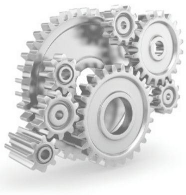
6th Edition
Revised & Updated

A Handbook on

Mechanical Engineering



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for

ESE, GATE, PSUs

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A Handbook on Mechanical Engineering

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Director's Message

B. Singh (Ex. IES)

During the current age of international competition in Science and Technology, the Indian participation through skilled technical professionals have been challenging to the world. Constant efforts and desire to achieve top positions are still required.

I feel every candidate has ability to succeed but competitive environment and quality guidance is required to achieve high level goals. At MADE EASY, we help you discover your hidden talent and success quotient to achieve your ultimate goals. In my opinion CSE, ESE, GATE & PSUs exams are tools to enter in to the main stream of Nation serving. The real application of knowledge and talent starts, after you enter in to the working system. Here at MADE EASY you are also trained to become winner in your life and achieve job satisfaction.

MADE EASY alumni have shared their winning stories of success and expressed their gratitude towards quality guidance of MADE EASY. Our students have not only secured All India First Ranks in ESE, GATE and PSUs entrance examinations but also secured top positions in their career profiles. Now, I invite you to become an alumnus of MADE EASY to explore and achieve ultimate goal of your life. I promise to provide you quality guidance with competitive environment which is far advanced and ahead than the reach of other institutions. You will get the guidance, support and inspiration that you need to reach the peak of your career.

I have a true desire to serve the Society and the Nation easing path of the education for the people of India.

After a long experience of teaching Mechanical Engineering over a period of time, MADE EASY team realised that there is a need of a good *Handbook* which can provide the crux of Mechanical Engineering in a concise form to the student to brush up the formulae and important concepts required for ESE, GATE, PSUs and other competitive examinations. This *handbook* contains all the formulae and important theoretical aspects of Mechanical Engineering. It provides much needed revision aid and study guidance before examinations.

B. Singh (Ex. IES)
CMD, MADE EASY Group

A Handbook on
Mechanical Engineering

Contents

Contents

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Fluid Mechanics and Hydraulic Machines

1

I Introduction

- **Ideal fluid:** A fluid is said to be ideal if it is assumed to be both incompressible and non-viscous. Its bulk modulus is infinite ($k = \infty$).
- **Real fluid:** Real fluids have viscosity, finite compressibility and surface tension.

Remember:

☑ It encounters zero resistance to motion.
☑ Ideal fluid has no surface tension.
☑ Ideal fluids are imaginary and do not exist in nature.

- **Specific weight or weight density**, $w = \dfrac{\text{Weight}}{\text{Volume}} = \dfrac{mg}{V} = \rho g$

  where, ρ = Density, g = Acceleration due to gravity
  S.I. unit of specific weight is N/m^3
  Specific weight of water = 9810 N/m^3

- **Specific volume,** $v = \dfrac{1}{\text{Density}} = \dfrac{1}{\rho}$

  Specific volume of the water: $v = \dfrac{1}{\rho} = \dfrac{1}{1000} = 0.001\ m^3/kg$

- **Specific gravity (s) or Relative density**

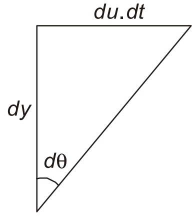
  Specific gravity, $s = \dfrac{\text{Density of fluid}}{\text{Density of standard fluid}}$

  $= \dfrac{\text{Specific weight of fluid}}{\text{Specific weight of standard fluid}}$

Remember:

☑ Specific gravity for water is 1.0 at 4°C and for mercury it is 13.6.
☑ Specific gravity varies with temperature, therefore it should be determined at a specified temperature (4°C or 25°C).

Newton's Law of Viscosity



$$\tau = \mu \frac{du}{dy} = \mu \frac{d\theta}{dt}$$

where, τ = Shear stress

μ = Coefficient of viscosity or absolute viscosity (or dynamic viscosity)

$\frac{du}{dy}$ = Velocity gradient

$\frac{d\theta}{dt}$ = Rate of angular deformation or Rate of shear strain

- For Newtonian fluid, coefficient of viscosity remains constant.

