RRB
Railway Recruitment Board

Junior Engineer

CBT 1 : 2019
Computer Based Test - Stage 1

- Mathematics
- General Intelligence & Reasoning

Comprehensive Theory with Solved Examples, Practice Sets & Previous Years Questions

MADE EASY Publications
**Preface**

*Railway Recruitment Board-Junior Engineer* has always been preferred by Engineers due to job stability. Indian Railways is one of the biggest Government employers in India. With the exam being just a few months away, it is time for the candidates planning to appear for the exam to pull up their socks and start their RRB-JE preparation.

The RRB-JE exam is conducted in two stages as shown in the table given below.

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*Note:* There shall be negative marking for incorrect answers in CBTs. Each question carries 1 mark and 1/3rd of the marks allotted for each question shall be deducted for each wrong answer. Candidates shortlisted in Stage 1 are called for Stage 2.

MADE EASY has taken due care to provide complete solution with accuracy. Apart from Railway Recruitment Board-Junior Engineer, this book is also useful for Public Sector Examinations and other competitive examinations for engineering graduates.

I have true desire to serve student community by providing good source of study and quality guidance. I hope this book will prove as an important tool to succeed in RRB-JE and other competitive exams. Any suggestion from the readers for improvement of this book is most welcome.

With Best Wishes

**B. Singh**

CMD, MADE EASY
Exam Syllabus
(Computer Based Test 2019-First Stage)

**Mathematics:** Number systems, BODMAS, Decimals, Fractions, LCM and HCF, Ratio and Proportion, Percentages, Mensuration, Time and Work, Time and Distance, Simple and Compound Interest, Profit and Loss, Algebra, Geometry, Trigonometry, Elementary Statistics, Square Root, Age Calculations, Calendar & Clock, Pipes & Cistern.

**General Intelligence and Reasoning:** Analogies, Alphabetical and Number Series, Coding and Decoding, Mathematical operations, Relationships, Syllogism, Jumbling, Venn Diagram, Data Interpretation and Sufficiency, Conclusions and Decision Making, Similarities and Differences, Analytical reasoning, Classification, Directions, Statement – Arguments and Assumptions etc.

**General Awareness:** Knowledge of Current affairs, Indian geography, culture and history of India including freedom struggle, Indian Polity and constitution, Indian Economy, Environmental issues concerning India and the World, Sports, General scientific and technological developments etc.

**General Science:** Physics, Chemistry and Life Sciences (up to 10th Standard CBSE syllabus).
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Learning Objectives

After completion of this chapter, you should have a thorough understanding of the following:

- Basic concept of Brackets
- Order
- Division and Multiplication
- Addition and Subtraction
- Algebraic formulae
- Kinds of questions which are asked in RRB-JE
- Methods of solving.

Introduction to the topic

With respect to preparation for the RRB-JE Examination, BODMAS is an important topic in the Mathematics part of CBT-1 with a total of 1 question in the last conducted exam. This chapter will give you a clear understanding of the definitions and the concepts, and help you develop a keen insight about different kinds of BODMAS questions asked in the RRB-JE.

BODMAS, even though is a part of number systems yet, it has its own importance as far as this examination is concerned.

Type of Questions

Question asked from this topic are based on concepts:
Basic concept of BODMAS, Brackets, Orders, Division, Multiplication, Addition, Subtraction and Algebraic formulae.

BODMAS is a useful acronym that lets you know which order to solve mathematical problems (or sums). It’s important that you follow the rules of BODMAS as without it your answers can be wrong.

The BODMAS acronym is for

- Brackets (parts of a calculation inside brackets always come first).
- Orders (numbers involving powers or square roots).
- Division.
- Multiplication.
- Addition.
- Subtraction.

Brackets

Start with anything inside brackets, going from left to right.

Ex. 1 \(4 \times (3 + 2) = ?\)

Sol. You need to do the operation, or sum, inside the brackets first, \(3 + 2\), then multiply the answer by 4.
\[
3 + 2 = 5 \\
4 \times 5 = 20
\]
If you ignored the brackets and did the sum \(4 \times 3 + 2\) you would get 14. You can see how the brackets make a difference to the answer.

Orders

Do anything involving a power or a square root next (these are also known as orders), again working from left to right if there is more than one.

Ex. 2 \(3^2 + 5 = ?\)

Sol. You need to do the power sum first, before you can add 5.
\[
3^2 = 3 \times 3 = 9 \\
9 + 5 = 14
\]
Division and Multiplication

Once you have done any parts of the sum involving brackets or powers the next step is division and multiplication.

