Indian Forest Service Main Examination
(2001-2018)

Mechanical Engineering
Paper-I

Also useful for Engineering Services Main Examination,
Civil Services Main Examination and
various State Engineering Services Examinations

MADE EASY Publications
Preface

Our country has a vast forest cover of near about 25% of geographical area and if man doesn't learn to treat trees with respect, man will become extinct; Death of forest is end of our life. Scientific management and judicial exploitation of forest becomes first task for sustainable development.

Engineer is one such profession which has an inbuilt word “Engineer – skillful arrangement” and hence IFS is one of the most talked about jobs among engineers to contribute their knowledge and skills for the arrangement and management for sustainable development.

In order to reach to the estimable position of Divisional Forest Officer (DFO), one needs to take an arduous journey of Indian Forest Service Examination. Focused approach and strong determination are the pre-requisites for this journey. Besides this, a good book also comes in the list of essential commodity of this odyssey.

I feel extremely glad to launch the revised edition of such a book which will not only make Indian Forest Service Examination plain sailing, but also with 100% clarity in concepts.

MADE EASY team has prepared this book with utmost care and thorough study of all previous years’ papers of Indian Forest Service Examination. The book aims to provide complete solution to all previous years’ questions with accuracy.

On doing a detailed analysis of previous years’ Indian Forest Service Examination question papers, it came to light that a good percentage of questions have been asked in Engineering Services, Indian Forest Services and State Services exams. Hence, this book is a one stop shop for all Indian Forest Service Examination, CSE, ESE and other competitive exam aspirants.

I would like to acknowledge efforts of entire MADE EASY team who worked day and night to solve previous years’ papers in a limited time frame and I hope this book will prove to be an essential tool to succeed in competitive exams and my desire to serve student fraternity by providing best study material and quality guidance will get accomplished.

With Best Wishes

B. Singh
CMD, MADE EASY Group
## Previous Years Solved Papers

### Indian Forest Service Main Examination

#### Mechanical Engineering

### Paper-I

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Syllabus

Paper - I


5. MANUFACTURING MANAGEMENT: Production Planning and Control, Forecasting-moving average, exponential smoothing, Operations scheduling; assembly line balancing. Product development, Breakeven analysis, Capacity planning. PERT and CPM. Control Operations: Inventory control-ABC analysis, EOQ model. Materials requirement planning, Job design, Job standards, work measurement, Quality management-Quality control Operations Research: Linear programming-Graphical and Simplex methods, Transportation and assignment models, Single server queuing model.

Value Engineering: Value analysis, cost/ value, Total quality management and forecasting techniques. Project management.

Q.1 Illustrate the use of discs of uniform strength in industry. Derive an expression for the profile of disc of uniform strength. How does it differ from those for disc of uniform thickness?

[IFS (Mains) 2001: 16 Marks]

Solution:

Use of disc: In applications such as turbine blades rotating at high speeds, it is often desirable to design for constant strength as the thickness varies exponentially which results in optimum material usage.

Rotors of steam or gas turbines, on the periphery of which blades are attached, are designed as disc of uniform strength, in which the stress developed due to centrifugal forces are equal and constant independent of radius. In order to achieve the objective of uniform strength throughout, thickness of the disc is varied along the radius. Consider a disc of radius \( R \), rotating at angular speed \( \omega \) about its axis. Take a small element \( abcd \) subtending an angle \( d\theta \) at the centre of the disc, disc is of uniform strength. Stress on faces \( ab, bc, cd \) and \( da \) of the small element is \( \sigma \). Thickness of the element at radius \( r \), is \( t \) and at radius \( r + dr \), thickness is \( t + dt \). (\( \rho \) is assumed as specific weight i.e., N/m\(^3\)).

Volume of element = \( r d\theta t dr \)

Mass of element = \( \rho \frac{rd\theta t dr}{g} \)

Centrifugal force, \( CF = \rho r \frac{d\theta dr}{g} \times \omega^2 r = \frac{\rho \omega^2 r^2 (d\theta dr)}{g} \)

Radial force on face \( ab = r d\theta t \sigma \)

Radial force on face \( cd = (r + dr)d\theta(t + dt) \sigma \)

Force on faces \( bc \) and \( da = \sigma t dr \)

where \( \sigma \) is uniform stress

Resolving all forces along the radial direction OCF:

\[
\frac{\rho \omega^2 r^2}{g} (t dr d\theta) + \sigma (r + dr)(t + dt) d\theta = \sigma rd\theta t + 2\sigma t dr \sin \frac{d\theta}{2}
\]

As \( d\theta \) is very small \( \left( \sin \frac{d\theta}{2} = \frac{d\theta}{2} \right) \) So,

\[
\frac{\rho \omega^2 r^2 t}{g} dr + \sigma r t + (\sigma dr t) + \sigma rd t + \sigma dr dt = \sigma t + \sigma dr t
\]

