Previous Years Solved Papers

Civil Services Main Examination (2001-2018)

Also useful for Engineering Services Main Examination and various State Engineering Services Examinations

Electrical Engineering Paper-I

Topicwise Presentation

Also useful for Engineering Services Main Examination and various State Engineering Services Examinations

MADE EASY Publications
Civil Service is considered as the most prestigious job in India and it has become a preferred destination by all engineers. In order to reach this estimable position every aspirant has to take arduous journey of Civil Services Examination (CSE). Focused approach and strong determination are the prerequisites for this journey. Besides this, a good book also comes in the list of essential commodity of this odyssey.

I feel extremely glad to launch the revised edition of such a book which will not only make CSE plain sailing, but also with 100% clarity in concepts.

MADE EASY team has prepared this book with utmost care and thorough study of all previous years papers of CSE. The book aims to provide complete solution to all previous years questions with accuracy.

On doing a detailed analysis of previous years CSE question papers, it came to light that a good percentage of questions have been asked in Engineering Services, Indian Forest Services and State Services exams. Hence, this book is a one stop shop for all CSE, ESE and other competitive exam aspirants.

I would like to acknowledge efforts of entire MADE EASY team who worked day and night to solve previous years papers in a limited time frame and I hope this book will prove to be an essential tool to succeed in competitive exams and my desire to serve student fraternity by providing best study material and quality guidance will get accomplished.

With Best Wishes

B. Singh (Ex. IES)
CMD, MADE EASY Group
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1. Circuit Element, Nodal and Mesh Analysis

Q.1 For the circuit shown below, find $i_1$, $i_2$, $i_3$ and $i_4$.

![Circuit Diagram]

Solution:

The circuit is redrawn as:

By Nodal equation,

\[-\frac{V_1}{100} - 2.5 - \frac{V_4}{10} + 0.2V_1 - \frac{V_4}{25} = 0\]

\[\Rightarrow \quad V_1 = \frac{250}{5} = 50 \text{ V}\]

\[i_4 = -\frac{V_1}{100} = -0.5 \text{ A}\]

\[i_1 = -\frac{V_1}{25} = -\frac{50}{25} = -2 \text{ A}\]

\[i_2 = i_1 + 0.2V_1 + i = -2 + 10 - 5 = 3 \text{ A}\]

\[i_3 = i - 2.5 + i_4 = -5 - 2.5 - 0.5 = -8 \text{ A}\]

\[i_1 = -2 \text{ A} ; \quad i_2 = 3 \text{ A} ; \quad i_3 = -8 \text{ A} ; \quad i_4 = -0.5 \text{ A}\]

Q.2 For the network shown below, find the current ratio transfer function given by $\alpha = \frac{I_2}{I_1}$.

![Circuit Diagram]
Solution:

Consider nodes (1) and (2). These two nodes constitute a super node.

\[ V_2 - V_1 = 2I_1 \]  
\[ ... (1) \]

Super node equation

\[ \frac{V_1}{1} - I_1 + \frac{V_2}{1} + \frac{V_2 - V_3}{2} = 0 \]
\[ ... (2) \]

\[ V_1 + V_2 + 0.5V_2 - 0.5V_3 = I_1 \]
\[ V_1 + 1.5V_2 - 0.5V_3 = I_1 \]

Node (3):

\[ \frac{V_3 - V_2}{2} - I_2 - \frac{I_1}{2} = 0 \]
\[ ... (3) \]

Put

\[ \frac{V_3 - V_2}{2} = 0.5I_1 + I_2 \]

\[ V_1 - I_1 + V_2 - 0.5I_1 - I_2 = 0 \]
\[ V_1 + V_2 = 1.5I_1 + I_2 \]
\[ ... (4) \]

From (1) and (4)

\[ V_2 = 1.75I_1 + 0.5I_2 \]

Also

\[ V_3 - V_2 = I_1 + 2I_2 \]
\[ V_3 = V_2 + I_1 + 2I_2 \]
\[ V_3 = 2.75I_1 + 2.5I_2 \]

Also

\[ I_2 = -V_3 \]