Multiplication and division rank equally, so you go from left to right in the sum, doing each operation in the order in which it appears.

Ex.3. 4 × 5 ÷ 2 + 7 = ?

Sol. You need to do division and multiplication first, but you have one of each.

Start from the left and work across to the right, which means that you start with 4 × 5 = 20. Then do the division, 20 ÷ 2 = 10.

Only then do you move to the addition:

10 + 7 = 17. The answer is 17.

Addition and Subtraction

The final step is to calculate any addition or subtraction. Again, subtraction and addition rank equally, and you simply move from left to right.

Ex.4. 4 + 6 − 7 + 3 = ?

Sol. You simply start on the left and work your way across.

4 + 6 = 10

10 − 7 = 3

3 + 3 = 6

Solved Examples

Q.1 (764 × ?) ÷ 250 = 382

(a) 115 (b) 145

(c) 135 (d) 125

Sol. (d)

\[
\frac{764 \times ?}{250} = 382
\]

\[
? = \frac{382 \times 250}{764} = 125
\]

Q.2 \(\frac{1}{4} \times (4856 \times 0.5) \times 12 = ?\)

(a) 7284 (b) 7462

(c) 7262 (d) 7414

Sol. (a)

\[
\frac{1}{4} \times (4856 \times 0.5) \times 12 = 7284
\]

Q.3 853 + ? ÷ 17 = 1000

(a) 2516 (b) 2482

(c) 2499 (d) 16147

Sol. (c)

\[
853 + \frac{?}{17} = 1000
\]

\[
\frac{?}{17} = 1000 - 853 = 147
\]

\[
? = 17 \times 147 = 2499
\]

Q.4 9643 − 7750 + ? = 4990

(a) 3079 (b) 3097

(c) 3090 (d) 4010

Sol. (b)

? = 4990 − 9643 + 7750 = 3097

Q.5 \(6156 \div \sqrt[3]{5} \times 53 = 4028\)

(a) 6889 (b) 6241

(c) 5929 (d) 6561

Sol. (d)

\[
6156 \times 53 = 4028
\]

\[
\Rightarrow \sqrt[3]{5} = 80.96 = 81 \Rightarrow ? = 6561
\]

Previous RRB-JE Questions

Q.1 Find the value of \(\frac{1}{3+\frac{2}{3+\frac{4}{3}}}\)

(a) \(\frac{148}{45}\) (b) 33

(c) \(\frac{38}{15}\) (d) \(\frac{143}{30}\)

[RRB (JE) 2014]

Sol. (a)

\[
3 + \frac{1}{3+\frac{2}{3+\frac{4}{3}}} = 3 + \frac{1}{3+\frac{2}{3+\frac{4}{3}}}
\]

\[
= 3 + \frac{1}{3 + \frac{6}{13}} = 3 + \frac{13}{45} = \frac{148}{45}
\]

Q.2 Solve the expression

\[
\frac{(2 + 3) \times 5 + 3 + \frac{1}{2}}{6 + 5 \times 4 + \frac{4}{5}}
\]
(a) 1  
(b) \( \frac{2}{5} \)  
(c) 31  
(d) 7

[RRA (JE) 2015]

Sol. (a) 
\[
\frac{(2 + 3) \times 5 + 3 + \frac{1}{2}}{6 + 5 \times 4 + \frac{4}{5}} = \frac{5 \times 5 + 3 \times 2}{6 + 5 \times 4 \times \frac{5}{4}} = \frac{25 + 6}{31} = \frac{31}{31} = 1
\]

Q.1 \( \frac{18}{3} + \frac{7}{2} = ? \)  
(a) 26 \( \frac{1}{3} \)  
(b) 19 \( \frac{1}{2} \)  
(c) 26 \( \frac{1}{6} \)  
(d) 25 \( \frac{2}{3} \)

Q.2 \( (38)^2 + (63)^2 + (x)^2 = 6089 \)  
(a) 26  
(b) 24  
(c) 28  
(d) 32

Q.3 \( -224 + (-314) \times (-9) = ? \)  
(a) -547  
(b) 2602  
(c) +547  
(d) -2602

Q.4 \( 8.7 + 6.2 \times 7.5 = ? \)  
(a) 55.04  
(b) 55.2  
(c) 66.48  
(d) 104.02

Q.5 64344 + 5239 + 4423 + 123 = ?  
(a) 74126  
(b) 74223  
(c) 74129  
(d) 75624

Q.6 135% of 342 - 342% of 13.5 = ?  
(a) 411.13  
(b) 412.23  
(c) 413.33  
(d) 415.53

