 600 \times 10^{-6} \times 150 \times 900}{0.078 \times 60 \times 3.5} = \textbf{4.39 ton}$$

1.5 A dense air refrigeration machine operating on Bell-Coleman cycle operates between 3.4 bar and 17 bar. The temperature of air after the cooler is 15°C and after the refrigerator is 6°C. If the refrigeration capacity is 6 tonnes, calculate:

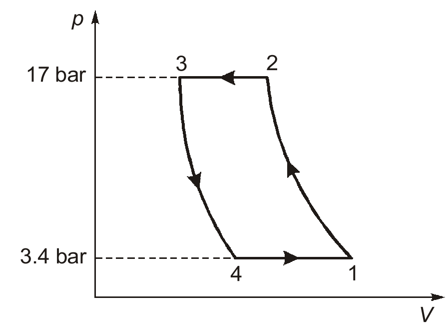
(i) temperature after compression and expansion
(ii) air circulation per min
(iii) work of compressor and expender
(iv) theoretical *COP*
(v) rate of water circulation required in the cooler in kg/min if the rise in temperature is limited to 30°C.

[10 marks : 2009]

Solution:

1 - 2 : Isentropic compression

3 - 4 : Isentropic expansion



$$T_3 = 15°C = 288 \text{ K}$$
$$T_1 = 6°C = 279 \text{ K}$$
Refrigeration capacity = 6 TR

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{3.4}{17}\right)^{\frac{0.40}{1.40}} = 0.6314$$

(i) $T_4 = 288 \times 0.6314 = 181.84$ K (Temperature after expansion)

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{17}{3.4}\right)^{\frac{0.40}{1.40}} = 1.5838$$

Temperature after compression

$$T_2 = 279 \times 1.5838 = 441.88 \text{ K}$$

Heat absorbed in cold chamber

$$Q = c_p (T_1 - T_4) = 1.005 \times (279 - 181.84) = \mathbf{97.646 \text{ kJ/kg}}$$

(ii) Air circulation per min $= \frac{6 \times 210}{97.646} = \mathbf{12.9 \text{ kg/min}}$

(iii) Work of compressor $= \frac{\gamma R}{\gamma - 1}(T_2 - T_1) \times \dot{m}$

$$= \frac{1.4}{0.4} \times 0.287\,(441.88 - 279) \times \frac{12.90}{60} = 35.177 \text{ kW}$$

Work of expander $= \frac{\gamma R}{\gamma - 1}(T_3 - T_4) \times \dot{m} = \frac{1.4}{0.4} \times (288 - 181.84) \times \frac{12.90}{60} \times 0.287$

$= \mathbf{22.927 \text{ kW}}$

(iv) $$COP = \frac{\text{Heat absorbed}}{\text{Net work done}}$$

Heat rejected $= c_p (T_2 - T_3) = 1.005\,(441.88 - 288)$

$= 154.65$ kJ/kg

Heat absorbed $= 97.646$ kJ/kg

Net work $= 154.65 - 97.646 = 57$ kJ/kg

$$COP = \frac{97.646}{57} = \mathbf{1.713}$$

(v) Rate of water circulation required if $\Delta T = 30°C$

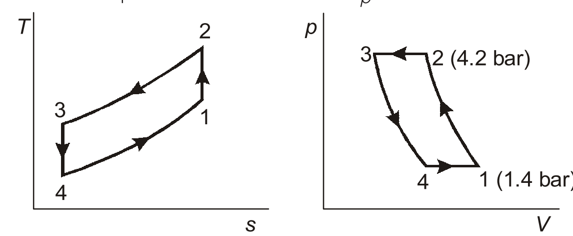
$$\dot{m}_w c_{pw} (\Delta T_w) = \dot{m}_R c_{pR} (\Delta T_R)$$
$$\dot{m}_w \times 4.2(30) = 12.9 \times 154.65$$
$$\dot{m}_w = \mathbf{15.833 \text{ kg/min}}$$

1.6 A Bell-Coleman refrigeration system is used to produce 10 tons of refrigeration. The cooler and refrigerator pressures are 4.2 bars and 1.4 bars. Air is cooled in the cooler to 45°C and temperature of air at the inlet of the compressor is –20°C. For an ideal cycle, calculate *COP*, mass of air circulated/min, theoretical piston displacement of compressor and power required per ton of refrigeration. Assume c_p for air as 1.005 kJ/kg-K. Find the cylinder dimensions if the compressor is single-acting single-cylinder with *L*/*D* ratio of 1.2 and runs at 600 rpm. [10 marks : 2010]

Solution:

Given: Refrigeration effect = 10 TR = 35 kW, T_3 = 45°C = 318 K, T_1 = –20°C = 253 K, c_p = 1.005 kJ/kgK

$$r_p = \frac{p_2}{p_1} = \frac{4.2}{1.4} = 3$$



Coefficient of performance

$$COP = \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}} - 1} = \frac{1}{(3)^{\frac{0.4}{1.4}} - 1}$$

$$= 2.7$$

From the isentropic process 1-2

$$\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_2}{T_1}$$

$$\left(\frac{4.2}{1.4}\right)^{\frac{0.4}{1.4}} = \frac{T_2}{253}$$

$$T_2 = 253(1.369) = 346.4 \text{ K}$$

Now

$$\frac{T_3}{T_4} = \frac{T_2}{T_1}$$

$$\frac{318}{T_4} = \frac{346.3}{253}$$

$$T_4 = \frac{318 \times 253}{346.3} = 232.32\text{K}$$

Since refrigeration capacity = 35 kW

$$35 = \dot{m}c_p(T_1 - T_4) = \dot{m}(1.005)(253 - 232.32)$$

$$\dot{m} = \frac{35}{20.778} = 1.684 \text{ kg/s}$$

$$\dot{m} = \mathbf{101.04 \text{ kg/min}}$$

$$\text{Power required} = \dot{m}c_p[(T_2 - T_1) - (T_3 - T_4)]$$

$$= 1.684 \times 1.005[346.3 - 253 - 318 + 232.32]$$

$$= \mathbf{12.89 \text{ kW}}$$

$$\text{Power required per ton} = \frac{12.89}{10} = 1.289 \text{ kW/TR}$$

Now

$$\rho_1 = \frac{p_1}{RT_1} = \frac{1.4 \times 10^5}{287 \times 253} = 1.928 \text{ kg/m}^3$$

Assuming volumetric efficiency as 100%

$$\frac{\pi}{4}D^2LN = \frac{\dot{m}}{\rho}$$

Since $L = 1.2D$

and $N = 600$ rpm

$$\frac{\pi}{4}D^2(1.2D)\times\frac{600}{60} = \frac{1.684}{1.928}$$

$$9.42\,D^3 = 0.873$$

$$D = 0.4526 \text{ m}$$

$$L = 0.5431 \text{ m}$$

Theoretical piston displacement

$$= \frac{\pi}{4}(45.26)^2(54.31) = 87377.33 \text{ cm}^3$$

1.7 In aircraft refrigerating unit using air cycle, 50 kg/min of air at 180 cm Hg gauge and 205°C are bled off the air compressor serving the jet engine of an airplane. The bled air is passed through a heat exchanger leaving at 175 cm Hg gauge and 75°C. At this point, it is expanded through a small cooling turbine to 20 cm Hg vacuum and –10°C. The air exhausted out of the plane is at 25°C. Assume c_p =1.0 kJ/kgK.

(i) Find the cooling in ton (refrigeration).

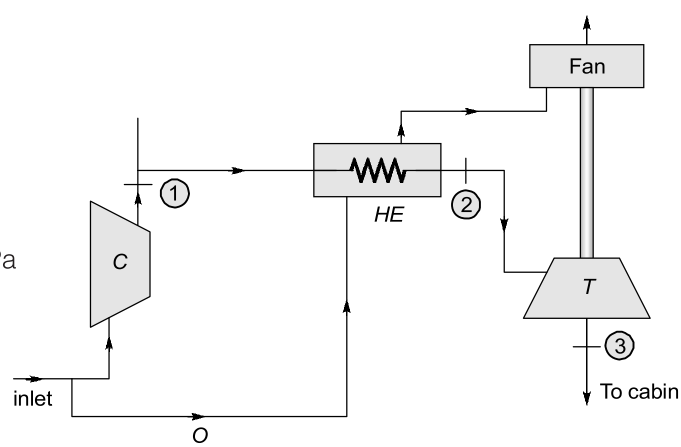
(ii) If the compressor receives air at stagnation state of 2 cm Hg gauge and 50 °C and if the small air-cooling turbine output serves the centrifugal fan for passing coolant air through the heat exchanger, determine the input power for the refrigerant plant.

(iii) What is the COP based on input power to bled off air?

[20 marks : 2017]

Solution:

Given: $\dot{m}_a = 50$ kg/min



Pressure at end of compression,

p_1 = 180 cm Hg gauge

p_1 = 180 + 76 = 256 cm of Hg

$$p_1 = \frac{256}{76}\times 101.325 = 341.3 \text{ kPa}$$

T_1 = 205 + 273 = 478 K

p_2 = 175 cm Hg gauge

= 175 + 76

= 251 cm of Hg

$$= \frac{251}{76}\times 101.325 = 334.64 \text{ kPa}$$

T_2 = 273 + 75 = 348 K

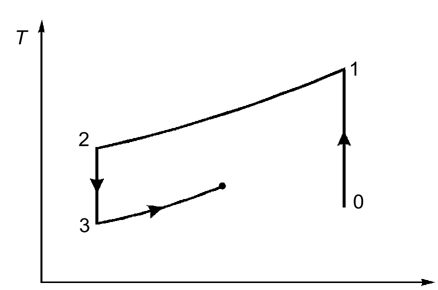
p_3 = 20 cm Hg vacuum = 76 – 20 = 56 cm of Hg

$$p_3 = \frac{56}{76}\times 101.325 = 74.66 \text{ kPa}$$

T_3 = 263 K

T_4 = cabin temperature = 298 K

c_p = 1 kJ/kg-K



(i) Cooling load $= \dot{m}_a \times c_p(T_4 - T_3) = \frac{50}{60}\times 1\times(298-263)$

$$= 29.16 \text{ kW} = \frac{29.16}{3.5} = 8.33 \text{ TR}$$