From (5),

\[ -I_2 = 2.75I_1 + 2.5I_2 \]
\[ -3.5I_2 = 2.75I_1 \]
\[ \alpha = \frac{I_2}{I_1} = \frac{-2.75}{3.5} = -0.786 \]

Q.3 Determine the power delivered by 6 A source.
Solution:

Consider node a and b

Node a:

\[ \frac{V_a - 5}{1} + \frac{V_a}{4} + 6 = 0 \]

\[ V_a \left(1 + \frac{1}{4}\right) = -1 \]

\[ \Rightarrow \quad V_a = -0.8 \text{ V} \]

Node b:

\[ \frac{V_b - 5}{2} + \frac{V_b}{3} - 6 = 0 \]

\[ V_b \left(\frac{1}{2} + \frac{1}{3}\right) = 8.5 \]

\[ \Rightarrow \quad V_b = 10.2 \text{ V} \]

Power supplied = \( (V_{ba}) (6) = (10.2 + 0.8) 6 \)

\[ P = 66 \text{ W} \]

Q.4 For the circuit shown in the figure determine \( V_o/I_s \) using nodal analysis.

Solution:

\[ V = I_s \] \hspace{1cm} ...(1)

Node (1),

\[ \frac{V}{1} + \frac{V - V_o}{1} - I_s = 0 \]

\[ 3V - V_o = I_s \] \hspace{1cm} ...(2)

Node (2),

\[ \frac{V_o}{1} + \frac{V_o - V}{1} + 3I_b = 0 \]

\[ 2V_o - V = -3I_b \] \hspace{1cm} ...(3)

From equation (1), \( I_s = V_0 \) put in equation (3)

\[ 2V_o - V = -3I_b \]

\[ 2V_o = -2 \text{ V} \]

\[ V = -V_o \]

\[ \Rightarrow \]

Putting, \( V = -V_o \) in equation (2)

\[ 3(-V_o) - V_o = I_s \]

\[ -4V_o = I_s \]

\[ \frac{V_o}{I_s} = -\frac{1}{4} = -0.25 \]
Q.5 Write the node equations for the network shown in figure. Assume node (2) as a reference node.

Solution:

Node 1:
\[
\frac{V_1 - V_3 - 50}{5} + \frac{V_4 - V_3}{j2} + \frac{V_1}{4} = 0
\]
\[V_3(0.2 + 0.25 - j0.5) - V_3(0.2 - j0.5) = 10 \quad ...(1)\]

Node 3:
\[
\frac{V_3 - V_4 + 50}{5} + \frac{V_2 - V_4}{j2} + \frac{V_2 + j50}{2} = 0
\]
\[V_3\left(\frac{1}{5} + \frac{1}{j2} - \frac{1}{j2}\right) - \frac{V_1}{5} - \frac{V_1}{j2} + 10 + j25 = 0\]
\[V_3(0.7) - V_3(-j0.5 + 0.2) = -10 - j25\]
\[V_1(0.2 - j0.5) - 0.7V_3 = 10 + j25 \quad ...(2)\]

Nodal equation given by (1) and (2).

Q.6 A storage battery has a no-load terminal voltage of 6 V. When the current through the battery is 100 A, the terminal voltage drops to 5 V. Show a pictorial representation of the battery as a constant current source.

Solution:

\[
E = \text{Open voltage of battery} = 6 \text{ V}
\]
with a current of 100 A
\[V_f = E - (100) R_i\]
\[V_f = 5 \text{ V} = 6 - 100 R_i\]
\[R_i = 10 \text{ m}\Omega\]

\[
I_s = \frac{E}{R_i} = \frac{6}{10 \times 10^{-3}} = 600 \text{ A}
\]

Constant current source,
Q.16  Find the value of $R_L$ so that the maximum power is consumed in it.

$$R_{th} = R_L = [60 \ || \ 30 + 60 \ || \ 40] = 44 \Omega$$

Solution:
For maximum power transfer load impedance should be equal to source impedance i.e.

Q.17  For the network shown below, determine $R_L$ which will receive maximum power.