Q.7 \( \sqrt{13.3225} = ? \)  
(a) 3.45  
(b) 3.55  
(c) 3.65  
(d) 3.75

Q.8 144 \times 7 + 612 \times 4 = ? \% of 12800  
(a) 24  
(b) 27  
(c) 30  
(d) 32

Q.9 \( \frac{1859}{7} = ? \)  
(a) 715  
(b) 725  
(c) 745  
(d) 775

Q.10 185% of 1359 + 18.5% of 1319 = ?  
(a) 2510  
(b) 2630  
(c) 2760  
(d) 2890

Solutions: BODMAS

1. (c) \( \frac{56}{3} + \frac{15}{2} = \frac{112 + 45}{6} = \frac{157}{6} = 26 \frac{1}{6} \)

2. (a) \( x^2 = 6089 - 1444 - 3969 = 676 \)  
\( x = 26 \)

3. (b) \( -224 + (-314) \times (-9) = -224 + 2826 = 2602 \)

4. (b) \( 8.7 + 6.2 \times 7.5 = 8.7 + 46.5 = 55.2 \)

5. (c) \( 64344 + 5239 + 4423 + 123 = 74129 \)

6. (d) \( ? = \frac{135 \times 342}{100} - \frac{342 \times 13.5}{100} = 461.7 - 46.17 = 415.53 \)

7. (c) \( \sqrt{13.3225} = 3.65 \)

8. (b) \( \frac{? \times 12800}{100} = 1008 + 2448 = 3456 \)  
\( ? = \frac{3456}{128} = 27 \)

9. (a) \( (?)^2 = 1859 \times 275 = 169 \times 11 \times 25 \times 11 \)  
\( (?)^2 = 25 \times 121 \times 169 \)  
\( ? = 5 \times 11 \times 13 = 715 \)

10. (c) \( ? = \frac{185 \times 1360}{100} + \frac{18.5 \times 1320}{100} = 2516 + 244.2 = 2760.2 \approx 2760 \)
Decimals

Learning Objectives

After completion of this chapter, you should have a thorough understanding of the following:

- Operations on decimal fractions
- Conversion of vulgar fraction into decimal fraction
- Multiplication of a Decimal Fraction by a Power of 10
- Comparison of fractions
- Recurring decimal - Pure and Mixed
- Division of decimal fraction by counting number
- Important formulae
- Kinds of questions which are asked in RRB-JE
- Methods of solving.

Introduction to the topic

With respect to preparation for the RRB-JE Examination, Decimals is an important topic in the Mathematics part of CBT-1. This chapter will give you a clear understanding of the definitions and the concepts, and help you develop a keen insight about different kinds of Decimals questions asked in the RRB-JE.

Type of Questions

Question asked from this topic are based on concepts:

Operations on decimal fractions, Conversion of vulgar fraction into decimal fraction, Comparison of fractions Recurring decimal - Pure and Mixed, Division of decimal fraction by counting number and Multiplication of a Decimal Fraction by a Power of 10.

1. Addition & Subtraction of Decimal Fractions

To add/subtract decimals,

(i) Write down the numbers, one under the other, with the decimal points lie in one column.
(ii) Now the numbers can be added normally (remember to put the decimal point in the answer).

Examples:

(a) \[1.3 + 0.24 = ?\]  
(b) \[1.25 + 0.1024 + 3 = ?\]

\[
\begin{array}{c}
1.3 \\
0.24 \\
\hline
1.54
\end{array}
\]
\[
\begin{array}{c}
1.25 \\
0.1024 \\
\hline
3
\end{array}
\]

\[
\begin{array}{c}
4.3524
\end{array}
\]

2. Multiplication of a Decimal Fraction by a Power of 10

Shift the decimal point to the right by as many places as the power of 10

For example:

(a) \[3.78 \times 10 = 37.8\]
(b) \[0.0043 \times 100000 = 430\]

3. Multiplication of Decimal Fractions

To multiply decimals,

(i) Multiply the given numbers considering them without decimal point.
(ii) In the product obtained, place the decimal point by starting at the right and moving a number of places equal to the sum of the decimal places in the given numbers.

Examples:

(a) \[4 \times .06 = ?\]

\[4 \times 6 = 24\]

Sum of the decimal places in the given numbers

\[= 0 + 2 = 2\]
Hence, answer = 0.24 

(b) \(2 \times .3 \times 0.005 = ?\) 
\[
2 \times 3 \times 5 = 30 \\
\text{Sum of the decimal places in the given numbers} \\
= 0 + 1 + 3 = 4 \\
\text{Hence, answer} = 0.0030 = 0.003

4. Division of Decimal Fraction by a Counting Number

To divide a decimal by a counting number,

(i) Divide the given numbers considering them without decimal point.