Dynamic Viscosity and Kinematic Viscosity

Due to viscosity, a fluid offers resistance to flow

(i) Dynamic Viscosity (μ):

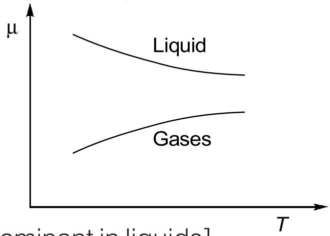
- It is defined as shear stress required to produce unit velocity gradient or deformation rate.
- Its SI unit is Pascal-second or **Ns/m^2 or kg/ms**
- Its CGS unit is poise
- 1 poise = Dyne-s/cm^2 = **0.1** Ns/m^2

(ii) Kinematic Viscosity, $\nu = \frac{\text{Dynamic viscosity } (\mu)}{\text{Mass density } (\rho)}$

- Its SI unit is m^2/s
- Its CGS unit is cm^2/s or stoke
- 1 stoke = cm^2/s = 10^{-4} m^2/s

Remember: ....

☑ Viscosity of **liquids** generally decreases with temperature whereas viscosity of **gases** increases with increase in temperature.



☑ Liquids with increasing order of viscosity are gasoline, water, crude oil, castor oil.

☑ Viscosity of **water** at 20°C is 1 centipoise.

☑ Viscosity is due to
- Intermolecular forces of cohesion [dominant in liquids]
- Transfer of molecular momentum between fluid layers [dominant in gases].

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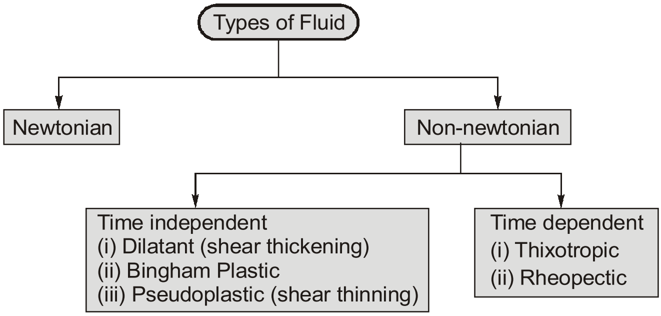
Types of Fluid

- 

- **Non-Newtonian Fluids:** Non-Newtonian fluids do not follow Newton's law of viscosity. The relation between shear stress and velocity gradient is

$$\tau = A\left(\frac{du}{dy}\right)^n + B$$

where A and B are constants depending upon type of fluid and the conditions of flow.

(i) **Dilatant or shear thickening fluids:** Fluids for which the apparent viscosity increases with the rate of deformation (such as solutions with suspended starch or sand).
Example: Butter, Quicksand.

(ii) **Bingham Plastic Fluids:** Materials such as toothpaste can resist a finite shear stress and thus behave as a solid, but deform continuously when the shear stress exceeds the yield stress and behave as a fluid.
Example: Sewage sludge, drilling mud, tooth paste and gel. These fluids always have certain minimum shear stress before they yield.

(iii) **Pseudoplastic or shear thinning fluids:** Fluids becoming less viscous as they are sheared harder, such as some paints, polymer solutions, and fluids with suspended particles.
Example: Paper pulp, Rubber solution, Lipsticks, Paints, Blood, Polymetric solutions etc.

(iv) **Thixotropic Fluids:** Thixotropy is a phenomenon where the viscosity of some fluids depends on the time of shear rate, and hence, thixotropy is a time-dependent phenomenon. The viscosity of thixotropic fluids decreases with time.
Example: Printer ink and Enamels.

(v) **Rheopectic Fluids:** Rheopectic fluids for which viscosity increases with stress over time. They also show an increased viscosity upon agitation meaning when the fluid is shaken, it becomes thick, or it may even solidify.

Example: Gypsum solution in water and Bentonite solution.

