GATE 2020

Computer Science & Information Technology

- Fully solved with explanations
- Analysis of previous papers
- Topicwise presentation
- Thoroughly revised & updated
Preface

Over the period of time the GATE examination has become more challenging due to increasing number of candidates. Though every candidate has ability to succeed but competitive environment, in-depth knowledge, quality guidance and good source of study is required to achieve high level goals.

The new edition of GATE 2020 Solved Papers: Computer Science & IT has been fully revised, updated and edited. The whole book has been divided into topicwise sections.

At the beginning of each subject, analysis of previous papers are given to improve the understanding of subject.

I have true desire to serve student community by way of providing good source of study and quality guidance. I hope this book will be proved an important tool to succeed in GATE examination. Any suggestions from the readers for the improvement of this book are most welcome.

B. Singh (Ex. IES)
Chairman and Managing Director
MADE EASY Group
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# Contents

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Syllabus:

Mathematical Logic: Propositional and first order logic.
Set Theory & Algebra: Sets, relations, functions, partial orders and lattices. Groups.
Combinatorics: Counting, recurrence relations, generating functions.
Graph Theory: Connectivity, matching, coloring.
Linear Algebra: Matrices, determinants, system of linear equations, eigenvalues and eigenvectors, LU decomposition.
Calculus: Limits, continuity and differentiability. Maxima and minima. Mean value theorem. Integration.

Analysis of Previous GATE Papers

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1.1 Indicate which of the following well-formed formula are valid:
(a) \((P \Rightarrow Q) \land (Q \Rightarrow R) \Rightarrow (P \Rightarrow R)\)
(b) \((P \Rightarrow Q) \Rightarrow (\neg P \Rightarrow \neg Q)\)
(c) \((P \land (\neg P \lor \neg Q)) \Rightarrow Q\)
(d) \(((P \Rightarrow R) \lor (Q \Rightarrow R)) \Rightarrow ((P \lor Q) \Rightarrow R)\)

[1990 : 2 Marks]

1.2 Which of the following predicate calculus statements are valid:
(a) \((\forall x) P(x) \lor (\forall x) Q(x) \Rightarrow (\forall x) \{P(x) \lor Q(x)\}\)
(b) \((\exists x) P(x) \land (\exists x) Q(x) \Rightarrow (\exists x) \{P(x) \land Q(x)\}\)
(c) \((\exists x) P(x) \lor Q(x) \Rightarrow (\forall x) P(x) \lor (\forall x) Q(x)\)
(d) \((\exists x) \{P(x) \lor Q(x)\} \Rightarrow (\exists x) P(x) \lor (\forall x) Q(x)\)

[1992 : 1 Mark]

1.3 Which of the following is an tautology:
(a) \((a \lor b) \rightarrow (b \land c)\)  
(b) \((a \land b) \rightarrow (b \lor c)\)
(c) \((a \lor b) \rightarrow (b \rightarrow c)\)  
(d) \((a \lor b) \rightarrow (b \lor c)\)

[1992 : 1 Mark]

1.4 The proposition \( p \land (\neg p \lor q) \) is
(a) A tautology  
(b) \( \equiv (p \land q) \)
(c) \( \equiv (p \lor q) \)  
(d) A contradiction

[1993 : 1 Mark]

1.5 Let \( p \) and \( q \) be propositions. Using only the truth table decide whether \( p \equiv q \) does not imply \( p \rightarrow \neg q \) is true or false.