(ii) In the quotient obtained, place the decimal point by starting at the right and moving a number of places equal to the sum of the decimal places in the given numbers.

Examples:

(a) \(0.04 \div 2 = ?\) 
\[
4 \div 2 = 2 \\
\text{Sum of the decimal places in the given numbers} \\
= 2 + 0 = 2 \\
\text{Hence, answer} = .02

(b) \(0.008 \div 4 = ?\) 
\[
8 \div 4 = 2 \\
\text{Sum of the decimal places in the given numbers} \\
= 3 + 0 = 3 \\
\text{Hence, answer} = 0.002

5. Division of Decimal Fraction/Counting Number by a Decimal Fraction

To divide a decimal/counting number by a decimal,

(i) Multiply both the dividend and the divisor by a suitable power of 10 to make the divisor a whole number.

(ii) Now divide the numbers normally.

Examples:

(a) \(0.04 \div 0.2 = \) 
\[
0.04 \times 10 = 0.4 \\
0.2 \times 10 = 2 \\
\text{Given expression} = \frac{0.4}{2} = 0.2

(b) \(0.0009 \div 0.03 = ?\) 
\[
0.0009 \times 100 = 0.09 \\
0.03 \times 100 = 3 \\
\text{Given expression} = \frac{0.09}{3} = 0.03

(c) \(\frac{3}{0.03} = ?\) 
\[
0.03 \times 100 = 300 \\
0.3 \times 100 = 300 \\
\text{Given expression} = \frac{3}{3} = 100

(d) \(\frac{40}{0.002} = ?\) 
\[
0.002 \times 10000 = 2 \\
40 \times 10^3 = 40000 \\
\text{Given expression} = \frac{40000}{2} = 20000

Solved Examples

Q.1 Evaluate \(\frac{(2.39)^2 - (1.61)^2}{2.39 - 1.61}\) 
(a) 2 (b) 4 (c) 6 (d) 8

Sol. (b)
Given expression = \(\frac{a^2 - b^2}{a - b} = \frac{(a + b)(a - b)}{a - b}\) 
= \((a + b) = (2.39 + 1.61) = 4\)

Q.2 What decimal of an hour is a second? 
(a) 0.0025 (b) 0.0256 (c) 0.00027 (d) 0.000126

Sol. (c)
Required decimal = \(\frac{1}{60 \times 60} = \frac{1}{3600} = 0.00027\)

Q.3 When 0.232323..... is converted into a fraction, then the result is: 
(a) \(\frac{1}{5}\) (b) \(\frac{2}{9}\) (c) \(\frac{23}{99}\) (d) \(\frac{23}{100}\)

Sol. (c)

\(0.232323... = \frac{23}{99}\)

Q.4 \(\frac{0.009}{?} = 0.01\) 
(a) 0.0009 (b) 0.09 (c) 0.9 (d) 9

Sol. (c)
Let, \(\frac{0.009}{x} = 0.01\) 
Then, \(x = \frac{0.009}{1} = \frac{0.9}{1} = 0.9\)

Q.5 The expression \((11.98 \times 11.98 + 11.98 \times x + 0.02 \times 0.02)\) will be a perfect square for \(x\) equal to 
(a) 0.02 (b) 0.2 (c) 0.4 (d) 0.9

Sol. (c)
Given expression = \((11.98)^2 + (0.02)^2 + 11.98 \times x\) 
For the given expression to be a perfect square, we must have 
\(11.98 \times x = 2 \times 11.98 \times 0.02\) or \(x = 0.04\)
**Solutions: Decimals**

1. **(d)**
   Let, \(3889 + 12.952 - x = 3854.002\).
   Then, \(x = (3889 + 12.952) - 3854.002 = 3901.952 - 3854.002 = 47.95\)

2. **(b)**
   \(4 \times 162 = 648\). Sum of decimal places = 6.
   So, \(0.04 \times 0.0162 = 0.000648 = 6.48 \times 10^{-4}\)

3. **(b)**
   Given expression = \(\frac{(a^2 - b^2)}{(a + b)(a - b)} = \frac{(a^2 - b^2)}{(a^2 - b^2)} = 1\)

4. **(c)**
   \(0.125125 \ldots = \frac{125}{999}\)

5. **(b)**
   \(2 \times 5 = 10\)
   Sum of decimal places = 4
   \(0.002 \times 0.5 = 0.001\)

6. **(c)**
   \(3.14 \times 10^6 = 3.14 \times 1000000 = 3140000\).