Neglecting \((dr \times dt)\) as negligible,

\[
\frac{\rho \omega^2 r^2 t}{g} dr + \sigma r dt = 0
\]
or
\[
\frac{dt}{t} = -\frac{\rho \omega^2 r}{g} \, dr
\]
\[
\int \frac{dt}{t} = -\int \frac{\rho \omega^2 r}{g} \, dr
\]
\[
\ln t = -\frac{\rho \omega^2 r^2}{2g} + \ln A \quad (A = \text{constant})
\]

or
\[
\ln \left( \frac{t}{A} \right) = -\frac{\rho \omega^2 r^2}{2gA} \quad \text{or} \quad \frac{t}{A} = e^{-\frac{\rho \omega^2 r^2}{2gA}}
\]

At centre, \( r = 0 \), and \( t = t_0 \)
\[
t_0 = A
\]
and profile thickness varies as,
\[
t = t_0 e^{-\frac{\rho \omega^2 r^2}{2gA}}
\]

Hence, the thickness of the disc is varied along the radius to achieve the objective of uniform strength. Whereas, disc of uniform thickness has constant thickness throughout the radius.

Q.2 With reference to Figure, neglect the weight of the bar and stopper. A load \( W \) is dropped from a height \( h \). Given \( W = 1.0 \) kN, area of cross-section of the bar = 20 mm\(^2\). Find the instantaneous stress developed in the bar when the weight \( W \) is dropped from a height \( h = 0 \).

[IFS (Mains) 2002 : 10 Marks]

Solution:

Let I.F. be the impact factor,
\[
\text{I.F.} = 1 + \sqrt{1 + \frac{2h}{\delta_{\text{static}}}} \quad \left\{ \delta_{\text{Static}} = \frac{WL}{EA} \right\}
\]

and
\[
\sigma_{\text{impact}} = \sigma_{\text{static}} \times \text{I.F.}
\]

when, \( (h = 0) \)
\[
\text{I.F.} = 1 + \sqrt{1 + 0} = 2
\]

and
\[
\sigma_{\text{static}} = \frac{W}{A}
\]

where
\[
W = 1.0 \, \text{kN}, \quad A = 20 \, \text{mm}^2
\]

\[
\therefore \quad \sigma_{\text{static}} = \frac{W}{A} = \frac{1 \times 10^3}{20} \, \text{N/mm}^2 = 50 \, \text{MPa}
\]

Therefore,
\[
\sigma_{\text{instantaneous}} = \sigma_{\text{impact}} = 2 \times \sigma_{\text{static}}
\]
\[
\sigma_{\text{impact}} = 2 \times 50 = 100 \, \text{MPa}
\]

Thus, instantaneous stress developed in the bar is 100 MPa

Q.3 As shown in figure, \( OE \) is a rigid beam, hinged at \( D \) and kept horizontal by two vertical supports \( AB \) and \( CD \).
If hot gases pass through the brass tube to raise its temperature by 150°C, find the load supported by both the vertical bars, given $OB = BD = DE$. Assume that before assembly, the brass bar is 1 mm too short and $W = 10$ kN.

[IFS (Mains) 2002 : 10 Marks]

Solution:

Given that,

$$OB = BD = DE = x \quad \text{(let)}$$

From $\Delta OBB'$ and $\Delta ODD'$

$$\frac{OB}{BB'} = \frac{OD}{DD'} \Rightarrow \frac{x}{BB'} = \frac{2x}{DD'}$$

$$DD' = 2BB'$$

$$\Rightarrow$$

$$\delta_{CD} = 2 \delta_{AB} \quad \text{...(i)}$$

$$A_b = \frac{\pi}{4} (15^2 - 10^2) = 98.17 \text{ mm}^2$$

$$A_s = \frac{\pi}{4} 5^2 = 19.63 \text{ mm}^2$$

FBD of beam $OE$:

Taking moment about $O$:

$$R_s \times x + R_b \times 2x = W \times 3x$$

$$\Rightarrow$$

$$R_s + 2R_b = 3 \times 10 = 30 \text{ kN} \quad \text{...(ii)}$$

Bar CD initially 1 mm short,

Net elongation in bar $CD = -1 + \text{Elongation due to temperature rise} + \text{Elongation due to load}$

$$= -1 + \alpha_b T_b L_b + \frac{R_b L_b}{A_b \times E_b}$$

$$= -1 + 20 \times 10^{-6} \times 150 \times 1.5 \times 10^3 + \frac{R_b \times 10^3 \times 1.5}{98.17 \times 110}$$

$$= -1 + 4.5 + 0.1389 R_b$$

$$\delta_{CD} = 3.5 + 0.1389 R_b \quad \text{...(iii)}$$

Net elongation in bar $AB = \frac{R_s \times L_s}{A_s \times E_s} = \frac{R_s \times 1.2 \times 10^3}{19.63 \times 200} = 0.3056 R_s \text{ mm}$

$$\delta_{AB} = 0.3056 R_s \text{ mm}$$

From equation (ii),

$$\delta_{AB} = (30 - 2R_b) \times 0.3056 \quad \text{...(iv)}$$

From equations (iii) and (iv) putting the values of $\delta_{AB}$ adn $\delta_{CD}$ in equation (i),

$$3.5 + 0.1389 R_b = 2 \times 0.3056 \times (30 - 2R_b)$$

$$\Rightarrow$$

$$R_b = 10.898 \text{ kN}$$

and

$$R_s = 30 - 2 \times 10.898 = 8.204 \text{ kN}$$

The load supported by brass bar = 10.898 kN

The load supported by steel bar = 8.204 kN

Q.4 Explain clearly the meanings of terms:

Elastic solid, non-elastic solid, inelastic solid, linearly elastic solid, and nonlinearly elastic solid.

