**18** *Years*

*Previous Solved Papers*

# GATE 2025

## Production & Industrial Engineering

- Fully solved with explanations
- Topicwise presentation
- Thoroughly revised & updated

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## GATE - 2025

## Production & Industrial Engineering

Topicwise Previous GATE Solved Papers (2007-2024)

*Editions*

1st Edition : 2020
2nd Edition : 2021
3rd Edition : 2022
4th Edition : 2023

**5th Edition : 2024**

# Preface

**B. Singh** (Ex. IES)

Over the period of time the GATE examination has become more challenging due to increasing number of candidates. Though every candidate has ability to succeed but competitive environment, in-depth knowledge, quality guidance and good source of study is required to achieve high level goals.

The new edition of **GATE 2025 Solved Papers : Production & Industrial Engineering** has been fully revised, updated and edited. The whole book has been divided into topicwise sections.

At the beginning of each subject, analysis of previous papers are given to improve the understanding of subject.

I have true desire to serve student community by way of providing good source of study and quality guidance. I hope this book will be proved an important tool to succeed in GATE examination. Any suggestions from the readers for the improvement of this book are most welcome.

**B. Singh (Ex. IES)**
Chairman and Managing Director
MADE EASY Group

# GATE-2025

## Production & Industrial Engg.

## CONTENTS

# Engineering Mathematics

UNIT I

## CONTENTS

# Engineering Mathematics

## *Syllabus*

**Linear Algebra :** Matrix algebra, systems of linear equations, eigen values and eigen vectors.

**Calculus :** Functions of single variable, limit, continuity and differentiability, mean value theorems, evaluation of definite and improper integrals, partial derivatives, total derivative, maxima and minima, gradient, divergence and curl, vector identities, directional derivatives, line, surface and volume integrals, Stokes, Gauss and Green's theorems.

**Differential Equations :** First order equations (linear and non-linear), higher order linear differential equations with constant coefficients, Cauchy's and Euler's equations, initial and boundary value problems, laplace transforms, solutions of one dimensional heat and wave equations and laplace equation.

**Complex Variables :** Analytic functions, Cauchy's integral theorem, Taylor series.

**Probability and Statistics :** Definitions of probability and sampling theorems, conditional probability, mean, median, mode and standard deviation, random variables, Poisson, normal and binomial distributions.

**Numerical Methods :** Numerical solutions of linear and non-linear algebraic equations, integration by trapezoidal and Simpson's rule, single and multi-step methods for differential equations.

# 1 Linear Algebra

**1.1** The angle (in degrees) between two planar vectors

$\bar{a} = \frac{\sqrt{3}}{2}i + \frac{1}{2}j$ and $\bar{b} = -\frac{\sqrt{3}}{2}i + \frac{1}{2}j$ is

(a) 30 (b) 60
(c) 90 (d) 120 **[2007 : 1 M]**

**1.2** $q_1$, ......$q_m$ are $n$-dimensional vectors, with $m < n$. This set of vectors is linearly dependent. $Q$ is the matrix with $q_1$, ...., $q_m$ as the columns. The rank of $Q$ is

(a) less than $m$ (b) $m$
(c) between $m$ and $n$ (d) $n$ **[2007 : 2 M]**

**1.3** The eigenvector pair of the matrix $\begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix}$ is

(a) $\begin{Bmatrix} 2 \\ 1 \end{Bmatrix}$ and $\begin{Bmatrix} 1 \\ -2 \end{Bmatrix}$ (b) $\begin{Bmatrix} 2 \\ 1 \end{Bmatrix}$ and $\begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$

(c) $\begin{Bmatrix} -2 \\ 1 \end{Bmatrix}$ and $\begin{Bmatrix} 1 \\ -2 \end{Bmatrix}$ (d) $\begin{Bmatrix} -2 \\ 1 \end{Bmatrix}$ and $\begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$

**[2008 : 2 M]**

**1.4** Inverse of the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is

(a) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix}$

**[2008 : 2 M]**

**1.5** The value of the determinant $\begin{vmatrix} 1 & 3 & 2 \\ 4 & 1 & 1 \\ 2 & 1 & 3 \end{vmatrix}$ is

(a) –28 (b) –24
(c) 32 (d) 36 **[2009 : 1 M]**

**1.6** The value of $x_3$ obtained by solving the following system of linear equations is

$x_1 + 2x_2 - 2x_3 = 4$
$2x_1 + x_2 + x_3 = -2$
$-x_1 + x_2 - x_3 = 2$

(a) –12 (b) –2
(c) 0 (d) 12 **[2009 : 2 M]**

**1.7** The value of $q$ for which the following set of linear algebraic equations

$2x + 3y = 0$
$6x + qy = 0$

can have non-trivial solution is

(a) 2 (b) 7
(c) 9 (d) 11 **[2010 : 1 M]**

**1.8** If $\{1, 0, -1\}^T$ is an eigenvector of the following matrix.

$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

then the corresponding eigenvalue is

(a) 1 (b) 2
(c) 3 (d) 5 **[2010 : 1 M]**

**1.9** If matrix $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ and matrix $B = \begin{bmatrix} 4 & 6 \\ 5 & 9 \end{bmatrix}$, the transpose of product of these two matrices, i.e., $(AB)^T$ is equal to