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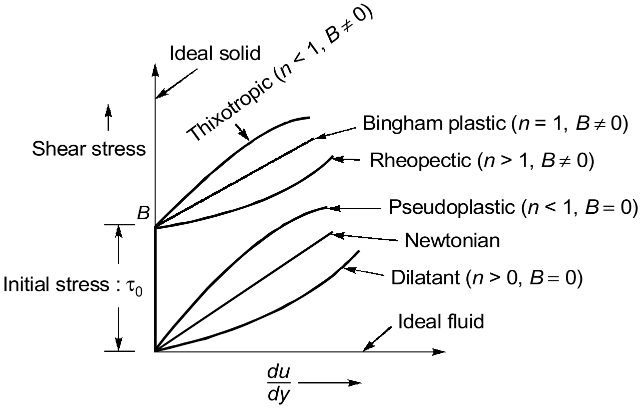


Compressibility (β), Isothermal Bulk Modulus (k_T) and Adiabatic Bulk Modulus (k_a)

- Compressibility (β)

  It is inverse of bulk modulus of elasticity, $k_{air} = 0.103$ MPa

  $k_{water} = 2060$ MPa, $k_{steel} = 20600$ MPa

$$\beta = \frac{1}{k} = \frac{-dv}{vdp} = \frac{d\rho}{\rho dp}$$

  where, k = Bulk modulus of elasticity; ρ = Density; v = Specific volume

- Isothermal bulk modulus (k_T)

$$k_T = p_{final} = \rho RT$$

- Adiabatic bulk modulus

$$k_a = \gamma\, p_{final} \qquad \text{Here, } \gamma = \frac{c_p}{c_v}$$

  c_p = Specific heat at constant pressure; c_v = Specific heat at constant volume

Remember: ....

☑ Liquids are considered incompressible because their value of k is very high.

☑ Gases are considered in compressible if Mach number (M < 0.2 to 0.3) because for this range percentage change in density are much less than 5%.

....

Surface Tension/Pressure Inside Drop, Bubble and Jet

Surface tension occurs at the interface of liquid and a gas or at the interface of two liquids. Surface tension is inversely proportional to temperature and it also acts when fluid is at rest.

- Liquid tends to minimize its surface area and hence the surface energy.
- Excess pressure inside drop (Solid like sphere)

$$\Delta P = \frac{4\sigma}{d}$$

- Excess pressure inside bubble

$$\Delta P = \frac{8\sigma}{d}$$

Remember:

☑ The pressure inside the droplet of soap bubble will be higher than p_{atm}.

☑ The higher the pressure inside the soap bubble, the smaller the size of soap bubble.

- Excess pressure inside jet

$$\Delta P = \frac{2\sigma}{d}$$

Remember:

☑ It is a **surface** phenomenon.

☑ It is force per unit length (N/m).

☑ For **water-air** interface at 20°C, its value is 0.0736 N/m and air-mercury interface, σ = 0.480 N/m.

☑ At critical point, liquid-vapour state are same, thus surface tension = 0.

☑ It is due to **cohesion** only.

☑ By increasing temperature, surface tension decreases.

☑ By adding impurity, surface tension decreases.

Capillary Action

- Height of water in capillary tube

$$h = \frac{4\sigma\cos\theta}{\rho g d}$$

where, h = Rise in capillary, σ = Surface tension of water
d = Diameter of tube
θ = Angle of contact between the liquid and the material.
θ = 0° for water and glass (clean)
θ = 128° for mercury and glass (clean)

	$F_{cohesion} < F_{adhesion}$	$F_{cohesion} > F_{adhesion}$
Level	Rises	Falls
θ	$< 90°$	$> 90°$
Ex.:	Water-glass	Mercury-glass

- When a liquid surface supports another liquid of density ρ_b, then rise in capillary is given as

$$h = \frac{4\sigma\cos\theta}{(\rho - \rho_b)gd}$$

- Capillary action is due to **both** adhesion and cohesion.
- For capillary action, diameter of tube should be **less** than 3 cm.
- Tubes of manometers are taken more than 6 mm in diameter in order to avoid the correction due to capillarity, because with increase in capillary diameter rise/fall decreases.

II Manometry

Pascal's Law

- The intensity of pressure at any point in a stationary fluid is same in all directions.

$$p_x = p_y = p_z$$

- Pressure varies **only with depth** in stationary fluids, whereas if fluids are in motion, pressure may vary in horizontal direction also.
- Fluid pressure is measured as Force/Area and it is expressed in Pascal (N/m^2) or bar.

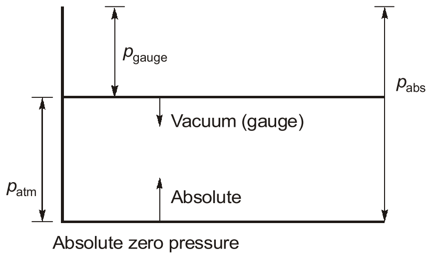
$$1 \text{ bar} = 10^5 \text{ N/m}^2 \text{ or Pa}$$
$$1 \text{ MPa} = 10 \text{ bar}$$

- Barometer shows **atmospheric** pressure.
- 1 kgf = 9.81 Newton.
- Pressure is a scalar quantity.