Illustrate your answer using a diagram showing the load-deformation relationship for the above solids.

[IFS (Mains) 2003 : 10 Marks]
Elastic solid: Solids which are capable of recovering size and shape after deformations.

Non-elastic solids: Solids which can not regain their original shape and after unloading.

Inelastic solids: Solids which do not deform under the application of force.

Linearly elastic solids: Solid which deform linearly proportionally to load. They also follow Hook's law, \( \sigma \propto \varepsilon \).

Non-linearly elastic solids: Elastic solids for which deformations are not linearly proportional to applied load. In these types of solid deformations follows power law.

Q. 5 A steel bar of diameter 60 mm and length 300 mm is subjected to an axial compressive load of 50 kN. To what diameter the middle one-third length of the bar be reduced in order to increase the stored energy by 50%?

[IFS (Mains) 2004 : 10 Marks]

Solution:

Given:

Initially, uniform steel bar,

\( d = 60 \text{ mm}, \ l = 300 \text{ mm}, \) Compressive load, \( P = 50 \text{ kN} \)

Strain energy stored, \( U_1 = \frac{P^2L}{2AE} \)

Let \( d_1 \) be the new reduced diameter of \((1/3)\)th length of the bar.

\[
U_2 = \frac{P^2}{2AE} \left( \frac{L}{3} \right) + \frac{P^2}{2AE} \left( \frac{L}{3} \right) + \frac{P^2}{2AE} \left( \frac{L}{3} \right) = \frac{P^2L}{3AE} + \frac{P^2L}{6AE}
\]

and

\[
U_2 = 1.5 \ U_1 \ (\text{Given})
\]

\[
\frac{P^2L}{3AE} + \frac{P^2L}{6AE} = 1.5 \times \frac{P^2L}{2AE}
\]

\[
\frac{1}{3A} + \frac{1}{6A_1} = \frac{1.5}{2A}
\]

\[
\frac{1}{6A_1} = \frac{0.4166}{A}
\]

\( A_1 = 0.4 \ A \)

\( d_1^2 = 0.4 \times (60)^2 \)

\( d_1 = 37.947 \text{ mm} \)

Thus, the diameter of middle 1/3 length be reduced to 37.947 mm
Q.6 A rectangular plate of thickness 10 mm carries tensile normal stresses of $\sigma_1 = 600\, \text{MPa}$ and $\sigma_2 = 200\, \text{MPa}$ on two perpendicular planes on which there are no shear stresses. Obtain the change in thickness of the plate. Take $E = 200 \times 10^9\, \text{Pa}$ and $G = 80 \times 10^9\, \text{Pa}$.

Solution:

Given: A rectangular plate of 10 mm thickness.
$\sigma_1 = 600\, \text{MPa}, \sigma_2 = 200\, \text{MPa}, \sigma_3 = 0, E = 200 \times 10^9\, \text{Pa}, G = 80 \times 10^9\, \text{Pa}$

As,
\[ G = \frac{E}{2(1+\nu)} \]
or
\[ 1 + \nu = \frac{E}{2G} = \frac{200 \times 10^9}{2 \times 80 \times 10^9} \]
Poisson’s ratio, $\nu = 0.25$

\[ \varepsilon_3 = \frac{1}{E} [\sigma_3 - \nu(\sigma_1 + \sigma_2)] = \frac{1}{200 \times 10^9} \times [0 - 0.25(600 + 200)] \quad (\because \sigma_3 = 0) \]
\[ \varepsilon_3 = -10^{-3} \]
\[ \delta = -10^{-3} \times 10 = -10^{-2} \, \text{mm} = -0.01 \, \text{mm} \]

Thus, the thickness of plate is reduced by 0.01 mm.

Q.7 Calculate the change in volume of a cube of steel with sides measuring 15 cm when it is immersed to a depth of 800 m in sea water which weighs 10 kN/m$^3$. Take $E = 200\, \text{GPa}$ and Poisson’s ratio $\nu = 0.28$.

Solution:

Given: Side of cube, $a = 15\, \text{cm}$, Depth, $h = 800\, \text{m}$, $w = 10\, \text{kN/m}^3$, $E = 200\, \text{GPa}$, $\nu = 0.28$

When a cube is immersed to a depth of 800 m in sea water, it is subjected to hydrostatic forces.