(a) $\begin{bmatrix} 28 & 19 \\ 34 & 47 \end{bmatrix}$ (b) $\begin{bmatrix} 19 & 34 \\ 47 & 28 \end{bmatrix}$

(c) $\begin{bmatrix} 48 & 33 \\ 28 & 19 \end{bmatrix}$ (d) $\begin{bmatrix} 28 & 19 \\ 48 & 33 \end{bmatrix}$

**[2011 : 1 M]**

**1.10** If $A$ (0, 4, 3), $B$ (0, 0, 0) and $C$ (3, 0, 4) are three points defined in $x$, $y$, $z$ coordinate system, then which one of the following vectors is perpendicular to both the line vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$?

(a) $16\hat{i} + 9\hat{j} - 12\hat{k}$ (b) $16\hat{i} - 9\hat{j} + 12\hat{k}$
(c) $16\hat{i} - 9\hat{j} - 12\hat{k}$ (d) $16\hat{i} + 9\hat{j} + 12\hat{k}$

**[2011 : 1 M]**

**1.11** The eigen values of the following matrix are

$$\begin{bmatrix} 10 & -4 \\ 18 & -12 \end{bmatrix}$$

(a) 4, 9 (b) 6, –8
(c) 4, 8 (d) –6, 8 **[2011 : 2 M]**

**1.12** $x + 2y + z = 4$
$2x + y + 2z = 5$
$x - y + z = 1$
The system of algebraic equations given above has
(a) a unique solution of $x = 1$, $y = 1$ and $z = 1$.
(b) only the two solutions of ($x = 1$, $y = 1$, $z = 1$) and ($x = 2$, $y = 1$, $z = 0$).
(c) infinite number of solutions.
(d) no feasible solution. **[2012 : 2 M]**

**1.13** For the matrix $A = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$, one of the normalized eigen vectors is given as

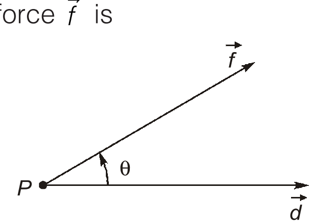
(a) $\begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}$ (b) $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$

(c) $\begin{pmatrix} \frac{3}{\sqrt{10}} \\ \frac{-1}{\sqrt{10}} \end{pmatrix}$ (d) $\begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix}$ **[2012 : 2 M]**

**1.14** The eigenvalues of a symmetric matrix are all
(a) complex with non-zero positive imaginary part
(b) complex with non-zero negative imaginary part
(c) real
(d) pure imaginary **[2013 : 1 M]**

**1.15** The system of equations, given below, has
$x + 2y + 4z = 2$
$4x + 3y + z = 5$
$3x + 2y + 3z = 1$
(a) a unique solution
(b) two solutions
(c) no solution
(d) more than two solutions **[2014 : 1 M]**

**1.16** If a constant force $\vec{f}$ applied on an object $P$, displaces it by a distance $\vec{d}$ inclined at an angle $\theta$ to the direction of force, $\vec{f}$, then the work done by the force $\vec{f}$ is




(a) $\text{div}(\vec{f} \times \vec{d})$ (b) $|\vec{f} \times (\text{curl } \vec{d})|$
(c) $|\vec{f} \times \vec{d}|$ (d) $\vec{f} \cdot \vec{d}$ **[2015 : 1 M]**

**1.17** The eigenvalues of the matrix
are $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$
(a) $i$ and $-i$ (b) 1 and –1
(c) 0 and 1 (d) 0 and –1 **[2016 : 1 M]**

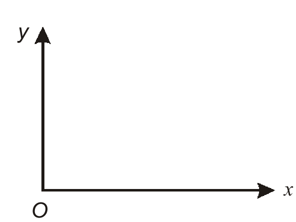
**1.18** The number of solutions of the simultaneous algebraic equations $y = 3x + 3$ and $y = 3x + 5$ is
(a) zero (b) 1
(c) 2 (d) infinite **[2016 : 1 M]**

**1.19** For two non-zero vectors $\vec{A}$ and $\vec{B}$, if $\vec{A} + \vec{B}$ is perpendicular to $\vec{A} - \vec{B}$, then
(a) the magnitude of $\vec{A}$ is twice the magnitude of $\vec{B}$
(b) the magnitude of $\vec{A}$ is half the magnitude of $\vec{B}$
(c) $\vec{A}$ and $\vec{B}$ cannot be orthogonal
(d) the magnitude of $\vec{A}$ and $\vec{B}$ are equal
**[2017 : 1 M]**

**1.20** For the orthogonal matrix $Q$, the valid equality is
(a) $Q^T = Q^{-1}$ (b) $Q = Q^{-1}$
(c) $Q^T = Q$ (d) $\det(Q) = 0$
**[2017 : 1 M]**