Solution:
Maximum power will be received by $R_L$ when the equivalent resistance of network across $R_L$ or internal resistance of Thevenin equivalent across $R_L$ is equal to $R_L$.

We need to find out $R_{th}$ for the current as circuit contains dependent voltage source, we can not find out $R_{th}$.
We will short all independent voltage source and will open all current source. Next we will place a voltage source of 1 V in place of $R_L$. Then we will find $I$ through this source.

$$R_{th} = \frac{1}{I}$$

Also,
\begin{align*}
-10I_1 + 5I + I - 1 &= 0 \\
10I + 5I + I - 1 &= 0
\end{align*}

⇒
\begin{align*}
I &= \frac{1}{16} A \\
R_{th} &= \frac{1}{I} = 16 \Omega
\end{align*}

For maximum power,$$R_L = R_{th} = 16 \Omega$$

Q.18  Use Thevenin’s theorem to find the current in 4 Ω branch of the network given in figure.

Q.19  Find the maximum power flowing into $R_L$.
Solution:

\( R_{\text{th}} \): To find \( R_{\text{th}} \), short all independent voltage source and open all independent current sources. Place 1 V in place of 4 Ω.

\[-10I_1 + 5I + I = 1\]
\[I = I_1\]
\[16I = 1\]

\[\Rightarrow\]
\[I = \frac{1}{16}\]
\[R_{\text{th}} = \frac{1}{I} = 16 \Omega\]

\( V_{\text{th}} \):

\[I_1 = -12 \text{ A}\]
\[10I_1 = 10(-12) = -120 \text{ V}\]

Voltage of node (x) = 12 + 12 \times 5 = 72 \text{ V}
Voltage of node a → 120 + 72 = 192 \text{ V}
Voltage of node b → 20 V
\[V_{ab} = 192 - 20 = 172 \text{ V}\]
\[V_{th} = V_{ab} = 172 \text{ V}\]

Current in 4 Ω = \[\frac{172}{16 + 4} = 8.6 \text{ A}\]
\[I = 8.6 \text{ A}\]

Q.19 Using Millman’s theorem, find the current-I in the 10-ohm resistor in figure.

[IAS-2011 : 20 marks]
Solution:

As per Millman’s theorem,

\[ V_{eq} = \sum_{i=1}^{n} \frac{V_i}{R_i} \]

\[ R_{eq} = \frac{1}{\sum_{i=1}^{n} \frac{1}{R_i}} \]

Millman’s equivalent,

\[ V_{eq} = \frac{E_1 + E_2}{R_1 + R_2} = \frac{24 + 20}{1 + 1} = \frac{12 + 10}{2} = 22 \text{ V} \]

\[ R_{eq} = \frac{1}{\frac{1}{2} + \frac{1}{2}} = 1 \Omega \]

\[ I_1 = \frac{22}{(1+2) + \frac{12 \times 9}{12 + 9}} = \frac{22}{3 + \frac{108}{21}} = 2.70 \text{ A} \]

\[ I = \frac{(9)I_1}{9 + 2 + 10} = \frac{9}{21}I_1 = \frac{2.7 \times 9}{21} = 1.16 \text{ A} \]

Current in 10 Ω resistor,

\[ I = 1.16 \text{ A} \]

Q.20 State and prove maximum power transfer theorem.

[IAS-2011 : 20 marks]

Solution:

**Maximum power transfer theorem**: An independent voltage source in series with an impedance \(Z_{th}\) or an independent current source in parallel with impedance \(Z_{th}\) delivers a maximum power to the load impedance \(Z_L\) when \(Z_L\) is equal to complex conjugate of \(Z_{th}\).

\[ Z_L = Z_{th} \]

\[ I = \frac{V_{th}}{Z_{th} + Z_L} = \frac{V_{th}}{(R_{th} + R_L) + j(X_{th} + X_L)} \]

Power,

\[ P = I^2 \cdot R_L = \frac{V_{th}^2 R_L}{(R_{th} + R_L)^2 + (X_{th} + X_L)^2} \]

...(1)