Pressure, $P = \rho gh = w \times h = 10 \times 10^3 \times 800\, \text{N/m}^2 = 8 \times 10^6\, \text{N/m}^2$

Bulk density, $K = \frac{E}{3(1-2\nu)} = \frac{200 \times 10^9}{3(1-2\times0.28)} = 151.51\, \text{GPa}$

Volumetric strain, $\varepsilon_v = \frac{P}{K} = \frac{8}{151.51 \times 10^9} = 0.0528 \times 10^{-3}$

$V = a^3 = (15)^3 = 3375\, \text{cm}^3$

$\varepsilon_v = \frac{\Delta V}{V}$.

Change in volume, $\Delta V = \varepsilon_v \times V = 0.0528 \times 10^{-3} \times 3375 = 0.1782\, \text{cm}^3$

Therefore, the change in volume of cube is 0.1782 $\text{cm}^3$.

Q.8 With the help of sketches, show the stress-strain relationship for both ductile and brittle materials. Indicate the following points on the relevant sketches:

Draw the stress-strain diagram for a low-carbon-steel show on it various events.

[IFS (Mains) 2006 : 10 Marks]
Solution:

Ductile material

When a yield point is not easily defined based on the shape of the stress-strain curve an offset yield point is arbitrarily defined. The value for this is commonly set at 0.002 (0.2%) plastic strain. This corresponding stress is proof stress.

For brittle material

These materials will fail with only little elongation after the proportional limit (point A) is exceeded, and the fracture stress (point B) is the same as ultimate stress.

Q.9 Determine the rise in temperature in order to induce buckling in a 1.0 metre long circular rod of diameter 40 mm. Assume the rod to be pinned at its ends and the coefficient of thermal expansion is \(20 \times 10^{-6}/\degree C\). Assume also uniform heating of the rod.

[IFS (Mains) 2006 : 8 Marks]

Solution:

Given: Length, \(L = 1 \text{ m}\), Diameter, \(d = 40 \text{ mm}\), \(\alpha = 20 \times 10^{-6}/\degree C\)

Let \(\Delta T\) be the rise in temperature

Thermal strain, \(\varepsilon_l = \alpha(\Delta T)\)

Thermal stress, \(\sigma_l = \varepsilon_lE = \alpha E(\Delta T)\)

Corresponding to the stress, load \((P)\)

\[ P = \sigma_l(A) \]
\[ P = \alpha EA(\Delta T) \]

For column to buckle,

For pin joints at ends, \(P_c = \frac{\pi^2EI}{L^2}\)
So,
\[ \frac{\pi^2 EI}{L^2} = \alpha EA(\Delta T) \]
\[ \frac{\pi^2}{64} \times \frac{\pi}{(40)^4} \times \frac{1}{(1000)^2} = 20 \times 10^{-6} \times \frac{\pi}{4} \times (40)^2 \times \Delta T \]
\[ \Delta T = 49.35^\circ C \]

Thus, the temperature rise required to induce buckling is 49.35°C.

Q.10 A compound cylinder is formed with inner diameter = 300 mm; the diameter at the junction = 400 mm and outer diameter = 500 mm. If the initial interference in diameters at the junction is 0.2 mm, find the radial pressure developed at the junction.

Find also the minimum temperature to which the outer cylinder is to be heated to slip it onto the inner cylinder.

Take \( E = 2 \times 10^5 \text{ N/mm}^2 \) and \( \alpha = 12.5 \times 10^{-6}^\circ \text{C} \).

*IFS (Mains) 2007 : 20 Marks*

**Solution:**

Given: \( D_1 = 300 \text{ mm}, \ D_2 = 400 \text{ mm}, \ D_3 = 500 \text{ mm}, \ \delta = \text{ initial interference}, \ P = \text{Radial pressure} \)

Due to interference, let us assume,
\[ \delta_i = \text{ increase in inner diameter of outer cylinder} \]
\[ \delta_o = \text{ decrease in outer diameter of inner cylinder} \]
\[ \delta = |\delta_i| + |\delta_o|, \ \text{i.e., without sign} \]
\[ \delta_i = \epsilon_i D_2 \quad (\epsilon_i = \text{tangential strain}) \]

\[ \delta_i = \frac{1}{E} (\sigma_i - v\sigma_r) D_2 = \frac{D_2 P}{E} \left( \frac{D_3^2 + D_2^2}{D_3^2 - D_2^2} + \nu \right) \quad \ldots(i) \]

\[ \therefore \quad \sigma_i = \text{Circumferential stress} = P \left( \frac{D_3^2 + D_2^2}{D_3^2 - D_2^2} \right) \]
\[ \sigma_r = -P \ (\text{radial stress}) \]

And in similar way,
\[ \delta_o = \epsilon_o D_2 \left( r_2 = \frac{D_2}{2} \right) \]
\[ = \frac{1}{E} (\sigma_i - v\sigma_r) D_2 \]
\[ \sigma_i = -P \left( \frac{D_2^2 + D_3^2}{D_3^2 - D_2^2} \right) \]
\( \sigma_r = -P \)
\[ \delta_o = -\frac{D_o P}{E} \left( \frac{D_o^2 + D_i^2}{D_o^2 - D_i^2} - \nu \right) \] ... (ii)