[1994 : 2 Marks]

1.6 If the proposition \( \neg p \Rightarrow q \) is true, then the truth value of the proposition \( \neg p \lor (p \Rightarrow q) \), where \( \neg \) is negation, \( \lor \) is inclusive or and \( \Rightarrow \) is implication, is
(a) True  
(b) Multiple valued
(c) False  
(d) Cannot be determined

[1995 : 2 Marks]

1.7 Which one of the following is false? Read \( \land \) as AND, \( \lor \) as OR, \( \neg \) as NOT, \( \rightarrow \) as one way implication and \( \leftrightarrow \) as two way implication.
(a) \((x \rightarrow y) \land x \rightarrow y\)
(b) \((\neg x \rightarrow y) \land (\neg x \rightarrow \neg y) \rightarrow x\)
(c) \((x \rightarrow (x \lor y))\)
(d) \((x \lor y) \leftrightarrow (\neg x \rightarrow \neg y)\)

[1996 : 2 Marks]

1.8 Let \( a, b, c, d \) be propositions. Assume that the equivalence \( a \leftrightarrow (b \lor \neg b) \) and \( b \leftrightarrow c \) hold. Then the truth-value of the formula \( (a \land b) \rightarrow (a \land c) \lor d \) is always
(a) True  
(b) False
(c) Same as the truth-value of \( b \)
(d) Same as the truth-value of \( d \)

[2000 : 2 Marks]

1.9 What is the converse of the following assertion?
I stay only if you go
(a) I stay if you go
(b) If I stay then you go
(c) If you do not go then I do not stay
(d) If I do not stay then you go

[2001 : 1 Mark]

1.10 Consider two well-formed formulas in propositional logic:
\( F_1 : P \Rightarrow \neg P \)
\( F_2 : (P \Rightarrow \neg P) \lor (\neg P \Rightarrow P) \)
Which of the following statements is correct?
(a) \( F_1 \) is satisfiable, \( F_2 \) is valid
(b) \( F_1 \) is unsatisfiable, \( F_2 \) is satisfiable
(c) \( F_1 \) is unsatisfiable, \( F_2 \) is valid
(d) \( F_1 \) and \( F_2 \) are both satisfiable

[2001 : 1 Mark]

1.11 “If \( X \) then \( Y \) unless \( Z \)” is represented by which of the following formulas in propositional logic?
\( (\neg \neg) \) is negation, \( \land \) is conjunction, and \( \Rightarrow \) is implication
(a) \((X \land \neg Z) \rightarrow Y\)  
(b) \((X \land Y) \rightarrow \neg Z\)
(c) \(X \rightarrow (Y \land \neg Z)\)  
(d) \((X \rightarrow Y) \land \neg Z\)

[2002 : 1 Mark]

1.12 Which of the following is a valid first order formula? (Here \( \alpha \) and \( \beta \) are first order formulae with \( x \) as their only free variable)
(a) \(((\forall x) [\alpha] \Rightarrow (\forall x) [\beta]) \Rightarrow (\forall x) [\alpha \Rightarrow \beta]\)
(b) \((\forall x) [\alpha] \Rightarrow (\exists x) [\alpha \land \beta]\)
(c) \((\forall x) [\alpha \lor \beta] \Rightarrow (\exists x) [\alpha] \Rightarrow (\forall x) [\alpha]\)
(d) \((\forall x) [\alpha \Rightarrow \beta] \Rightarrow ((\forall x) [\alpha] \Rightarrow (\forall x) [\beta])\)

[2003 : 2 Marks]
1.13 Consider the following formula \( \alpha \) and its two interpretations \( I_1 \) and \( I_2 \).

\[ \alpha : (\forall x) [P_x \iff (\forall y) [Q_{xy} \iff \neg Q_{yy}]] \]

\[ \Rightarrow (\forall x) \neg P_x \]

\( I_1 \) : Domain : the set of natural numbers

\[ P_x = 'x' \text{ is a prime number' \} \]

\[ Q_{xy} = 'y' \text{ divides } x' \]

\( I_2 \) : Same as \( I_1 \), except that \( P_x = 'x' \text{ is a composite number.'} \]

Which of the following statements is true?