7. **(d)**
   Given expression = \(\frac{8 - 2.8}{1.3} = \frac{5.2}{1.3} = \frac{52}{13} = 4\)

8. **(b)**
   Sum of decimal places = 7.
   Since the last digit to the extreme right will be zero (since \(5 \times 4 = 20\)), so there will be 6 significant digits to the right of the decimal point.

9. **(a)**
   \(\frac{0.0203 \times 2.92}{0.0073 \times 14.5 \times 0.7} = \frac{203 \times 292}{73 \times 145 \times 7} = \frac{4}{5} = 0.8\)

10. **(c)**
    \(\frac{4.036}{0.04} = \frac{403.6}{4} = 100.9\)
Learning Objectives

After completion of this chapter, you should have a thorough understanding of the following:

- Proper and Improper fractions
- Mixed fractions
- Elements of a fractions
- Simplification of fractions
- Comparison of fractions
- Multiplication rules for two fractions
- Operation of fractions
- Important formulae
- Kinds of questions which are asked in RRB-JE
- Methods of solving.

Introduction to the topic

With respect to preparation for the RRB-JE Examination, Fractions has been seen to be one of the important topics in the Mathematics part of CBT-1. This chapter will give you a clear understanding of the definitions and the concepts, and help you develop a keen insight about different kinds of Fractions asked in the RRB-JE.

Type of Questions

Question asked from this topic are based on concepts:
Types of fractions, Conversion into decimals, Equating denominators, Operations of fractions - addition, subtraction, multiplication, division, Multiplication rules for two fractions, Elements of a fractions, Simplification of fractions and Comparison of fractions.

The three types of fractions are

- Proper fraction
- Improper fraction
- Mixed fraction

1. **Proper fraction**

Fractions whose numerators are less than the denominators are called proper fractions. (Numerator < denominator)

For examples:
23, 34, 45, 56, 67, 29, 58, 25, etc are proper fractions.

Two parts are shaded in the above diagram. Total number of equal parts is 3. Therefore, the shaded part can be represented as $\frac{2}{3}$ in fraction. The numerator (top number) is less compared to the denominator (bottom number). This type of fraction is called proper fraction.

**Note:** The value of a proper fraction is always less than 1.

2. **Improper fraction**

Fractions with the numerator either equal to or greater than the denominator are called improper fraction. (Numerator = denominator or, Numerator > denominator)

Fractions like $\frac{5}{4}$, $\frac{17}{5}$, $\frac{5}{2}$ etc. are not proper fractions.

These are improper fractions. The fraction $\frac{7}{7}$ is an improper fraction.

The fractions $\frac{5}{4}$, $\frac{3}{2}$, $\frac{8}{3}$, $\frac{6}{10}$, $\frac{13}{15}$, $\frac{9}{20}$, $\frac{13}{14}$, $\frac{17}{17}$ are the examples of improper fractions. The top number (numerator) is greater than the bottom number (denominator). Such type of fraction is called improper fraction.
Notes:
(i) Every natural number can be written as a fraction in which 1 is its denominator. For example, \(\frac{2}{1} = 2\), \(\frac{25}{1} = 25\), \(\frac{53}{1} = 53\), etc. So every natural number is an improper fraction.
(ii) The value of an improper fraction is always equal to or greater than 1.

3. Mixed fraction
A combination of a proper fraction and a whole number is called a mixed fraction.

\[ 1\frac{1}{3}, 2\frac{1}{3}, 3\frac{2}{5}, 4\frac{2}{5}, \text{ and } 11\frac{1}{10} \] are examples of mixed fraction.

Two \(\frac{1}{2}\)'s make a whole.

\[ \frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1 \]

Ex.1 What will you get if you add one more \(\frac{1}{2}\) to a whole?

Sol. \[ \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2} = \frac{1}{2} \]

Now, you have three half or you can say that you have a whole and a half or \(\frac{1}{2}\).

Number such as \(1\frac{1}{2}\) is a mixed number.

In other words:
A fraction which consists of two parts:
(i) a natural number and
(ii) a proper fraction, is called a mixed fraction, e.g., \(3\frac{2}{5}, 7\frac{3}{4}\), etc.

In \(3\frac{2}{5}\), 3 is the natural number part and \(\frac{2}{5}\) is the proper fraction part. In fact, \(3\frac{2}{5}\) means \(3 + \frac{2}{5}\).