Here, negative sign represents contraction
Adding equations (i) and (ii)

\[ \delta = |\delta_i| + |\delta_o| \]

As,
\[ \delta = \frac{PD_o}{E} \left[ \frac{2D_o^2(D_o^2 - D_i^2)}{(D_o^2 - D_i^2)(D_o^2 - D_i^2)} \right] \]

\[ 0.2 = \frac{P(400)}{2 \times 10^5} \left[ \frac{2(400)^2(500^2 - 300^2)}{(500^2 - 400^2)(400^2 - 300^2)} \right] \]

\[ P = 12.30 \text{ N/mm}^2 \]

Let \((\Delta T)\) be the temperature to which the outer cylinder is heated to slip it onto the inner cylinder.

\[ \varepsilon_c = \frac{\Delta}{D_o} = \left( \frac{0.2}{400} \right) \]
\[ \alpha = 12.5 \times 10^{-6}^\circ C \]

As,
\[ \alpha(\Delta T) = \varepsilon_c \]

\[ \Delta T = \frac{5 \times 10^{-4}}{12.5 \times 10^{-6}} = 40^\circ C \]

Q.11 An axial tensile load of 100 kN is applied to a steel rod of 38 mm diameter and 500 mm long. Calculate the change in volume of the rod, if \( E = 200 \text{ GPa} \) and \( \nu = 0.26 \).

[IFS (Mains) 2010 : 5 Marks]

Solution:

Given: Rod diameter, \( d = 38 \text{ mm} \), Length, \( l = 500 \text{ mm} \), \( E = 200 \text{ GPa} \), \( \nu = 0.26 \), \( P = 100 \text{ kN} \)

\[ \sigma_i = \frac{P}{A} \]
\[ A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (38)^2 = 1133.54 \text{ mm}^2 \]

\[ \sigma_i = \frac{P}{A} = \frac{100 \times 10^3}{1133.54} = 88.21 \text{ MPa} \]

\[ \sigma_i = E \varepsilon_l \]

\[ \varepsilon_l = \frac{\sigma_i}{E} = \frac{88.219}{200 \times 10^3} = 0.441 \times 10^{-3} \]

Let \( \Delta l \) be the change in length.
\[ \Delta D \] be the change in diameter.

\[ \varepsilon_D = -\nu \varepsilon_l = -0.26 \times 0.441 \times 10^{-3} \]

\[ \varepsilon_D = -0.1146 \times 10^{-3} \] (negative sign indicates diameter reduces.)

As,
\[ V = \frac{\pi}{4} D^2 l \]
\[ \frac{\Delta V}{V} = \varepsilon_v = 2 \varepsilon_D + \varepsilon_l \]

\[ \varepsilon_v = 0.2118 \times 10^{-3} \]
Q.12 A steel tube, 24 mm external diameter and 18 mm internal diameter, encloses a copper rod 15 mm diameter to which it is rigidly joined at each end. If at a temperature of 30°C there is no longitudinal stress, calculate the stresses in the rod and tube, when the temperature is raised to 200°C. Given:

**For steel:**
- \( E_s = 210 \text{ GPa} \)
- \( \alpha_s = \text{Coefficient of thermal expansion} = 11 \times 10^{-6}/^\circ C \)

**For copper:**
- \( E_c = 100 \text{ GPa} \)
- \( \alpha_c = 18 \times 10^{-6}/^\circ C \)

\[ \Delta V = V_e - V = 120 \text{ mm}^3 \]

\[ V = \frac{\pi}{4} D^2 l = \frac{\pi}{4} (38)^2 \times 500 = 567057.47 \text{ mm}^3 \]

**Solution:**

Given:

- Steel tube: \((d_s)^2_o = 24 \text{ mm}, (d_s)^2_i = 18 \text{ mm}, E_s = 210 \text{ GPa}, \alpha_s = 11 \times 10^{-6}/^\circ C\)
- \(d_c = 15 \text{ mm}, E_c = 100 \text{ GPa}, \alpha_c = 18 \times 10^{-6}/^\circ C\)

When temperature is raised to 200°C. Let \(\sigma_s, \sigma_c\) be the stresses in the steel tube and rod respectively.

\[ \Delta T = 200 - 30 = 170^\circ C \]

As,

\[ P_s = P_c \]
\[ \sigma_s A_s = \sigma_c A_c \]

(Copper rod will be in compression and steel tube will be in tension)

\[ \sigma_s \times \frac{\pi}{4} (24^2 - 18^2) = \sigma_c \times \frac{\pi}{4} (15)^2 \]

\[ (1.12)\sigma_s = \sigma_c \]

\(...(i)\)

Also, strain in tube = strain in rod.

\[ \frac{\alpha_s (\Delta T)}{E_s} + \frac{\sigma_s}{E_s} = \frac{\alpha_c (\Delta T)}{E_c} - \frac{\sigma_c}{E_c} \]

\[ \frac{\sigma_s}{E_s} + \frac{\sigma_c}{E_c} = (\alpha_c - \alpha_s) \Delta T = (18 \times 10^{-6} - 11 \times 10^{-6}) \times 170 \]

\[ \frac{\sigma_s}{E_s} + \frac{1.12 \sigma_s}{E_c} = 1.19 \times 10^{-3} \]