(a) \( I_1 \) satisfies \( \alpha \), \( I_2 \) does not
(b) \( I_2 \) satisfies \( \alpha \), \( I_1 \) does not
(c) Neither \( I_1 \) nor \( I_2 \) satisfies \( \alpha \)
(d) Both \( I_1 \) and \( I_2 \) satisfy \( \alpha \)

[2003 : 2 Marks]

1.14 The following resolution rule is used in logic programming: Derive clause \( (P \lor Q) \) from clauses \( (P \lor R), (Q \lor \neg R) \)

Which of the following statements relating to this rule is FALSE?

(a) \( (P \lor R) \land (Q \lor \neg R) \Rightarrow (P \lor Q) \) is logically valid
(b) \( (P \lor Q) \Rightarrow (P \lor R) \land (Q \lor \neg R) \) is logically valid
(c) \( (P \lor Q) \) is satisfiable if and only if \( (P \lor R) \land (Q \lor \neg R) \) is satisfiable
(d) \( (P \lor Q) \Rightarrow \text{FALSE} \) if and only if both \( P \) and \( Q \) are unsatisfiable

[2003 : 2 Marks]

1.15 Identify the correct translation into logical notation of the following assertion. Some boys in the class are taller than all the girls

Note: Taller \((x, y)\) is true if \(x\) is taller than \(y\).

(a) \((\exists x)(\forall y)(x < y)\) (b) \((\exists x)(\forall y)(y < x)\)

[2004 : 1 Mark]

1.17 Let \( p, q, r \) and \( s \) be four primitive statements. Consider the following arguments:

\[ \begin{align*}
P : [\neg (p \lor q) \land (r \Rightarrow s) \land (p \lor r)] & \Rightarrow (\neg s \Rightarrow q) \\
Q : [\neg (p \land q) \land [q \Rightarrow (p \Rightarrow r)] & \Rightarrow \neg r \\
R : [(q \land r) \Rightarrow p] \land (\neg q \lor p) & \Rightarrow r \\
S : [p \land (p \Rightarrow r) \land (q \lor \neg r)] & \Rightarrow q
\end{align*} \]

Which of the above arguments are valid?

(a) \( P \) and \( Q \) only
(b) \( P \) and \( R \) only
(c) \( P \) and \( S \) only
(d) \( P, Q, R \) and \( S \)

[2004 : 2 Marks]

1.18 The following propositional statement is

\[ (P \Rightarrow (Q \lor R)) \Rightarrow ((P \land Q) \Rightarrow R) \]

(a) Satisfiable but not valid
(b) Valid
(c) A contradiction
(d) None of the above

[2004 : 2 Marks]

1.19 Let \( P, Q \) and \( R \) be three atomic propositional assertions. Let \( X \) denote \((P \lor Q) \Rightarrow R\) and \( Y \) denote \((P \Rightarrow R) \lor (Q \Rightarrow R)\). Which one of the following is a tautology?

(a) \( X = Y \)
(b) \( X \Rightarrow Y \)
(c) \( Y \Rightarrow X \)
(d) \( \neg Y \Rightarrow X \)

[2005 : 2 Marks]

1.20 What is the first order predicate calculus statement equivalent to the following? Every teacher is liked by some student

(a) \( (\forall x)[\text{teacher}(x) \Rightarrow \exists y \text{ student}(y) \land \text{likes}(y, x)] \)
(b) \( (\exists x)[\text{teacher}(x) \Rightarrow \exists y \text{ student}(y) \land \text{likes}(y, x)] \)
(c) \( \exists y (\forall x) [\text{teacher}(x) \Rightarrow [\text{student}(y) \land \text{likes}(y, x)]] \)
(d) \( \forall y (\exists x) [\text{teacher}(x) \land \exists y \text{ student}(y) \land \text{likes}(y, x)] \)

[2005 : 2 Marks]