For maximum power,

\[ \frac{\partial P}{\partial X_L} = 0 \]
\[ \frac{\partial P}{\partial X_L} = \frac{-2(V_{th}^2)R_L(X_L + X_{th})}{[(R_L + R_{th})^2 + (X_{th} + X_L)^2]^2} = 0 \]

\[ \Rightarrow \]

\[ X_{th} + X_L = 0 \]
\[ X_{th} = -X_L \]

Putting, \[ X_{th} = -X_L \] in equation (1)

\[ P = \frac{V_{th}^2 R_L}{(R_{th} + R_L)^2} \]

For maximum power transfer, \[ \frac{\partial P}{\partial R_L} = 0 \]

\[ \frac{\partial P}{\partial R_L} = \frac{V_{th}^2 (R_{th} + R_L)^2 - 2(V_{th})^2 R_L (R_{th} + R_L)}{(R_{th} + R_L)^4} = 0 \]

\[ (R_{th} + R_L)^2 - 2R_L = 0 \]

So,

\[ R_{th} = R_L \]
\[ X_{th} = -X_L \]
\[ Z_L = Z_{th} \]

Q.21 Show that there can be no value of \( R_L \) in the circuit given in figure that will make it resonant.

![Circuit Diagram]

Solution:

\[ Y_{in} = \frac{1}{R_L + j10} + \frac{1}{4 - j5} = \frac{R_L - j10}{R_L^2 + 100} + \frac{4 + j5}{41} \]

\[ = \frac{R_L}{R_L^2 + 100} + \frac{4}{41} + \frac{j5}{41} - \frac{j10}{R_L^2 + 100} \]

For resonance \( Y_{in} \) must be real.

So,

\[ \frac{5}{41} = \frac{10}{R_L^2 + 100} \]

\[ 5R_L^2 + 500 = 410, \quad R_L^2 = -18 \]

Equation doesn’t satisfy for any value of \( R_L \).
So circuit can’t be resonant.

Q.22 State Tellegen’s theorem for network analysis.

![Circuit Diagram]

Solution:

Tellegen’s theorem is applicable for any lumped network having elements which are linear or non-linear active or passive, time varying or time-invariant. The theorem is based on KCL and KVL. It is completely independent of the nature of the element.

Consider a network with ‘b’ branches.

\( V_1, V_2, \ldots, V_b, i_1, i_2, \ldots, i_b \) are branch voltage and current then

\[ \sum_{K=1}^{b} V_{K}i_{K} = 0 \]
So Tellegen’s theorem:

- Not concerned with type of elements.
- Based on KCL and KVL.
- Reference direction of branch voltage and current are arbitrary.

Proof:

\[
\begin{bmatrix}
V_1, V_2, V_3, \ldots, V_p \\
n_1, n_2, n_3, \ldots, n_b
\end{bmatrix} \quad \text{Branch voltage and current}
\]

\[e_1, e_2, e_3, \ldots, e_p \rightarrow \text{node voltages}\]

Let \(K\)th branch connect nodes \(p\) and \(q\) with voltage \(e_p\) and \(e_q\). Then

\[
V_K i_K = (e_p - e_q) i_{pq} \quad \ldots(1)
\]

\[
V_K i_K = (e_q - e_p) i_{qp} \quad \ldots(2)
\]

Add equation (1) and (2),

\[
V_K i_K = \frac{1}{2} \left[ (e_p - e_q) i_{pq} + (e_q - e_p) i_{qp} \right]
\]

\[
\sum_{K=1}^{b} V_K i_K = \frac{1}{2} \sum_{p=1}^{b} \sum_{q=1}^{b} (e_p - e_q) i_{pq}
\]

\[
= \frac{1}{2} \sum_{p=1}^{b} e_p \sum_{q=1}^{b} i_{pq} - \frac{1}{2} \sum_{q=1}^{b} e_q \sum_{p=1}^{b} i_{pq} \quad \ldots(3)
\]

As per KCL algebraic sum of current is zero

\[
\sum_{p=1}^{b} i_{pq} = \sum_{p=1}^{b} i_{pq} = 0
\]

Putting in (3),

\[
\sum_{K=1}^{b} V_K i_K = 0
\]

Q.23 In the network shown in figure, determine the value of current through 1 \(\Omega\) resistance connected between terminals \(A\) and \(B\). Verify the answer using superposition theorem also.