**1.21** Vector triple product $a \times (b \times c)$ of three vectors $a$, $b$ and $c$ is given by
(a) $(a \cdot c)b - (a \cdot b)c$ (b) $(b \cdot c)a - (a \cdot c)b$
(c) $(a \cdot b)c - (a \cdot c)b$ (d) $(b \cdot c)a - (a \cdot b)c$
**[2018 : 1 M]**

**1.22** Considering the coordinate system shown in the figure, a force of magnitude 10 kN has $x$-component of –6 kN. Possible $y$-component (s) of the force is/are




(a) +8 kN only (b) +5 kN only
(c) +8 kN and –8 kN (d) +5 kN and –5 kN
**[2018 : 1 M]**

**1.23** The diagonal elements of a 3-by-3 matrix are –10, 5 and 0, respectively. If two of its eigenvalues are –15 each, the third eigenvalue is _______
**[2018 : 1 M]**

**1.24** For any real, square and non-singular matrix $\vec{B}$, the det $\vec{B}^{-1}$ is

(a) zero (b) $(\det\vec{B})^{-1}$

(c) $-(\det\vec{B})$ (d) $\det\vec{B}$ [2019 : 1 M]

**1.25** The vector that is normal to the surface $2xz^2 - 3xy - 4x = 7$ at the point (1, –1, 2) is

(a) $2\hat{i} - 3\hat{j} + 8\hat{k}$ (b) $2\hat{i} + 3\hat{j} + 4\hat{k}$

(c) $7\hat{i} - 3\hat{j} + 8\hat{k}$ (d) $7\hat{i} - 5\hat{j} + 8\hat{k}$

**[2019 : 1 M]**

**1.26** The divergence of the vector $\vec{v} = y^2\hat{i} + z^2\hat{j} + x^2\hat{k}$ is

(a) 0 (b) $2z$

(c) $2y$ (d) $2x$ **[2020 : 1 M]**

**1.27** For the matrix $\begin{bmatrix} 1 & 5 \\ 3 & 3 \end{bmatrix}$, the eigen vectors are

(a) $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 3 \end{bmatrix}$ (b) $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 5/3 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ -3 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -5/3 \\ 1 \end{bmatrix}$

**[2020 : 2 M]**

**1.28** The eigen values of matrix, $A = \begin{bmatrix} 8 & 3 \\ 2 & 7 \end{bmatrix}$ are 5 and 10. For matrix $B = A + \alpha I$, where $\alpha$ is a constant $I$ is 2 × 2 identify matrix, its eigenvalues are

(a) 5, 10 (b) $5 + \alpha$, $10 + \alpha$

(c) $5 - \alpha$, $10 - \alpha$ (d) $5\alpha$, $10\alpha$ **[2021 : 2 M]**

**1.29** Matrix A as product of two other matrices is given by

$$A = \begin{bmatrix} 3 \\ 2 \end{bmatrix}\begin{bmatrix} 1 & 4 \end{bmatrix}.$$

The value of det(A) is_____. [round off to nearest integer] **[2022 : 1 M]**

**1.30** If a matrix is squared, then

(a) both eigenvalues and eigenvectors are retained.

(b) eigenvalues get squared but eigenvectors are retained.

(c) both eigenvalues and eigenvectors must change.

(d) eigenvalues are retained but eigenvectors change. **[2022 : 2 M]**

**1.31** Given matrices

$$A = \begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 3 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix};$$

$B$ is skew-symmetric matrix of $A$. $B_{13}$ is

(a) –3 (b) –2

(c) 2 (d) 3 **[2023 : 1 M]**

**1.32** Transformation matrix to translate a point P from (10, 15) to (15, 25) is

(a) $\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 10 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

**[2023 : 1 M]**

**1.33** For the matrix $\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix}$, eigen value corresponding to the eigenvector $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$ is ________ (in integer).

**[2023 : 2 M]**

**1.34** If $A = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$ is a matrix such that $A^2 = I$, where $I$ is an identity matrix, then which of the following is TRUE?

(a) $1 + a^2 + bc = 0$ (b) $1 - a^2 + bc = 0$

(c) $1 - a^2 - bc = 0$ (d) $1 + a^2 - bc = 0$

**[2024 : 2 Marks]**

■■■■

## Answers Linear Algebra

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.1** | (d) | **1.2** | (a) | **1.3** | (a) | **1.4** | (a) | **1.5** | (b) | **1.6** | (b) | **1.7** | (c) |
| **1.8** | (a) | **1.9** | (d) | **1.10** | (a) | **1.11** | (b) | **1.12** | (c) | **1.13** | (b) | **1.14** | (c) |
| **1.15** | (a) | **1.16** | (d) | **1.17** | (a) | **1.18** | (a) | **1.19** | (d) | **1.20** | (a) | **1.21** | (a) |
| **1.22** | (c) | **1.23** | (25) | **1.24** | (b) | **1.25** | (c) | **1.26** | (a) | **1.27** | (d) | **1.28** | (b) |
| **1.29** | (0) | **1.30** | (b) | **1.31** | (c) | **1.32** | (a) | **1.33** | (1) | **1.34** | (c) | | |