Absolute Pressure

Absolute pressure is measured with reference to absolute zero. Absolute pressure cannot be negative

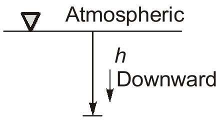
Absolute pressure = Gauge pressure + Local atmospheric pressure



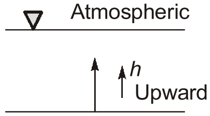
- $p_{gauge} = \rho g h$
  Here, ρ = Density of fluid,
- **Gauge** pressure can be positive, negative or zero.
- Atmospheric pressure varies with **altitude**, **temperature** and **local** conditions.
- At **mean sea level** atmospheric pressure is 1.01×10^5 Pa or 1 bar or 10.3 m of height of water or 76 cm height of **mercury**.

Hydrostatic Law

- For h measured downward, $\dfrac{dp}{dh} = w$



- For upward h, $\dfrac{dp}{dh} = -w$



Negative sign implies that pressure increases in a direction in which 'h' decreases.

Remember:

☑ Hydrostatic pressure shows linear variation with depth below the free surface.

☑ Hydrostatic law can be applied to both incompressible and compressible fluid provided the local density is taken into account.

Conversion of one Fluid Column to Another Fluid Column

$$\sigma_1 h_1 = \sigma_2 h_2, \qquad S_1 h_2 = S_2 h_2$$

where, ρ = Density of fluid, S = Relative density

Remember:

☑ Piezometer is not used for measurement of high pressure and gas pressure, it is suitable for **small** and **positive** pressure measurement.

☑ The manometric liquid should have **high density** and **low vapour pressure**.

☑ Simple manometer/U-tube manometer can measure both **positive** and **negative** pressure.

☑ Aneroid/Mercury barometer is used to measure **local** atmospheric pressure on **absolute** scale.

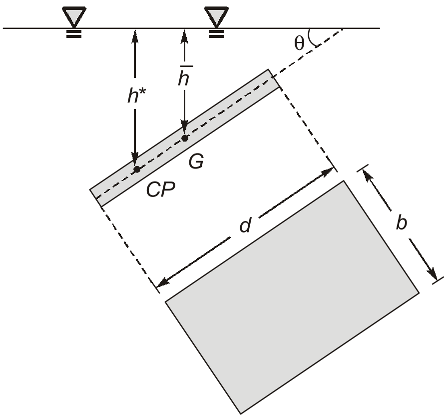
☑ Inclined single column manometer is used to increase sensitivity by a factor of $\dfrac{1}{\sin\theta}$

☑ Inverted column U-tube manometer is used for measuring liquid pressure only. [$S_{manometric} < S_{liquid}$]
☑ Density of mercury = 13600 kg/m^3
☑ Density of air = 1.24 kg/m^3

III Hydrostatic Force

Hydrostatic Force on Submerged Surface

Case	Force	Center of pressure (h^*)
Horizontal position	$\rho g A\bar{h}$	$h^* = \bar{h}$
Vertical position	$\rho g A\bar{h}$	$h^* = \bar{h} + \frac{I_G}{A\bar{h}}$
Inclined position	$\rho g A\bar{h}$	$h^* = \bar{h} + \frac{I_G}{A\bar{h}}\sin^2\theta$



Note:

☑ Depth of centre of pressure is independent of density of the fluid.

$$I_G = \frac{bd^3}{12} \quad \text{(For rectangular plate)}$$

$$I_G = \frac{\pi}{64}d^4 \quad \text{(For circular plate)}$$

where, A = Area of surface touching fluid

I_G = Area moment of inertia about centroidal axis and parallel to free axis.