![Network Diagram](image)

Solution:

KVL in loop 1:

\[
4 = 2I + 3(2 + I) + 1(2 + I) = 2I + 6 + 3I + 2 + I
\]

\[
4 = 6I + 8
\]

\[
6I = -4
\]

\[
\Rightarrow \quad I = -\frac{2}{3} \text{A}
\]

Current in 1 \(\Omega\) resistor,

\[
(2 + I) = 2 - \frac{2}{3} = \frac{4}{3} \text{A}
\]
Verification by superposition:

Only 4 V source:

\[ I_1 = \frac{4}{6} = \frac{2}{3} \text{ A} \]

Only 2 A current source,

\[ I_2 = \frac{2(2)}{2 + 4} = \frac{2}{3} \text{ A} \]

\[ I = I_1 + I_2 = \frac{2}{3} + \frac{2}{3} = \frac{4}{3} \text{ A} \]

Q.24 For the circuit shown in figure, find the current through 5 \( \Omega \) resistor by using Thevenin's theorem and verify the same by using superposition theorem.

Solution:

Let's remove 5 \( \Omega \) resistor and calculate \( R_{th} \) and \( V_{th} \)

**Calculation of \( R_{th} \):**

All voltage sources will be shorted,

\[ R_{th} = 10 \Omega \]
Calculation of $V_{th}$:

Applying KVL in the path shown,

$V_A - 100 - 20 + 50 = V_B$

$V_{th} = V_A - V_B = 70$ V

Thevenin’s equivalent:

$V_{th} = 70$ V

Verification by superposition:

Current due to $E_1$ in resistor $R$,

$I_1 = \frac{100}{15} = 6.67$ A

Current due to $E_2$,

$I_2 = 0$ A

Current will flow through shorted branch. Hence,

Current due to $E_3$,

$I_3 = \frac{20}{15} = 1.33$ A

Current due to $E_4$,

$I_4 = 0$ A
Current due to $E_5$:

$$I_5 = \frac{50}{15} = 3.33 \text{ A}$$

Current due to $E_6$:

$$I_6 = 0 \text{ A}$$

Applying superposition theorem:

Current through resistor,

$$R = I_1 + I_2 + I_3 + I_4 + I_5 + I_6$$

$$= 6.67 + 0 + 1.33 + 0 - 3.33 + 0 = 4.67 \text{ A}$$

Hence verified, as result is same both by Thevenin and superposition theorem.

Q.25 Using Thevenin’s theorem, find the current through the 40 $\Omega$ resistor connected between terminals $a$ and $b$ in figure.

![Diagram](IAS-2018 : 10 marks)

Solution:

Using Thevenin’s theorem, to find the current through the 40 $\Omega$ resistor, we consider Thevenin’s voltage ($V_{th}$) and Thevenin’s resistance ($R_{th}$).

Here, $V_{th}$ is found by open circuiting the circuit across 40 $\Omega$ resistor i.e. terminal $ab$. And $R_{th}$ is found by considering the equivalent resistance across terminal $ab$ by removing 40 $\Omega$ resistance.

To find $V_{th}$:

$$V_{th} = V_{ab} = V_a - V_b = \frac{1}{4} \times 220 - \frac{600}{1000} \times 220$$

$$V_{th} = 55 - 132 = -77 \text{ V}$$

The negative sign of $V_{th}$ shows that the potential of point $b$ is higher than that of point $a$.

To find $R_{th}$:

$$R_{th} = R_{ab} = \frac{3 \times 1}{3 + 1} \text{k}\Omega + \frac{0.4 \times 0.6}{0.4 + 0.6} \text{k}\Omega = 0.99 \text{k}\Omega$$

The Thevenin’s equivalent circuit

The current through 40 $\Omega$ resistance, $I$ is

$$I = \frac{V_{th}}{R_{th} + 40 \Omega} = \frac{-77}{0.99 + 0.04} \text{ mA}$$

$$= -74.757 \text{ mA}$$

So, the current through 40 $\Omega$ resistance is 74.757 mA and current flows from point $b$ to point $a$. 
\[ P(t) = \frac{V_m I_m}{2} \left[ \cos \theta + \cos(2\omega t - \theta) \right] \]

\[ P(t) = V_{rms} I_{rms} \cos \theta + V_{rms} I_{rms} \cos (2\omega t - \theta) \] ...