## Explanations Linear Algebra

**1.1 (d)**

Given: $\bar{a} = \frac{\sqrt{3}}{2}i + \frac{1}{2}j$

and $\bar{b} = -\frac{\sqrt{3}}{2}i + \frac{1}{2}j$

Angle between $\bar{a}$ and $\bar{b}$

$$= \cos^{-1}\frac{ab}{|a|\,|b|} = \cos^{-1}\frac{\left(\frac{\sqrt{3}}{2}i + \frac{1}{2}j\right)\left(\frac{-\sqrt{3}}{2}i + \frac{1}{2}j\right)}{\sqrt{\frac{3}{4}+\frac{1}{4}}\sqrt{\frac{3}{4}+\frac{1}{4}}}$$

$$= \cos^{-1}\left(-\frac{1}{2}\right) = 120°$$

**1.2 (a)**

The rank of matrix $m \times n$ is min{$m$, $n$}. Here, vector is linearly dedendent. ∴ Rank is less than $m$.

**1.3 (a)**

Let given matrix $A = \begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix}$

Characteristic equation $|A - \lambda I| = 0$

$$\begin{vmatrix} 3-\lambda & 4 \\ 4 & -3-\lambda \end{vmatrix} = 0$$

$$(3-\lambda)(-3-\lambda) - 16 = 0$$

$$-9 - 3\lambda + 3\lambda + \lambda^2 - 16 = 0$$

$$\lambda^2 - 25 = 0$$

$$\lambda = \pm 5$$

Eigen vector for eigen value $\lambda = -5$

$$\begin{bmatrix} 3+5 & 4 \\ 4 & -3+5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$8x_1 + 4x_2 = 0$$

$$4x_1 + 2x_2 = 0$$

∴ Eigen vector is $K\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

Eigen vector for $\lambda = 5$

$$\begin{bmatrix} 3-5 & 4 \\ 4 & -3-5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$-2x_1 + 4x_2 = 0$$

$$4x_1 - 8x_2 = 0$$

∴ Eigen vector is $K\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

**1.4 (a)**

Given matrix $A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

As we know that

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$|A| = 0(0-0) - 1(1-0) + 0(0-0) = -1$$

$$\text{Adj } A = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{\text{adj } A}{|A|} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**1.5 (b)**

The value of the given determinant

$$= 1\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} - 3\begin{vmatrix} 4 & 1 \\ 2 & 3 \end{vmatrix} + 2\begin{vmatrix} 4 & 1 \\ 2 & 1 \end{vmatrix}$$

$$= 1(3-1) - 3(12-2) + 2(4-2)$$

$$= 2 - 3(10) + 2(2)$$

$$= 6 - 30 = -24$$

**1.6 (b)**

Given system of linear equations

$$x_1 + 2x_2 - 2x_3 = 4$$

$$2x_1 + x_2 + x_3 = -2$$

$$-x_1 + x_2 - x_3 = 2$$

$$x_3 = \frac{\Delta_3}{\Delta} = \frac{\begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & -2 \\ -1 & 1 & 2 \end{bmatrix}}{\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix}} = \frac{12}{-6} = -2$$

**1.7 (c)**

For non-trivial solution

$$r < n$$

$$|A| = 0$$

$$\begin{vmatrix} 2 & 3 \\ 6 & 9 \end{vmatrix} = 0$$

$$29 - 18 = 0$$

$$9 = 9$$

**1.8 (a)**

As we know that

$$AX = \lambda X$$

$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \lambda\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \lambda\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$\therefore$ $$\lambda = 1$$

**1.9 (d)**

$$AB = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 4 & 6 \\ 5 & 9 \end{bmatrix} = \begin{bmatrix} 28 & 48 \\ 19 & 33 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 28 & 19 \\ 48 & 33 \end{bmatrix}$$

Hence, option (d) is right.

**1.10 (a)**

Given : $A(0, 4, 3)$, $B(0, 0, 0)$ and $C(3, 0, 4)$

$$\therefore \overrightarrow{BA} = 0\hat{i} - 4\hat{j} - 3\hat{k}$$

$$\overrightarrow{BC} = -3\hat{i} - 4\hat{k}$$

The vector perpendicular to $\overrightarrow{BA}$ and $\overrightarrow{BC}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -4 & -3 \\ -3 & 0 & -4 \end{vmatrix} = 16\hat{i} + 9j - 12\hat{k}$$

**1.11 (b)**

Characteristic equation $[A - \lambda I] = 0$

$$\begin{bmatrix} 10-\lambda & 4 \\ -18 & -12-\lambda \end{bmatrix} = 0$$

$$(10-\lambda)(-12-\lambda) + 72 = 0$$

$$-120 - 10\lambda + 12\lambda + \lambda^2 + 72 = 0$$

$$\lambda^2 + 2\lambda - 48 = 0$$

$$\lambda^2 + 8\lambda - 6\lambda - 48 = 0$$

$$\lambda(\lambda + 8) - 6(\lambda + 8) = 0$$

$$\lambda = 6, -8$$

**1.12 (c)**

$$[A|\,B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 2 & 1 & 2 & 5 \\ 1 & -1 & 1 & 1 \end{array}\right]$$