Note: A mixed number is formed with a whole number and a fraction.

Property 1:
A mixed fraction may always be converted into an improper fraction.

\[ 3\frac{1}{2} = \frac{3\times2+1}{2} = \frac{6+1}{2} = \frac{7}{2} \]

Multiply the natural number by the denominator and add to the numerator. This new numerator over the denominator is the required fraction.

Property 2:
An important fraction can be always be converted into a mixed fraction.

Divide the numerator by the denominator to get the quotient and remainder. Then the quotient is the natural number part and the remainder over the denominator is the proper fraction part of the required mixed fraction.

Ex.2 \(\frac{43}{6}\) can be converted into a mixed fraction as follows:

\[ \frac{6\frac{7}{42}}{1} \]

Dividing 43 by 6, we get quotient = 7 and remainder = 1.

Therefore, \(\frac{43}{6} = 7\frac{1}{6}\)

Elements of a fraction

The fraction \(\frac{a}{b}\) is composed of a numerator \(a\) and a denominator \(b\).

Equivalent fractions

It is important to remember that there are many ways to represent the same fraction. For example, the fractions \(\frac{1}{2}\) and \(\frac{2}{4}\) are equivalent. But how do we go from one fraction to another and conserve the equivalence relation?

A fraction remains equivalent if the numerator and the denominator are multiplied or divided by the same number.

e.g. \(\frac{2}{3} = \frac{2\times5}{3\times5} = \frac{10}{15}\) and \(\frac{24}{30} = \frac{24\div6}{30\div6} = \frac{4}{5}\).
Simplification of a fraction
A fraction is written in its simplified form if the numerator and the denominator have no common factor. In other words, it is impossible to find a number that is a divisor to both the numerator and the denominator in a fraction’s simplified form.

For example, the fraction $\frac{120}{200}$ is not written in its simplified form since there are numbers that divide both 120 and 200. The largest common divisor (factor) of 120 and 200 is 40, where

$$\frac{120}{200} = \frac{120 \div 40}{200 \div 40} = \frac{3}{5}$$

Since we divided the numerator and the denominator by the same number (40), the fraction $\frac{3}{5}$ is equivalent to $\frac{120}{200}$. In addition, $\frac{3}{5}$ is the simplified form of $\frac{120}{200}$ since no other common factor exists for 3 and 5.

A simplification can be done in many steps if we do not recognize at once the largest common factor for the numerator and the denominator.

For example:

$$\frac{108}{144} = \frac{108 \div 2}{144 \div 2} = \frac{54}{72} = \frac{54 \div 9}{72 \div 9} = \frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

$$\frac{108}{144} = \frac{108 \div 36}{144 \div 36} = \frac{3}{4}$$

In the end, no matter how many steps taken, the same simplified form will be found...

Rules for adding and subtracting fractions

$$\frac{a}{b} \pm \frac{c}{d} = \frac{a \pm c}{b}$$

The symbol $\pm$, which is read "plus or minus", indicates that this rule applies both to sums and subtractions.

For example:

$$\frac{3}{8} + \frac{7}{8} = \frac{3+7}{8} = \frac{10}{8} = \frac{10 \div 2}{8 \div 2} = \frac{5}{4}$$

$$\frac{5}{6} + \frac{7}{6} = \frac{5+7}{6} = \frac{12}{6} = \frac{12 \div 6}{6 \div 6} = \frac{2}{1}$$

Note that the adding and subtracting rule for fractions is applicable only if both fractions have the same denominator. However, this will generally not be the case. We will need to rewrite the fractions into equivalent fractions with a common denominator.

For example evaluate the following sum $\frac{2}{5} + \frac{1}{3}$.

These fractions cannot be added together before rewriting them with a common denominator. The smallest common multiple of 3 and 5 is 15. 15 will therefore be the common denominator.

$$\frac{2}{5} + \frac{1}{3} = \frac{2 \times 3}{5 \times 3} + \frac{1 \times 5}{3 \times 5} = \frac{6}{15} + \frac{5}{15} = \frac{6 + 5}{15} = \frac{11}{15}$$

Note: When possible, it may be useful to simplify fractions before proceeding with addition or subtraction. Such a simplification will facilitate finding a common denominator.

Multiplication rule for two fractions

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} = \frac{ac}{bd}$$

It is important to note that contrary to sums, the multiplication rule does not impose constraints to the denominator values. This means they do not need to be common.