Instantaneous power consists of two components:
1. \( V_{rms} I_{rms} \cos \theta \)
2. \( V_{rms} I_{rms} \cos (2\omega t - \theta) \)

To calculate the average power in a single phase circuit we observe from equation (1) that it contains a cosine term which varies with time and average value of cosine function over a cycle is zero. So second term will not appear in average power.

\[ P_{av} = V_{rms} I_{rms} \cos \theta \]

Q.53 Two impedances \( Z_1 = 5 \, \Omega \) and \( Z_2 = (5 - jX_c) \, \Omega \), are connected in parallel and this combination is connected in series with \( Z_2 = (6.25 + j1.25) \, \Omega \). Determine the value of capacitance of \( X_c \) to achieve resonance if the supply is 100 V, 50 Hz.

[IAS-2016 : 10 marks]

Solution:

From the given data circuit is as under:

![Circuit Diagram]

Equivalent impedance of above circuit

\[ \frac{(5)(5 - jX_c)}{5 + 5 - jX_c} + 6.25 + j1.25 = \frac{(25 - j5X_c)(10 + jX_c)}{100 + X_c^2} + 6.25 + j1.25 \]

\[ = \frac{5X_c^2 + 250 - j25X_c}{100 + X_c^2} + 6.25 + j1.25 \]

For resonance imaginary part of the equivalent impedance = 0

\[ \frac{-25X_c}{100 + X_c^2} + 1.25 = 0 \]

Simplifying,

\[ X_c^2 + 100 - 20 \times C - (X_c - 10)^2 = 0 \]

\[ \Rightarrow X_c = 10 \]

Now,

\[ X_c = \frac{1}{\omega C} \]

\[ \omega = 2\pi f \]

\[ f = 50 \text{ Hz (given)} \]

So,

\[ \frac{1}{(2\pi)(50)(C)} = 10 \]

\[ C = \frac{1}{1000\pi} = 318.3 \mu F \]
Q.54  Consider the figure below:

$N_1$ and $N_2$ are two 2-port networks connected in parallel on both input port side as well as output port side, to form a composite 2-port network $N$ as indicated. $N_1$ and $N_2$ are defined by the Z-parameters as below:

\[
[Z_{n_1}] = \begin{bmatrix} 4 & 3 \\ 3 & 5 \end{bmatrix} \phantom{\Omega}, \quad [Z_{n_2}] = \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix} \phantom{\Omega}
\]

Obtain the Z-parameters for the composite 2-port network $N$.

[IAS-2002 : 20 marks]

Solution:

For parallel network, \[ [Y] = [Y_1] + [Y_2] \]

\[ [Y_{n_1}] = [Z_{n_1}]^{-1} \text{ or } [Z_{n}] = [Y_{n}]^{-1} \]

\[ [Y_{n_1}] = [Z_{n_1}]^{-1} = \frac{1}{Z_{n_1}} \text{adj} [Z_{n_1}] = \frac{1}{11} \begin{bmatrix} 5 & -3 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 0.45 & -0.27 \\ -0.27 & 0.36 \end{bmatrix} \]

\[ [Y_{n_2}] = [Z_{n_2}]^{-1} = \frac{1}{8} \begin{bmatrix} 4 & -2 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 0.5 & -0.25 \\ -0.25 & 0.375 \end{bmatrix} \]

\[ [Y_n] = [Y_{n_1}] + [Y_{n_2}] = \begin{bmatrix} 0.95 & -0.52 \\ -0.52 & 0.735 \end{bmatrix} \]

\[ [Z_n] = [Y_n]^{-1} \]

\[ [Y_n]^{-1} = (0.95)(0.735) - (-0.52)^2 = 0.698 - 0.270 = 0.427 \]

\[ [Z_n] = \frac{1}{0.427} \begin{bmatrix} 0.735 & 0.52 \\ 0.52 & 0.95 \end{bmatrix} = \begin{bmatrix} 1.72 & 1.22 \\ 1.22 & 2.22 \end{bmatrix} \]

Q.55  Determine y-parameters of the network shown in the below figure using z-parameters.