$\bar{h}$ = Vertical distance of *CG* of body from free surface.

w = Specific weight

θ = Angle at which the surface is inclined with the free surface

Pressure Force on Curved Surface

- **Horizontal Force (F_H):** Horizontal component of the resultant hydrostatic force 'F_x' of curved surface should be computed by projecting the surface upon a vertical plane and multiplying the projected area by the pressure at its own centroid of projected area.

$$F_H = \rho g A\bar{x}$$

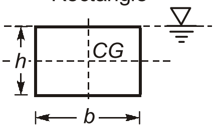
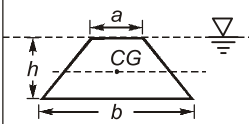
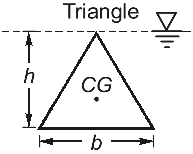
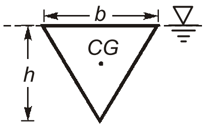
- **Vertical Force (F_V):** Vertical component of force 'F_y' is equal to the weight of the liquid block lying above the curved surface upto free surface.

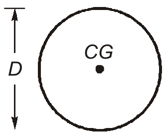
$$F_H = \rho \forall g$$

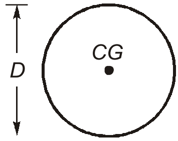
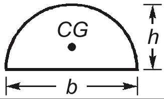
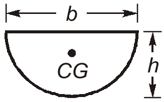
- **Resultant Force,** $F = \sqrt{(F_H)^2 + (F_V)^2}$

  Angle of line of action of resultant force with the horizontal is given by $\tan\theta = \dfrac{F_y}{F_x}$

Depth of Center of Pressure for Some Vertical Plane Surfaces from Liquid Surface

Surface	C.G.($\bar{h}$)	C.P.(h^*)
Rectangle 	$\frac{h}{2}$	$\frac{2h}{3}$
Trapezium 	$\frac{a+2b}{a+b}\cdot\frac{h}{3}$	$\frac{a+3b}{a+2b}\cdot\frac{h}{2}$
Triangle (a) 	$\frac{2h}{3}$	$\frac{3h}{4}$
(b) 	$\frac{h}{3}$	$\frac{h}{2}$

Surface	C.G.($\bar{h}$)	C.P.(h^*)
Circle 	$\frac{D}{2}$	$\frac{5D}{8}$

Circle 	$\frac{D}{2}$	$\frac{5D}{8}$
Parabola (a) 	$\frac{3h}{5}$	$\frac{5h}{7}$
(b) 	$\frac{2h}{5}$	$\frac{4h}{7}$

Remember:

- ☑ In case of vertical surface, when depth of immersion (h) is very large, then centre of pressure ≈ centre of gravity or $h^* \approx \bar{h}$.
- ☑ Magnitude of hydrostatic forces on a plane surface does not change with rotation in a horizontal plane as $\bar{h}$ remains same. Since $\frac{I_G \sin^2\theta}{A\bar{x}}$ is always +ve, hence $\bar{h}$ is always greater than $\bar{x}$. i.e. centre of pressure is always located below G.

IV Buoyancy and Floatation

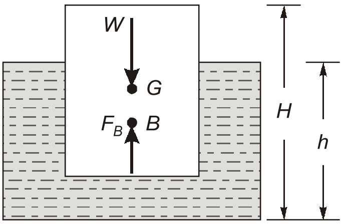
Archimedes Principle

When a body is submerged either fully or partially, then it is acted upon by a force of buoyancy vertically up which is equal to weight of liquid displaced by the body.

Remember:

- ☑ This force of buoyancy always acts through the centroid of liquid displaced.
- ☑ Centre of buoyancy is that point through which buoyant force acts.

Principal of Flotation



At equilibrium

$$F_B = W$$

$$\rho_{Body} \times V = \rho_{fluid} \times \forall$$

$$H\rho_{Body} = h\rho_{fluid}$$

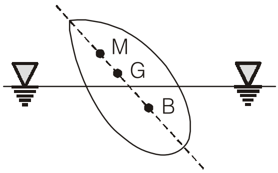
(for constant cross - section area)

Here, H = Height of body

h = Height of body that is submerged in fluid

Condition for Equilibrium for Floating/Submerged Body

- In case of **floating body**, metacenter should be above centre of gravity. GM Positive $\Rightarrow$ M above G, $BM > BG$ (Stable equilibrium)



Stable equilibrium

$$GM = BM - BG = \frac{I_{min}}{\forall_{immersed}} - BG$$

- For neutral equilibrium, GM = 0, 'M' coincides with 'G'.
- For unstable equilibrium, GM < 0, 'G' is above 'M' for floating body.
- In case of **submerged body**, center of buoyancy should be above centre of gravity.
  $\Rightarrow$ B above G (Stable equilibrium), G above B (unstable equilibirum) G and B coincide (neutral equilibrium).
- Distance between metacenter and centre of buoyancy (BM) = $\frac{I_{min}}{V_{immersed}}$ (Metacentric radius)
  Here,
  I_{min} = Moment of inertia of top view of floating body about longitudinal axis
  V = Volume of body immersed in liquid

Remember:

☑ Metacentric height for rolling condition will be less than metacentric height for pitching condition.