Applying $R_3 \to R_3 - R_1$ and $R_2 \to R_2 - 2R_1$

$$[A|\,B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & -3 & 0 & -3 \\ 0 & -3 & 0 & -3 \end{array}\right]$$

Applying $R_3 \to R_3 - R_2$

$$[A|\,B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & -3 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\rho(A) = \rho(A|\,B) < n$$

$\therefore$ Infinite number of solutions.

**1.13 (b)**

Characteristic equation $|A - \lambda I| = 0$

$$\begin{vmatrix} 5-\lambda & 3 \\ 1 & 3-\lambda \end{vmatrix} = 0$$

$$(5-\lambda)(3-\lambda) - 3 = 0$$

$$15 - 5\lambda - 3\lambda + \lambda^2 - 3 = 0$$

$$\lambda^2 - 8\lambda + 12 = 0$$

$$\lambda - 2\lambda - 6\lambda + 12 = 0$$

$$\lambda(\lambda - 2) - 6(\lambda - 2) = 0$$

$$\lambda = 2, 6$$

$$(A - 2I)X = 0$$

$$\begin{bmatrix} 3 & 3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_1 + x_2 = 0$$

Hence, required vector is $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$

**1.14 (c)**

Eigen value of symmetric matrix is real.

**1.15 (a)**

$$x + 2y + 4z = 2$$

$$4x + 3y + z = 5$$

$$3x + 2y + 3z = 1$$

$$[A|\,B] = \left[\begin{array}{ccc|c} 1 & 2 & 4 & 2 \\ 4 & 3 & 1 & 5 \\ 3 & 2 & 3 & 1 \end{array}\right]$$

Applying $R_2 \to R_2 \to 4R_1$ and $R_3 \to R_3 \to 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 2 \\ 0 & -5 & -15 & -3 \\ 0 & -4 & -9 & -14 \end{array}\right]$$

Applying $R_3 \to 5R_3 - 4R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 2 \\ 0 & -5 & -15 & -3 \\ 0 & 0 & 15 & 58 \end{array}\right]$$

# General Engineering

UNIT

II

## CONTENTS

# General Engineering

## *Syllabus*

***Engineering Materials:*** Structure, physical and mechanical properties, and applications of common engineering materials (metals and alloys, semiconductors, ceramics, polymers, and composites – metal, polymer and ceramic based); Iron-carbon equilibrium phase diagram; Heat treatment of metals and alloys and its influence on mechanical properties; Stress-strain behavior of metals and alloys.

***Applied Mechanics:*** Engineering mechanics – equivalent force systems, free body concepts, equations of equilibrium; Trusses; Strength of materials – stress, strain and their relationship; Failure theories; Mohr's circle (stress); Deflection of beams, bending and shear stresses; Euler's theory of columns; Thick and thin cylinders; Torsion.

***Theory of Machines and Design:*** Analysis of planar mechanisms, cams and followers; Governors and fly wheels; Design of bolted, riveted and welded joints; Interference/shrink fit joints; Friction and lubrication; Design of shafts, keys, couplings, spur gears, belt drives, brakes and clutches; Pressure vessels.

***Thermal and Fluids Engineering:*** Fluid mechanics – fluid statics, Bernoulli's equation, flow through pipes, laminar and turbulent flows, equations of continuity and momentum, capillary action; Dimensional analysis; Thermodynamics – zeroth, first and second laws of thermodynamics, thermodynamic systems and processes, calculation of work and heat for systems and control volumes; Air standard cycles; Heat transfer – basic applications of conduction, convection and radiation.

# 1 Engineering Materials

**1.1** Which one of the following cooling methods is best suited for converting Austenite steel into very fine Pearlite steel?
(a) Oil quenching (b) Water quenching
(c) Air cooling (d) Furnace cooling
[2007 : 1 M]

**1.2** Which one of the following is a heat treatment process for surface hardening?
(a) Normalising (b) Annealing
(c) Carburising (d) Tempering [2008 : 1 M]

**1.3** When 0.8% carbon eutectoid steel is slowly cooled from 750°C to room temperature,
(a) austenite transforms to pearlite
(b) pearlite transforms to austenite
(c) austenite transforms to martensite
(d) pearlite transforms to martensite [2008 : 1 M]

**1.4** A typical Fe-C alloy containing greater than 0.8% *C* is known as
(a) Eutectoid steel (b) Hypoeutectoid steel
(c) Mild steel (d) Hypereutectoid steel
[2009 : 1 M]

**1.5** The capacity of a material to absorb energy when deformed elastically, and to release it back when unloaded is termed as
(a) toughness (b) resilience
(c) ductility (d) malleability [2009 : 1 M]