[IAS-2008 : 14 marks]
Solution:

The network is composed of two network is parallel,

![Diagram of the network](image)

**Network (I) z-parameter**

\[ z_{11} = \frac{V_1}{I_{11}} \big|_{I_2=0} = \frac{s+1}{s} = \left( \frac{s+1}{s} \right) \]

**Network symmetrical,**

\[ z_{11} = z_{22} = \left( \frac{s+1}{s} \right) \]

\[ z_{12} = \frac{V_1}{I_{12}} \big|_{I_2=0} \]

\[ V_1 = (I_2) (1) = I_2 \]

\[ z_{12} = 1 = z_{21} \text{ (Network is reciprocal)} \]

\[ [z_j] = \begin{bmatrix} \frac{s+1}{s} & 1 \\ 1 & \frac{s+1}{s} \end{bmatrix} \]

\[ [y_j] = [z_j]^{-1} = \frac{2s+1}{s^2} \begin{bmatrix} \frac{s+1}{s} & -1 \\ -1 & \frac{s+1}{s} \end{bmatrix} \]

**Network (II) z-parameter**

\[ z_{11} = z_{22} = \frac{V_1}{I_{11}} \big|_{I_2=0} = \frac{s+1}{s} \text{ (Symmetrical)} \]

\[ z_{21} = z_{12} = \frac{V_1}{I_{12}} \big|_{I_2=0} = \frac{1}{s} \text{ (Reciprocal)} \]

\[ [y_{11}] = [z_{11}]^{-1} = \begin{bmatrix} \frac{s+1}{s} & \frac{1}{s} \\ \frac{1}{s} & \frac{s+1}{s} \end{bmatrix}^{-1} = \frac{s+2}{s} \begin{bmatrix} \frac{s+1}{s} & -\frac{1}{s} \\ -\frac{1}{s} & \frac{s+1}{s} \end{bmatrix} \]

For parallel network,

\[ [y] = [y_j] + [y_{11}] \]

\[ [y] = \begin{bmatrix} \frac{(2s+1)(s+1)}{s^3} + \frac{(s+1)(s+2)}{s^2} - \left( \frac{2s+1}{s^2} \right) - \frac{s+2}{s^2} \\ -\frac{(2s+1)}{s^2} - \frac{s+2}{s^2} - \frac{(2s+1)(s+1)}{s^2} - \frac{(s+1)(s+2)}{s^2} \end{bmatrix} \]

\[ [y] = \begin{bmatrix} \frac{s^3 + 5s^2 + 5s + 1}{s^3} - \frac{3(s+1)}{s^2} \\ -\frac{3(s+1)}{s^2} - \frac{s^3 + 5s^2 + 5s + 1}{s^3} \end{bmatrix} \]
Q.56  A 2-port network with \( z \)-parameters given by \( z = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \) is connected with a similar 2-port network in cascade. Determine the overall \( z \)-parameters for the composite network.

\[ \text{[IAS-2008 : 6 marks]} \]

Solution:

In cascade connection transmission parameter are added. We will first convert \( z \)-parameter into \( [T] \) parameter and then we will add upto \( [T] \) parameter. To get \( [z] \) parameter we will convert combined \( T \) parameter into \( z \)-parameter.

\[
\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \Delta z \\ z_{21} & z_{22} \\ z_{21} & z_{21} \end{bmatrix}
\]

\( z_{11} = z_{21} = z_{12} = z_{22} = 1 \)
\( \Delta z = 0 \)

\[
\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}
\]

Net \([A] = [A_1][A_2] = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \]

\( A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \)

\( \Delta T = 1 \)

\[
[z] = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} A & \Delta T \\ C & D \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}
\]

Overall \( z \)-parameter of combination.