☑ For rolling movement of a ship, the centroidal axis about which the second moment is taken is the longitudinal one, while for pitching movement, the appropriate axis is the transverse one.

Time Period of Oscillation

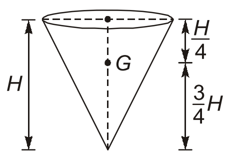
If a floating body oscillates then its time period of transverse oscillation is given by

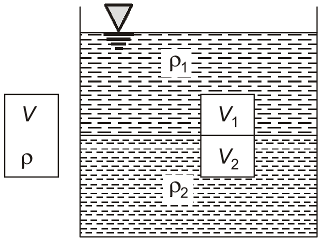
$$T = 2\pi\sqrt{\frac{K_G^2}{g \cdot GM}}$$

Here, K_G = Least radius of gyration, GM = Meta-centric height

For cone the center of gravity lies at $\frac{3}{4}H$ from the pointed end.

- As GM ↑, T ↓ stability ↑
- As GM ↓, T ↑ comfort for passengers in ship ↑





$$\frac{V_2}{V} = \frac{\rho - \rho_1}{\rho_2 - \rho_1}$$

V Fluid Kinematics

- Lagrangian concept is for single fluid particle
- Eulerian concept is for particular section or point. It is applicable where concept of continuum is valid.

Steady and Unsteady Flow

If the fluid and flow characteristics (such as density, velocity, pressure, etc.) **at a point** do not change with time, the flow is said to be steady flow.

$$\frac{dv}{dt} = 0, \frac{dp}{dt} = 0, \frac{d\rho}{dt} = 0$$ **(For steady flow.)**

- It is applicable for **all** properties.
- If the fluid and flow variables at a point or cross-section **change** with time, the flow will be **unsteady**.

Uniform and Non-Uniform Flow

If the velocity vector at all points in the flow is same **at any instant of time**, the flow is **uniform flow.** If the velocity vector **varies** from point to point at any instant of time, the flow will be **non-uniform**.

$$\frac{dv}{ds} = 0$$

- It is applicable **only** for velocity.

Laminar and Turbulent Flow

In laminar flow, the particles move in layers sliding smoothly over the adjacent layers while in turbulent flow, particles have the random and erratic movement, intermixing in the adjacent layers.

Streakline

- It is the line formed by joining all the particles which have crossed a given point over a period.
- When a dye is injected in a liquid or smoke in a gas so as to trace the subsequent motion of fluid particles passing a fixed point, the path followed by the dye or smoke is called the streakline.

Pathline

A pathline is a curve traced by a single fluid particle during its motion.

Timeline

Line formed by joining all the particles which are present adjacent to each other in a flow field at a given instant of time.

Streamline

A streamline is an imaginary line drawn in a flow field such that a tangent drawn at any point on this line represents the direction of velocity vector at that point.

Remember:

- ☑ There is no **velocity** component normal to stream lines.
- ☑ Streakline, pathline and streamline are Identical in a steady few.
- ☑ There is no discharge across a streamline.
- ☑ Stream line can't be drawn for divergent portion of C-D nozzle because

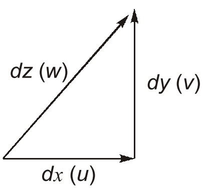
  M > 1 and $\frac{d\rho}{\rho} > 5\%$, so it is compressible flow, and stream line is only drawn for incompressible flow.