(From equation (i), \(\sigma_c = 1.12 \sigma_s\))

\[ \sigma_s \left[\frac{1}{210 \times 10^3} + \frac{1.12}{100 \times 10^8}\right] = 1.19 \times 10^{-3} \]

\[ \sigma_s = 74.55 \text{ MPa} \]

and

\[ \sigma_c = 1.12 \sigma_s = 1.12 \times 74.55 = 83.5 \text{ MPa} \]
Q.13 A thin cylindrical shell is 5 m long, has 200 mm internal diameter and has thickness of metal 10 mm. It is filled completely with a fluid at atmospheric pressure. If an additional 25000 mm³ fluid is pumped in, find the pressure inside the shell and hoop stress developed. Find also the changes in diameter and length. Take \( E = 200 \text{ GPa} \) and \( \nu = 0.3 \).

Solution:

Given: \( d_1 = 200 \text{ mm}, t = 10 \text{ mm}, l = 5 \text{ m}, \) Volume pumped in \( (V_p) = 25000 \text{ mm}^3 \), \( E = 200 \text{ GPa} \), \( \nu = 0.3 \)

Volumetric strain, \( \varepsilon_v = 2 \varepsilon_D + \varepsilon_I \)

\[
\varepsilon_D = \frac{PD}{4tE} [2 - \nu] ; \quad \varepsilon_I = \frac{PD}{4tE} [1 - 2\nu]
\]

\[
\varepsilon_v = \frac{dV}{V} = \frac{PD}{4tE} [5 - 4\nu]
\]

\[
dV = 25000 \text{ mm}^3
\]

\[
V = \left( \frac{\pi}{4} d_1^2 \right) l = \frac{\pi}{4} (200)^2 \times 5 \times 10^3 = 157 \times 10^6 \text{ mm}^3
\]

\[
\varepsilon_v = \frac{dV}{V} = \frac{25 \times 10^3}{157 \times 10^6} = 0.1592 \times 10^{-3}
\]

\( 0.1592 \times 10^{-3} = \frac{P \times 200}{4 \times 10 \times 200 \times 10^3} [5 - 4 \times (0.3)] \)

\( P = 1.675 \text{ MPa} \)

Pressure in the shell, \( P = 1.675 \text{ MPa} \)

Hoop stress, \( \sigma_h = \frac{Pd_1}{2t} = \frac{1.675 \times 200}{2 \times 10} = 16.757 \text{ MPa} \)

\[
\varepsilon_{\text{hoop}} = \frac{PD}{4tE} [2 - \nu] = \frac{1.675 \times 200}{4 \times 10 \times 200 \times 10^3} [2 - 0.3] = 7.11875 \times 10^{-6}
\]

Change in diameter \( (\delta D) = 7.11875 \times 10^{-5} \times 200 = 0.1424 \text{ mm} \)

\[
\varepsilon_{\text{long}} = \frac{PD}{4tE} [1 - 2\nu] = \frac{1.675 \times 200}{4 \times 10 \times 200 \times 10^3} [1 - 2 \times 0.3] = 1.675 \times 10^{-5}
\]

Change in length \( (\delta l) = 1.675 \times 10^{-5} \times 5 = 8.375 \times 10^{-5} \text{ m} = 0.08375 \text{ mm} \)

Q.14 A circular steel rod tapers uniformly from 40 mm diameter to 150 mm diameter in a length of 400 mm. How much the bar will elongate under an axial pull of 40 kN? Take \( E = 200 \text{ GPa} \).

Solution:

Given: \( d_1 = 40 \text{ mm}, \ d_2 = 150 \text{ mm}, \ l = 400 \text{ mm}, \ P = 40 \text{ kN}, \ E = 200 \text{ GPa} \)

Here,

\[
d_s = d_1 + (d_2 - d_1) \frac{x}{l}
\]

Thus, the deflection of section \( dx \) is,

\[
d\delta = \frac{P (dx)}{E \pi d_s^2}
\]
\[ \delta = \int_{0}^{l} \frac{Pdx}{E \left( d_1 + \frac{x}{l} (d_2 - d_1) \right)^2} \]

(Integrating both sides from 0 to l)

\[ \delta = \frac{-PIa}{\pi E (d_2 - d_1)} \left[ \frac{1}{d_1 + \frac{x}{l} (d_2 - d_1)} \right]_0^l \]

\[ \delta = \frac{PIa (d_2 - d_1)}{\pi E d_1 d_2} = \frac{4Pl}{\pi Ed_1 d_2} \]

Let \( \delta \) be the elongation,

\[ \delta = \frac{4Pl}{\pi a d_2 E} = \frac{4 \times 40 \times 10^3 \times 400}{\pi \times 40 \times 150 \times 200 \times 10^3} = 0.0169 \text{ mm} \]

Q. 15 A tensile test specimen having a diameter of 12.7 mm was loaded up to a load of 76 kN and its diameter was measured as 12 mm. Compare true stress and strain with engineering stress and strain. 