**1.6** Anisotropy in rolled components is caused by
(a) change in dimensions
(b) scale formation
(c) closure of defects
(d) grain orientation [2009 : 1 M]

**1.7** Eutectic composition of iron-carbon alloy always corresponds to its
(a) lowest melting temperature
(b) highest melting temperature
(c) least carbon percentage
(d) higher fracture toughness [2010 : 1 M]

**1.8** As the weight percentage of carbon increases in plain carbon steel, its
(a) weldability decreases
(b) ductility improves
(c) tensile strength decreases
(d) formability improves [2010 : 1 M]

**1.9** Austempering is a heat treatment process that is aimed at obtaining
(a) martensitic steel
(b) bainitic steel
(c) tempered martensitic steel
(d) austenitic steel [2010 : 1 M]

**1.10** Which of the following is a surface (two-dimensional) imperfection in the crystal structure of common metals?
(a) Vacancy (b) Dislocation
(c) Grain boundary (d) Inclusion [2011 : 1 M]

**1.11** Match the following materials with their most appropriate application:

| Material | | Application |
|---|---|---|
| 1. Low carbon steel | P. | Machine tool base |
| 2. Stainless steel | Q. | Aircraft parts |
| 3. Gray cast iron | R. | Kitchen utensils |
| 4. Titanium alloys | S. | Car body panels |

(a) 1-P, 2-R, 3-Q, 4-S (b) 1-P, 2-R, 3-S, 4-Q
(c) 1-S, 2-Q, 3-P, 4-R (d) 1-S, 2-R, 3-P, 4-Q
[2011 : 2 M]

**1.12** During normalizing process of steel, the specimen is heated
(a) between the upper and lower critical temperature and cooled in still air.
(b) above the upper critical temperature and cooled in furnace.
(c) above the upper critical temperature and cooled in still air.
(d) between the upper and lower critical temperature and cooled in furnace.
[2012 : 1 M]

**1.13** For a ductile material, toughness is a measure of
(a) resistance to scratching
(b) ability to absorb energy upto fracture
(c) ability to absorb energy till elastic limit
(d) resistance to indentation [2013 : 1 M]

**1.14** Consider the following statements:
(P) Hardness is the resistance of a material to indentation.
(Q) Elastic modulus is a measure of ductility.
(R) Deflection depends on stiffness.
(S) The total area under the stress-strain curve is a measure of resilience.
Among the above statements, the correct ones are
(a) *P* and *Q* only (b) *Q* and *S* only
(c) *P* and *R* only (d) *R* and *S* only
[2016 : 1 M]

**1.15** With reference to Iron-Carbon equilibrium phase diagram, the crystal structure of 0.3% plain carbon steel at 1,100°C is
(a) HCP (b) BCT
(c) BCC (d) FCC [2017 : 1 M]

**1.16** When austenite decomposes upon cooling into two phases-ferrite and cementite, the reaction is called
(a) Eutectic (b) Eutectoid
(c) Peritectic (d) Peritectoid
[2018 : 1 M]

**1.17** Match the crystal structure in Column A with the corresponding packing fractions in Column B of the table

| Column A | | Column B | |
|---|---|---|---|
| 1 | Simple cubic | *P* | 0.74 |
| 2 | Hexagonal close-packed | *Q* | 0.68 |
| 3 | Body-centered cubic | *R* | 0.52 |
| 4 | Face-centered cubic | | |

(a) 1-*P*, 2-*R*, 3-*Q*, 4-*Q* (b) 1-*R*, 2-*P*, 3-*R*, 4-*Q*
(c) 1-*R*, 2-*P*, 3-*Q*, 4-*P* (d) 1-*P*, 2-*R*, 3-*P*, 4-*Q*
[2019 : 1 M]

**1.18** Group I lists phases of steel and Group II lists crystal structures in the table below :
Group I
P. Ferrite
Q. Austenite
R. Martensite
Group II
1. Hexagonal Close Packed (HCP)
2. Body Centered Cubic (BCC)
3. Body Centered Tetragonal (BCT)
4. Face Centered Cubic (FCC)

Match the phase with the corresponding crystal structure.
(a) P-2, Q-4, R-3 (b) P-4, Q-2, R-3
(c) P-2, Q-4, R-1 (d) P-4, Q-2, R-1
[2020 : 1 M]

**1.19** Pearlite microstructure in an eutectoid steel consists of alternating layers of two phases, namely a ferrite and
(a) Bainite (b) Cementite
(c) Martensite (d) Austenite [2021 : 1 M]

**1.20** Which one of the following metals has a face-centered cubic (FCC) structure?
(a) Chromium (b) Magnesium
(c) Aluminium (d) Alpha iron [2022 : 1 M]

**1.21** A eutectoid steel with 100% austenite is cooled from a temperature of 750°C to a room temperature of 35°C. Match the cooling methods with transformed structures.
Cooling method
P. Water quenching
Q. Oil quenching
R. Air cooling
S. Furnace cooling
Transformed structure
1. Coarse pearlite
2. Fine pearlite
3. Martensite
4. Very fine pearlite
(a) P-3, Q-4, R-1, S-2
(b) P-1, Q-2, R-3, S-4
(c) P-2, Q-3, R-4, S-1
(d) P-3, Q-4, R-2, S-1 [2022 : 1 M]