1.21 Let \( P(x) \) and \( Q(x) \) be arbitrary predicates. Which of the following statements is always TRUE?

(a) \( (\forall x)(P(x) \lor Q(x)) \Rightarrow ((\forall x)P(x)) \lor ((\forall x)Q(x)) \)
(b) \( (\forall x)(P(x) \Rightarrow Q(x)) \Rightarrow ((\forall x)P(x)) \Rightarrow (\forall x)Q(x) \)
(c) \( (\forall x)(P(x) \Rightarrow (\forall x)Q(x))) \Rightarrow ((\forall x)P(x) \Rightarrow Q(x)) \)
(d) \( ((\forall x)P(x)) \Rightarrow ((\forall x)Q(x))) \Rightarrow ((\forall x)(P(x) \Rightarrow Q(x))) \)

[IT-2005 : 2 Marks]

1.22 Consider the following first order logic formula in which \( R \) is a binary relation symbol.

\[ \forall x \forall y \ (R(x, y) \Rightarrow R(y, x)) \]

The formula is
(a) Satisfiable and valid
(b) Satisfiable and so is its negation
(c) Unsatisfiable but its negation is valid
(d) Satisfiable but its negation is unsatisfiable

[IT-2006 : 2 Marks]

1.23 Which of the first order predicate calculus statements given below correctly expresses the following English statement? Tigers and lions attack if they are hungry or threatened.
(a) \( \forall x \ [(\text{tiger}(x) \land \text{lion}(x)) \rightarrow (\text{hungry}(x) \lor \text{threatened}(x)) \rightarrow \text{attacks}(x)] \)
(b) \( \forall x \ [(\text{tiger}(x) \lor \text{lion}(x)) \rightarrow (\text{hungry}(x) \lor \text{threatened}(x)) \land \text{attacks}(x)] \)
(c) \( \forall x \ [(\text{tiger}(x) \lor \text{lion}(x)) \rightarrow (\text{attacks}(x) \land (\text{hungry}(x) \lor \text{threatened}(x))) \]
(d) \( \forall x \ [(\text{tiger}(x) \lor \text{lion}(x)) \rightarrow (\text{hungry}(x) \lor \text{threatened}(x)) \rightarrow \text{attacks}(x)] \)

[2006 : 2 Marks]

1.24 Consider the following propositional statements:
\( P_1 : (A \land B) \rightarrow C = ((A \land C) \lor (B \rightarrow C)) \)
\( P_2 : (A \lor B) \rightarrow C = ((A \lor C) \land (B \rightarrow C)) \)
Which one of the following is true?
(a) \( P_1 \) is a tautology, but not \( P_2 \)
(b) \( P_2 \) is a tautology, but not \( P_1 \)
(c) \( P_1 \) and \( P_2 \) are both tautologies
(d) Both \( P_1 \) and \( P_2 \) are not tautologies

[2006 : 2 Marks]

1.25 A logical binary relation \( \odot \), is defined as follows:
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Let \( \sim \) be the unary negation (NOT) operator, with higher precedence, than \( \odot \). Which one of the following is equivalent to \( A \land B \)?
(a) \( \sim (A \odot B) \)
(b) \( \sim (A \land B) \)
(c) \( \sim (A \odot B) \)
(d) \( \sim (A \land B) \)

[2006 : 2 Marks]

1.26 Let \( \text{Graph}(x) \) be a predicate which denotes that \( x \) is a graph. Let \( \text{Connected}(x) \) be a predicate which denotes that \( x \) is connected. Which of the following first order logic sentences DOES NOT represent the statement; “Not every graph is connected”?
(a) \( \sim \forall x \ (\text{Graph}(x) \rightarrow \text{Connected}(x)) \)
(b) \( \exists x \ (\text{Graph}(x) \land \sim \text{Connected}(x)) \)
(c) \( \sim \forall x \ (\neg \text{Graph}(x) \lor \text{Connected}(x)) \)
(d) \( \forall x \ (\text{Graph}(x) \Rightarrow \sim \text{Connected}(x)) \)