Equation of stream line

Tangent to stream line gives velocity

$$\vec{V} \times \overrightarrow{dx} = 0$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u & v & w \\ dx & dy & dz \end{vmatrix} = 0$$



$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

Here, u, v, w = Components of velocity in x, y, z direction

Continuity Equation (Conservation of Mass)

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \quad \text{(for compressible flow)}$$

Here, ρ = Density, A = Area, V = Velocity

- For incompressible fluid, density will be constant, thus continuity equation will be

$$A_1 V_1 = A_2 V_2 \quad \text{for incompressible flow}$$

General Continuity Equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

$$\frac{d\rho}{dt} + \rho(\nabla \vec{V}) = 0 \quad \text{(Vector form)}$$

Special Case:

- If flow is steady, then $\left(\frac{\partial \rho}{\partial t} = 0\right)$

  Thus continuity equation will be

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

- For steady and incompressible flow, the density will be constant thus

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\text{Div. } \vec{V} = 0$$

Note:

☑ Any mathematical function will become a flow only when it satisfies the continuity equation.

Total Acceleration of fluid

$$a_x = \underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}}_{\text{Convective acceleration}} + \underbrace{\frac{\partial u}{\partial t}}_{\text{Temporal or local acceleration}}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

- Total Acceleration = Convective acceleration with respect to space + local acceleration with respect to time

Type of flow	Convective Acceleration	Temporal Acceleration
Steady and uniform	0	0
Steady and non- uniform	Exists	0
Unsteady and uniform	0	Exists
Unsteady and non- uniform	Exists	Exists

Tangential acceleration - Magnitude of velocity changes
Normal acceleration - Direction of velocity changes

Rotational Component/Vorticity/Circulation

- Rotational component (ω)

$$\omega = \frac{1}{2}\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}; \qquad \omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

For irrotational flow, $\omega_x = \omega_y = \omega_z = 0$

- **Vorticity (ξ) :** 2D, Scalar quantity

$$\xi = 2 \times \text{ Rotational component}$$

- Circulation (Γ)

It is line integral of tangential component of velocity around a closed curve.

$$\Gamma = \text{Vorticity about centroid of area} \times \text{Area}$$

$$\Rightarrow \text{Vorticity} = \frac{\Gamma}{A} \text{ [circulation density]}$$

Remember:

☑ In irrotational flow, the vorticity is zero, at all points in the flow region while for rotational flow, vorticity is non-zero.

☑ Flow outside the boundary layer has irrotational characteristic while that within the boundary layer has rotational characteristic.

Velocity Potential Function (ϕ)

It is defined as a scalar function of space and time such that its negative derivative along any direction gives the component of velocity along that direction.

$$u = \frac{-\partial\phi}{\partial x}, \quad v = \frac{-\partial\phi}{\partial y}, \quad w = \frac{-\partial\phi}{\partial z}$$

Remember:

- ☑ Negative sign implies flow always occurs in the direction of decreasing potential.
- ☑ Velocity potential function exists only for irrotational flow and potential flow.
- ☑ Laplace equation is given by $\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0$, $\nabla^2\phi=0$
- ☑ If velocity potential function satisfies Laplace equation, then it also satisfies continuity equation and hence the **flow** is **possible**.

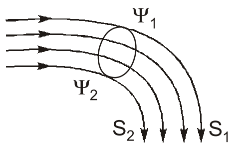
Streamline Function (ψ)

Defined only for 2D, steady and incompressible flow. Its partial derivative w.r.t. any direction gives the component of velocity at right angle to the direction.

$$v = \frac{\partial\psi}{\partial x}, \quad u = -\frac{\partial\psi}{\partial y}$$

- If Ψ exists, then it automatically satisfies continuity equation and flow can be rotational or irrotational.
- If stream function satisfies Laplace equation, then it is case of **irrotational flow**. $\nabla^2\psi=0$
- Discharge per **unit** length $=\psi_1-\psi_2$

  $\psi_1 > \psi_2$



Flow net

It is an imaginary grid formed by drawing a series of equipotential lines and streamlines.

Cauchy-Riemann equation

$$-\frac{\partial\phi}{\partial x} = -\frac{\partial\psi}{\partial y}=u$$

$$-\frac{\partial\phi}{\partial y} = \frac{\partial\psi}{\partial x}=v$$

$$\psi = \int -u.dy + \int v(\text{terms not containing } y).dx$$

$$\phi = \int -u.dx - \int v(\text{terms not containing } x).dy$$

Remember:

- ☑ Equipotential lines and constant stream function lines are **orthogonal except at stagnation point.**