Q.57  State the conditions when a 2-port network is reciprocal and when it is symmetrical. Under what condition \( y \)-parameters of a 2-port network do not exist?

\[ \text{[IAS-2008 : 4 marks]} \]

Solution:

Reciprocal 2-port network: In this type of network, the ratio of excitation to response is invariant to the inter change of excitation port and response port.

Symmetrical 2-port network: In this type of network, the input and output ports can be interchanged without altering the port voltages and currents.

\( y \)-parameters do not exist for networks in which the two ports have same voltage. For these networks, the \( z \)-parameters form 0 singular matrix.

Q.58  Two-T networks are connected in cascade as shown in the below figure. Find a single T equivalent for the network.

\[ \text{[IAS-2008 : 6 marks]} \]
Solution:

Let's find $ABCD$ parameters for the network

\[ V_1 = A V_2 + B I_2 \]
\[ I_1 = C V_2 + D I_2 \]

\[ A = 2 \]
\[ C = \frac{1}{R} \]
\[ B = 3R \]
\[ D = 2 \]

Now, for the network,

\[
\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 2 & 3R \\ \frac{1}{R} & 2 \end{bmatrix} \begin{bmatrix} 2 & 3R \\ \frac{1}{R} & 2 \end{bmatrix} = \begin{bmatrix} 7 & 12R \\ 4 & 7 \end{bmatrix}
\]

For,

\[
\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}
\]

Let the equivalent $T$-network be,

\[ A = \frac{V_1}{V_3} \bigg|_{I_3=0} = \frac{R_1 + R_2}{R_2} = 7 \]

\[ R_1 = 6R_2 \]
\[ B = \frac{V_1}{I_3} \bigg|_{I_3=0} \]

\[ 12 = (R_1 + R_2) \frac{R_1}{R_2} + R_1 \]

\[ 12 = 6(R_1 + R_2) + R_1 \]

\[ 12 = 6(7R_2) + 6R_2 \]

\[ 12 = 48R_2 \]

\[ R_2 = \frac{1}{4} \Omega \]

\[ R_1 = \frac{3}{2} \Omega \]

\[ \therefore \]

Q.59 A Bridge-T network is made up of four capacitances, each having a value of 1 Farad. Determine $y$-parameters of this network, assuming that this Bridge-T network can be treated as parallel interconnection of two two-port networks.

Solution:

Bridge T-section:
It can be considered as two network in parallel

\[ \begin{align*}
    y_{11} &= \frac{I_1}{V_1} = s = y_{22} \quad \text{and} \quad y_{12} &= \frac{I_1}{V_2} = -s = y_{21} \\
    y_{-11} &= \frac{I_1}{V_2} = \frac{2s}{3} \\
    I_1 &= 1 + \frac{1}{s} = \frac{3}{2s} \\
    I_2 &= \frac{2s}{3} \\
    I &= \frac{2s}{3} + \frac{1}{s} = \frac{s}{3} \\
    I &= \frac{1}{s} + \frac{1}{s} = \frac{s}{3} \\
    y_{12} &= \frac{-s}{3} = y_{21} \quad \text{(reciprocal)}
\end{align*} \]

\[ [y] = \begin{bmatrix}
    \frac{2s}{3} & \frac{s}{3} \\
    \frac{s}{3} & \frac{2s}{3} \\
    \frac{-s}{3} & \frac{3}{3}
\end{bmatrix}, \quad [y]_{II} = \begin{bmatrix}
    s & -s \\
    -s & s
\end{bmatrix} \]

\[ [y] \text{ for Bridge-T } = [y]_{I} + [y]_{II} \]

\[ [y] = \begin{bmatrix}
    \frac{2s}{3} + s & -s - \frac{s}{3} \\
    -s - \frac{s}{3} & \frac{2s}{3} + s
\end{bmatrix} = \begin{bmatrix}
    \frac{5s}{3} & -\frac{4s}{3} \\
    -\frac{4s}{3} & \frac{5s}{3}
\end{bmatrix} \]