**1.22** Match the engineering materials at room temperature with the given crystal structures.

| Engg. material | | Crystal structure | |
|---|---|---|---|
| P. | Si | 1. | FCC |
| Q. | Fe | 2. | HCP |
| R. | Al | 3. | Diamond Cubic |
| S. | Zn | 4. | BCC |

(a) P-3, Q-4, R-1, S-2 (b) P-2, Q-1, R-4, S-3
(c) P-2, Q-4, R-1, S-3 (d) P-3, Q-1, R-4, S-2
[2023 : 2 M]

**1.23** As per the Fe-C phase diagram, the microstructure of plain carbon steel with 0.4 wt.% carbon at room temperature contains
(a) proeutectoid ferrite and pearlite
(b) proeutectoid cementite and pearlite
(c) ferrite and austenite
(d) austenite and cementite [2023 : 2 M]

**1.24** Which one of the following pure metals has the hexagonal close packed (HCP) crystal structure at room temperature?
(a) Magnesium (b) Iron
(c) Aluminium (d) Copper
[2024 : 1 Mark]

**1.25** For a mild steel specimen subjected to uniaxial tensile load, which of the following is/are TRUE?
(a) The engineering stress-strain curve is linear within the elastic limit.
(b) The specimen fails in cup and cone type fracture.
(c) The true stress is always more than the engineering stress at any finite strain.
(d) The specimen does not regain its original dimensions after complete unloading from an initial stress above the yield stress.
[2024 : 1 Mark]

**1.26** In the iron-carbon equilibrium phase diagram, the temperature and composition of the eutectoid point are 727°C and 0.77 weight % carbon, respectively. If a steel specimen with 1.2 weight % carbon is cooled from 1000°C to the room temperature, then the fraction of pro-eutectoid cementite phase in the steel is ____ (Rounded off to 2 decimal places)
(a) 0.07 (b) 0.93
(c) 0.18 (d) 0.12
[2024 : 2 Marks]

## Answers Engineering Materials

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.1 | (a) | 1.2 | (c) | 1.3 | (a) | 1.4 | (d) | 1.5 | (b) | 1.6 | (d) | 1.7 | (a) |
| 1.8 | (a) | 1.9 | (b) | 1.10 | (c) | 1.11 | (d) | 1.12 | (c) | 1.13 | (b) | 1.14 | (c) |
| 1.15 | (d) | 1.16 | (b) | 1.17 | (c) | 1.18 | (a) | 1.19 | (b) | 1.20 | (c) | 1.21 | (d) |
| 1.22 | (a) | 1.23 | (a) | 1.24 | (a) | 1.25 | (b, c, d) | 1.26 | (a) | | | | |

## Explanations Engineering Materials

**1.1 (a)**

Very fine pearlite is obtained when austenite is oil quenched. Martensite is obtained when water quenched. Fine pearlite is obtained in air cooling and coarse pearlite is obtained when furnace cooled.

**1.2 (c)**

Carburising is a process used to impart hardness on surface of component.

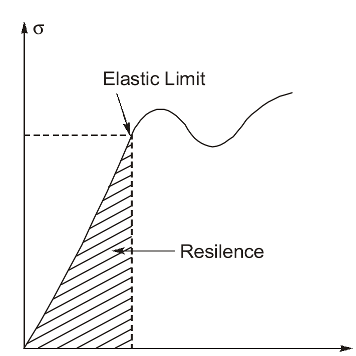
**1.3 (a)**

When 0.8% carbon eutectoid steel is slowly cooled from 750°C to room temperature, austenite is transferred to pearlite which is alternatie lamella of ferrite and cementite.

**1.4 (d)**

The iron carbon alloy that contain more than 0.8% C is known as hypereutectoid steel.

**1.5 (b)**




The capacity of a material to absorb energy when deformed elastically and to release it back when unloaded is called as resilience.

**1.6 (d)**

Anisotrophy in rolled components is observed due to grain orientation.

**1.7 (a)**

Eutectic composition of iron carbon alloy is ledeburite and it has lowest possible melting point of all iron-cementite alloys. As it can be seen in iron-carbon equilibrium diagram.

**1.8 (a)**

As carbon % is increase weldability decreases as its tendency to form martensite increases. Its ductility is reduced and brittleness increases as carbon % increases. Hardness and tensile strength increases, formability is reduced.

**1.9 (b)**

Austempering is a heat treatment process used to manufacture Bainite.

**1.10 (c)**

Grain boundary is a surface imperfection, vacancy, dislocation and inclusion are point defects in crystal.

**1.11 (d)**

1. Low carbon steel is used for car body panels.
2. Stainless steel is used for kitchen utensils.
3. Grey cast iron has property to damp vibration so used for machine tool base.
4. Titanium alloys are used for making aircraft parts.