[IFS (Mains) 2015 : 10 Marks]

Solution:

Given: Initial diameter, \( d_1 = 12.7 \text{ mm} \), Load, \( P = 76 \text{ kN} \), Diameter recorded, \( d = 12 \text{ mm} \),

Initial area, \( A_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} (12.7)^2 = 126.61 \text{ mm}^2 \)

Area at that recorded diameter, \( A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (12)^2 = 113.04 \text{ mm}^2 \)

Engineering stress, \( \sigma_{\text{eng}} = \frac{P}{A} = \frac{76 \times 10^3}{126.61} = 600.268 \text{ MPa} \)

True stress, \( \sigma_{\text{true}} = \frac{P}{A} = \frac{76 \times 10^3}{113.04} = 672.328 \text{ MPa} = 672.328 \text{ MPa} \)

So, \( \sigma_{\text{true}} > \sigma_{\text{eng}} \) and \( \frac{\sigma_{\text{true}}}{\sigma_{\text{eng}}} = 1.12 \)

Engineering strain \( \epsilon_{\text{eng}} = \frac{\Delta L}{L} = \frac{L_f - L_i}{L_i} = \left( \frac{A_i}{A_f} - 1 \right) \)

\[ = \left( \frac{126.61}{113.04} - 1 \right) = 0.12 \]

True strain \( \epsilon_{\text{true}} = \ln \left( \frac{L_f}{L_i} \right) \)

\[ = \ln \left( \frac{A_i}{A_f} \right) = \ln \left( \frac{126.61}{113.04} \right) = 0.1133 \]

\[ \frac{\epsilon_{\text{true}}}{\epsilon_{\text{eng}}} = \frac{0.1133}{0.12} = 0.9447 \]
Q.16 A rod of 1 m length is kept at a temperature of 30°C. Find the expansion of the rod when the temperature is raised to 80°C. If this expansion is prevented, find the stress induced in the material of the rod. Take: \( E = 100 \text{ GPa} \) and \( \alpha = 0.000012/\text{°C} \).

\[ (\text{IFS Mains)} \ 2017 : 8 \text{ Marks} \]

**Solution:**

Given, Length of rod, \( L = 1 \text{ m} \), Temperature, \( T_1 = 30^\circ \text{C}, \ E = 100 \text{ GPa}, \ \alpha = 0.000012/\text{°C} \)

Expansion of the rod when temperature is raised to \( T_2 = 80^\circ \text{C}, \ \delta_i = \alpha(\Delta T)L \quad (\therefore \ \varepsilon_i = \alpha \Delta T) \)

\[ \Delta T = T_2 - T_1 = 80 - 30 = 50^\circ \text{C} \]

\[ \delta_i = 0.000012 \times 50 \times 1000 = 0.6 \text{ mm} \]

When this expansion is prevented, stress induced in the rod,

\[ \sigma_c = E\alpha (\Delta T) = 100 \times 10^3 \times 0.000012 \times 50 = 60 \text{ MPa} \]

Expansion of the rod = 0.6 mm, the stress induced in rod = 60 MPa

Q.17 A bar of 2 m length is rigidly fixed to a support at top section where diameter is 50 mm and remains constant up to a length of 1 m. For the remaining portion, the diameter is 25 mm. If a weight of 1000 N falls freely through 100 mm and lands uniformly on a rigid collar at the lowermost cross-section, calculate the stress and extension in the bar. Take \( E = 2.1 \times 10^5 \text{ N/mm}^2 \).

\[ (\text{IFS Mains)} \ 2018 : 10 \text{ Marks} \]

**Solution:**

Given:

Total length of bar = 2 m

Area of top section, \( A_1 = \frac{\pi}{4} \alpha_1^2 = \frac{\pi}{4} (50)^2 = 625\pi \text{ mm}^2 \)

Area of lower section, \( A_2 = \frac{\pi}{4} \alpha_2^2 = \frac{\pi}{4} (25)^2 = 156.25\pi \text{ mm}^2 \)

Height of load from the collar, \( h = 100 \text{ mm} \)

Weight of load, \( W = 1000 \text{ N} \)

\[ \delta_{st} = \frac{Wl_1 + Wl_2}{A_1E + A_2E} \]

\[ \delta_{st} = \frac{1000}{625\pi \times 2.1 \times 10^5} + \frac{1000}{156.25\pi \times 2.1 \times 10^5} \]

\[ \delta_{st} = 0.012126 \text{ mm} \]

We know that

Impact factor, I.F. = \[ 1 + \sqrt{1 + \frac{2h}{\delta_{st}}} \]

\[ = 129.43 \]

Extension in the bar = I.F. \times \delta_{st}

\[ = 129.43 \times 0.012126 = 1.5695 \text{ mm} \]

Stress in 50 mm diameter bar, \( \sigma_1 = \frac{W}{A_1} \times \text{I.F.} \)

\[ = \frac{1000}{625\pi} \times 129.43 = 65.918 \text{ MPa} \]

Stress in 25 mm diameter bar = \[ \frac{W}{A_2} \times \text{I.F.} = \frac{1000}{156.25\pi} \times 129.43 = 263.672 \text{ MPa} \]
2. Shear Force and Bending Moment Diagram

Q.18 Discuss the general features of Shear Force and Bending Moment diagrams in case of simply supported beams to which various types of loads are applied.