For example:

$$\frac{4}{7} \times \frac{3}{11} = \frac{4 \times 3}{7 \times 11} = \frac{12}{77}$$

$$\frac{-3}{2} \times \frac{5}{4} = \frac{-3 \times 5}{2 \times 4} = \frac{-15}{8}$$

Note: It may be useful to simplify fractions before multiplying. In addition to simplifying each fraction individually, simplifying the denominator of one fraction with the numerator of the other fraction is permitted, provided that both have common factors.

Division rule of two fractions

$$\frac{a}{b} \div \frac{c}{d} = \frac{a \times \frac{d}{c}}{b \times \frac{d}{c}} = \frac{ad}{bc}$$

The rule allows us to transform a division into a multiplication.

For example:

$$\frac{2}{7} \div \frac{3}{8} = \frac{2 \times 8}{7 \times 3} = \frac{16}{21}$$

Pay attention to the following notation that describes the same division:

$$\frac{2/7}{3/8} = \frac{2 \times 8}{7 \times 3} = \frac{16}{21}$$
A few final remarks

- Working with fractions does not modify the priority of operations.
  For example:
  \[
  \frac{2}{3} + \frac{4}{5} \times \frac{2}{3} = \frac{2}{3} + \frac{8}{15} + \frac{8}{15} = \frac{10}{15} + \frac{8}{15} = \frac{18}{15} = \frac{6}{5}
  \]

- A whole number can always be written as a fraction if an operation is to be done between the number and a fraction.
  For example:
  \[
  8 \frac{5}{3} = \frac{8}{1} + \frac{5}{3} = \frac{24}{3} + \frac{5}{3} = \frac{39}{3} = \frac{13}{1}
  \]

- Avoid working with mixed numbers... transform them into simple fractions.
  For example:
  \[
  4 \frac{2}{7} = 4 + \frac{2}{7} = \frac{28}{7} + \frac{2}{7} = \frac{30}{7}
  \]

**Solved Examples**

**Q.1** \[2 \times ? - 6 = \frac{676}{26} \] What will come in place of question mark?

(a) 9  
(b) 15/26  
(c) 16  
(d) 26

**Sol.** (c)

\[
2 \times x - 6 = \frac{676}{26}
\]
\[
\therefore \quad 2 \times x - 6 = 26
\]
\[
2 \times x = 32
\]
\[
\therefore \quad x = 16
\]

**Q.2** An integer is 10 more than its one-third part. The integer is

(a) 15  
(b) 12  
(c) 18  
(d) 25

**Sol.** (a)

Let integer be \( M \)

\[
\Rightarrow \quad M - \frac{M}{3} = 10
\]
\[
\therefore \quad M = 15
\]

**Q.3** Convert 0.88 into vulgar fraction

(a) 80/99  
(b) 44/45  
(c) 22/25  
(d) 8/9

**Sol.** (d)

\[
0.88 = \frac{88}{90} = \frac{8}{9}
\]

**Q.4** Which of the following fraction is the smallest?

(a) 12/14  
(b) 13/19  
(c) 17/21  
(d) 7/8

**Sol.** (b)

\[
\frac{12}{14} = 0.857 \quad ; \quad \frac{13}{19} = 0.684
\]
\[
\frac{17}{21} = 0.8095 \quad \text{and} \quad \frac{7}{8} = 0.875
\]

Since 0.684 is the smallest, so 13/19 is the smallest fraction.

**Q.5** Which of the following fraction is greater than 2/3 and less than 4/5?

(a) \( \frac{1}{2} \)  
(b) \( \frac{9}{10} \)  
(c) \( \frac{3}{4} \)  
(d) \( \frac{5}{6} \)

**Sol.** (c)

\[
\frac{1}{2} = 0.5 \quad ; \quad \frac{2}{3} = 0.66 \quad ; \quad \frac{4}{5} = 0.8 \quad ; \quad \frac{9}{10} = 0.9
\]
\[
\frac{3}{4} = 0.75 \quad , \quad \frac{5}{6} = 0.833,
\]

Clearly, 0.8 lies between 0.75 and 0.833

3/4 lies between 2/3 and 4/5

**PRACTICE SET: QUESTIONS**

**Q.1** When Rs 250 added to 1/4th of a given amount of money makes it smaller than 1/3rd of the given amount of money by Rs 100. What is the given amount of money?

(a) 350  
(b) 600  
(c) 4200  
(d) 3600

**Q.2** A boy was asked to find the value of \( \frac{7}{12} \) of a sum of money. Instead of multiplying the sum by \( \frac{7}{12} \), he divided it by \( \frac{7}{12} \) and thus his answer exceeded the correct value by Rs.95. Find the correct value?