**1.12 (c)**

Normalizing is a heat treatment process in which steel is heated 48–50°C above its upper critical temperature and if required holding at that temperature and then cooling in still air at room temperature.

**1.13 (b)**

Toughness is defined as ability of material to absorb energy during plastic deformation upto fracture.
It is total area of stress strain curve upto fracture.

**1.14 (c)**

(P) Hardness is the resistance of a material to indentation $\rightarrow$ correct

(Q) Elastic modulus is a measure of ductility $\rightarrow$ wrong

(R) Deflection depends on stiffness $\rightarrow$ correct because if a material undergo more strain (deflection) in elastic region means stiffness is low.

(S) The total area under the stress-strain curve is a measure of resilience $\rightarrow$ Wrong because it is called toughness.

**1.15 (d)**

By the reference of Iron-carbon equilibrium phase diagram FCC (Face Centroid Cubic) is the crystal structure of 0.3% plain carbon steel at 1100°C.

**1.16 (b)**

$M$ = Solid, $L$ = Liquid
Eutectic reaction $\rightarrow L1 \rightarrow M1 + M2$
Eutectoid reaction $\rightarrow M1 \rightarrow M2 + M3$
Peritectic reaction $\rightarrow L1 + M1 \rightarrow M2$
Peritectoid reaction $\rightarrow M1 + M2 \rightarrow M3$
In ques. Austenite (solid) decomposes into ferrite (solid) and cementite (solid).
So, it is Eutectoid reaction.

**1.17 (c)**

Atomic packing fraction (APF) is defined as follows:

$$\text{APF} = \frac{\text{Volume of the atoms in unit cell}}{\text{Volume of unit cell}}$$

Volume of the atoms in unit cell

$$= \text{Number of effective atoms} \times \frac{4}{3}\pi r^3$$

Volume of unit cell = $a^3$
For simple cubic,

$$\text{APF} = \frac{1\times\frac{4}{3}\pi r^3}{a^3}$$

We know that
for simple cubic, no. of effective atom = 1
and $a = 2r$

$$\text{APF} = 1\times\frac{4}{3}\frac{\pi r^3}{(2r)^3} = \frac{4}{3}\frac{\pi r^3}{8r^3} = \frac{\pi}{6} = 0.52$$

Similarly, for BCC, FCC and HCP, APF comes out to 0.68, 0.74 and 0.74 respectively. So 1–$R$, 2–$P$, 3–$Q$, 4–$P$. Correct option is (c).

**1.18 (a)**

Given : Group I with phase of steel and Group II with its crystal structure.
To find : A match between Group I and Group II.

P : Ferrite : Ferrite is BCC iron phase with very limited solubility for carbon.

Q : Austenite : Crystal structure of Austenite is FCC, i.e., Face Centered Cubic.

R : Martensite : Martensite has body-centered tetragonal crystal structure.

**1.20 (c)**

| Metals | | Structure |
|---|---|---|
| Chromium | → | BCC (at room temperature) |
| Magnesium | → | HCP |
| Aluminium | → | FCC |
| Alpha iron | → | BCC |

**1.21 (d)**

As cooling rate increases, the structure so obtained will become harder and harder thus,

- By water quenching martensite is formed
- By oil quenching very fine pearlite is formed
- By air cooling fine pearlite is formed
- By furnace cooling coarse pearlite is formed

**1.24 (a)**

The crystal structure of pure metals at room temperature are given as follows:

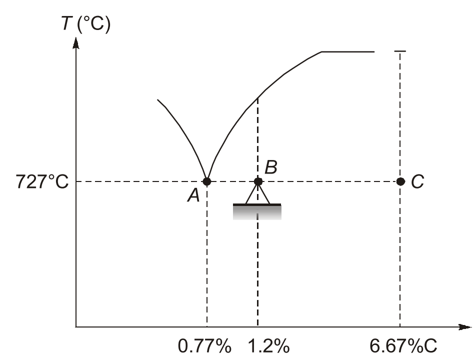
- Magnesium - HCP
- Iron - BCC
- Aluminium - FCC
- Copper - FCC

**1.25 (b, c, d)**

- The engineering stress-strain curve is linear within the proportional limit.
- Mild steel is a ductile material and hence it fails in cup and cone type fracture.
- Since the instantaneous area of cross-section is always less than the original one, true stress is always more than the engineering stress at any finite strain.
- The specimen can regain its original dimensions after complete unloading from an initial stress below the yield stress.

**1.26 (a)**

Given : composition of specimen, $C_0$ = 1.2%C
Composition of eutectoid point, $C_e$ = 0.77%C
Composition of cementite, $C_{Fe_3C}$ = 6.67%C
Applying lever rule just above 727°C,




$$f_{\text{Pro-Fe}_3\text{C}} = \frac{AB}{AC} = \frac{C_0 - C_e}{C_{Fe_3C} - C_e} = \frac{1.2 - 0.77}{6.67 - 0.77}$$

$$\simeq 0.07$$

■■■■