[IFS (Mains) 2001 : 10 Marks]

Solution:

Simply supported beams subjected to various types of loads:

1. Concentrated load:

   Shear force is constant and bending moment varies linearly along the length of the beam.

2. Uniform load

   Shear force varies linearly and bending moment varies parabolically along the length of the beam.

3. Uniformly varying load

   Shear force varies parabolically and bending moment varies cubically along the length of the beam.

where,

\[ x = \frac{L}{\sqrt{3}} = 0.577 \, L \]

\[ (B.M.)_{\text{max}} = \frac{wL^2}{9\sqrt{3}} \]
Q.19 A cantilever ABC of length 3.0 m carries two concentrated loads of 2.0 kN and 1.2 kN at B and C as shown in the figure. The cross-section of the cantilever is given below. For AB portion, breadth is 12 cm and for BC portion, breadth is 6 cm. Depth is uniform all through and is 10 cm. E for beam material may be taken as 200 GPa. Find the slope and deflection at the free end C of the beam.

Solution:

Given: A cantilever beam ABC

\[ h_{AB} = h_{BC} = \text{Uniform} = 10 \text{ cm}, \quad b_{AB} = 12 \text{ cm}, \quad b_{BC} = 6 \text{ cm}, \quad E = 200 \text{ GPa} \]

Using \( \frac{M}{EI} \) diagram to calculate slope and deflection at free end.

\[ M + 1.2x = 0 \quad (M = -1.2x) \quad (\text{for BC portion}) \]

\[ I_{BC} = \frac{1}{12}bh^3 = \frac{1}{12} \times 6 \times (10)^3 = 500 \text{ cm}^4 = 5 \times 10^{-8} \text{ m}^4 \]

\[ (EI)_{BC} = 200 \times 10^9 \times 500 \times 10^{-8} = 10^6 \text{ N-m}^2 \]

\[ I_{AB} = \frac{1}{12}bh^3 = \frac{1}{12} \times 12 \times (10)^3 = 10^3 \text{ cm}^4 = 10^3 \times 10^{-8} \text{ m}^4 \]

\[ (EI)_{AB} = 200 \times 10^9 \times 10^3 \times 10^{-8} = 2 \times 10^6 \text{ (N-m}^2) \]

\[ \left( \frac{M}{EI} \right) \text{ diagram} \]
\[
\theta_{AC} = \text{Slope at free end 'C' = Area of} \left( \frac{M}{EI} \right) \text{diagram}
\]
\[
= \left\{ \frac{1}{2} \times 1.5 \times 1.8 \times 10^{-3} + (0.9 \times 10^{-3} \times 1.5) \right\}
\]
\[
+ \left\{ \frac{1}{2} \times 1.5 \times 2.4 \times 10^{3} \right\}
\]

\[
\theta_c = 4.5 \times 10^{-3} \text{ radians (C.W.)}
\]

\[
\delta_c = \text{deflection at free end 'C'}
\]
\[
= \left[ A_1 \bar{X}_1 + A_2 \bar{X}_2 + A_3 \bar{X}_3 \right]
\]
\[
\delta_c = \left[ \frac{1}{2} \times 1.5 \times 1.8 \times 10^{-3} \times \frac{2}{3} \times (1.5) \right] + \left[ 0.9 \times 10^{-3} \times 1.5 \times 2.25 \right]
\]
\[
+ \left[ \frac{1}{2} \times 1.5 \times 2.4 \times 10^{-3} \times \left( \frac{5}{3} \times 1.5 \right) \right]
\]
\[
\delta_c = 8.8875 \times 10^{-3} \text{ m}
\]

Q.20 A cantilever of length 1.2 m carries a UDL of 4 kN/m run and a concentrated load of 10 kN at the free end. The cross-section of the cantilever is rectangular and having a width of 40 mm and a depth of 100 mm.

(i) Draw S.F. and B.M. diagrams for the cantilever and obtain the value of S.F. and B.M. at a section 1 m from the free end

(ii) Obtain normal stress and shear stress distribution on a section 1 m from the free end, and

(iii) Calculate the maximum deflection of the cantilever and indicate the location where it occurs.

[IFS (Mains) 2004 : 10 + 15 + 5 = 30 Marks]

Solution:

Given:

A cantilever beam subjected to UDL and a concentrated load.

\[
\begin{align*}
\text{Shear force, } V &= 4x + P = 4x + 10 \\
\end{align*}
\]

and

\[
M + 4x \left( \frac{x}{2} \right) +Px = 0
\]

\[
M = -(2x^2 + 10x)
\]

When

\[
x = 1.2 \text{ m from free end}
\]

\[
V = 4 \times 1.2 + 10 = 14.8 \text{ kN}
\]