(a) 95  
(b) 49  
(c) 84  
(d) None of these

**Q.3** Eight people are planning to share equally the cost of a rental car, if one person withdraws from the arrangement and the others share equally the entire cost of the car, then the share of each of the remaining persons increased by?

(a) 3/2  
(b) 7/8  
(c) 5/7  
(d) None of these
Q.3 2 trees are there. One grows at 3/5 of the other. In 4 years total growth of the trees is 8 ft. what growth will smaller tree have in 2 years?
(a) 1.2   (b) 1.3   (c) 1.5   (d) 2

\[ 144K - \frac{49K}{84} = 95 \Rightarrow K = 84 \]
\[ \frac{7}{12K} \times 84 = 49 \]

Q.5 In a pair of fractions, fraction A is twice the fraction B and the product of two fractions is 2/25. What is the value of fraction A?
(a) 1/5   (b) 1/25   (c) 2/5   (d) data inadequate

Q.6 A fraction in which nominator is 4 less than denominator. When 10 is added to the denominator the fraction becomes 1/8. What is the original fraction?
(a) 1/4   (b) 2/9   (c) 2/6   (d) None of these

Q.7 \( x \) % of \( y \) is \( y \)% of ?
(a) \( x/y \)   (b) \( 2y \)   (c) \( x \)   (d) Cannot be determined

Q.8 Express the 4% as a fraction.
(a) 11/25   (b) 1/25   (c) 10/25   (d) 13/25

Q.9 The difference between the numerator and the denominator of a fraction is 5. If 5 is added to its denominator, the fraction is decreased by \( \frac{1}{4} \).

Find the value of the fraction.
(a) \( \frac{1}{6} \)   (b) \( \frac{2}{1} \)   (c) \( 3\frac{1}{2} \)   (d) None of these

Q.10 Which of the following is larger than \( 3/5 \)?
(a) 39/50   (b) 1/2   (c) 59/100   (d) 3/10

Solutions: Fractions

1. (c)
   - Let the given amount be \( \mathbf{r} \mathbf{x} \)
   \[ \frac{x}{3} - \frac{x}{4 + 250} = 100 \]
   \[ x = \mathbf{r} 4200 \]

2. (b)
   - Let, \( \text{Sum} = \mathbf{r} \mathbf{K} \)
   \[ \frac{12}{7} \mathbf{K} - \frac{7 \mathbf{K}}{12} = 95 \]

3. (d)
   - Original share of one person = \( 1/8 \)
   - New share of one person = \( 1/7 \)
   - Increase = \( 1/7 \times 1/8 = 1/56 \)
   - Required fractions = \( (1/56)/(1/8) = 1/7 \)

4. (c)
   - The big tree grows 8 ft in 4 years \( \rightarrow \) the big tree grows 4ft in 2 years when we divide
   \[ 4\mathbf{F}/5 = 8 \times 3 = 2.4 \]
   \[ 4\left(x + \frac{3}{5}x\right) = \frac{88x}{5} = 2x = \frac{5}{4} \]
   - After 2 years \( x = \left(\frac{3}{5}\right) \times \left(\frac{5}{4}\right) \times 2 = 1.5 \)

5. (c)
   - \( A = 2B \); \( B = 1/2A \)
   - \( AB = 2/25 \)
   - \( (1/2)A^2 = 2/25 \); \( A^2 = 4/25 \)
   - \( A = 2/5 \)

6. (c)
   - Let nominator be \( x \), denominator be \( x + 4 \) and fraction be \( y \) then \( y = \frac{x}{x + 4} \) on adding 10 to denominator \( y = \frac{1}{8} \)
   \[ \Rightarrow \frac{x}{x+14} = \frac{1}{8} \]
   \[ \Rightarrow x = 2 \]
   \[ \therefore \ y = 2/6 \]

7. (c)
   - \( x \)% of \( y = \frac{(x/100)y}{(y/100)} = x \)
   - \( x \)% of \( x \)
   - So, \( x \) is the answer.

8. (b)
   - \( 4/100 = 1/25 = 0.04 \)

9. (b)
   - Let the denominator be \( x \).
   - Then, numerator = \( x + 5 \)
   - Now, \( \frac{x+5}{x} - \frac{(x+5)}{x} = \frac{5}{4} + 1 = \frac{9}{4} = 2 \frac{1}{4} \)

10. (a)
    - \( 3/5 = 6/10, 1/2 = 5/10, 39/50 = 7.8/10, 59/100 = 5.9/10 \)
    - \( 7.8/10 > 6/10 \), so, \( 39/50 > 3/5 \).