[2007 : 2 Marks]

1.27 Which of the following is TRUE about formulae in Conjunctive Normal Form?
(a) For any formula, there is a truth assignment for which at least half the clauses evaluate to true.
(b) For any formula, there is a truth assignment for which all the clauses evaluate to true.
(c) There is a formula such that for each truth assignment at most one-fourth of the clauses evaluate to true.
(d) None of the above

[2007 : 2 Marks]

1.28 Which one of these first-order logic formulae is valid?
(a) \( \forall x \ (P(x) \land Q(x)) \Rightarrow ((\forall x P(x)) \Rightarrow (\forall x Q(x))) \)
(b) \( \exists x \ (P(x) \lor Q(x)) \Rightarrow ((\exists x P(x)) \Rightarrow (\exists x Q(x))) \)
(c) \( \exists x \ (P(x) \land Q(x)) \Rightarrow ((\exists x P(x)) \land (\exists x Q(x))) \)
(d) \( \forall x \exists y \ P(x, y) \Rightarrow \exists y \forall x P(x, y) \)

[IT-2007 : 2 Marks]

1.29 Let \( \text{fsa} \) and \( \text{pda} \) be two predicates such that \( \text{fsa}(x) \) means \( x \) is a finite state automaton, and \( \text{pda}(y) \) means, that \( y \) is a pushdown automaton. Let equivalent be another predicate such that equivalent \( (a, b) \) means \( a \) and \( b \) are equivalent. Which of the following first order logic statement represents the following:
Each finite state automaton has an equivalent pushdown automaton.
(a) \( \forall x \ (\text{fsa}(x) \Rightarrow (\exists y \ (\text{pda}(y) \land \text{equivalent}(x, y))) \)
(b) \( \sim \forall x \ (\sim \exists y \ (\text{fsa}(x) \Rightarrow (\text{pda}(y) \land \text{equivalent}(x, y)))) \)
(c) \( \forall x \ (\exists y \ (\text{fsa}(x) \land \text{pda}(y) \land \text{equivalent}(x, y))) \)
(d) \( \forall x \ (\exists y \ (\text{fsa}(x) \land \text{pda}(x) \land \text{equivalent}(x, y))) \)

[2008 : 1 Mark]

1.30 Which of the following first order formulae is logically valid? Here \( a(x) \) is a first order formula with \( x \) as a free variable, and \( \beta \) is a first order formula with no free variable.
(a) \([\beta \rightarrow (\exists x, \alpha(x))] \rightarrow [\forall x, \beta \rightarrow \alpha(x)]\)
(b) \([\exists x, (\beta \rightarrow \alpha(x))] \rightarrow [\beta \rightarrow (\forall x, \alpha(x))]\)
(c) \([\exists x, \alpha(x)] \rightarrow [\beta \rightarrow (\forall x, \alpha(x))]\)
(d) \([\forall x, \alpha(x)] \rightarrow [\beta \rightarrow (\forall x, \alpha(x))]\)

[IT-2008 : 2 Marks]

1.31 Which of the following is the negation of
\([\forall x, \alpha \rightarrow (\exists y, \beta \rightarrow (\forall u, \exists u, \gamma))]\)
(a) \([\exists x, \alpha \rightarrow (\forall y, \beta \rightarrow (\exists u, \forall v, \gamma))]\)
(b) \([\exists x, \alpha \rightarrow (\forall y, \beta \rightarrow (\forall u, \forall v, \neg \gamma))]\)
(c) \([\forall x, \neg \alpha \rightarrow (\exists y, \neg \beta \rightarrow (\forall u, \exists u, \neg \gamma))]\)
(d) \([\forall x, \alpha \land (\exists y, \beta \land (\exists u, \forall v, \neg \gamma))]\)

[IT-2008 : 2 Marks]

1.32 P and Q are two propositions. Which of the following logical expressions are equivalent?
1. \(P \lor \neg Q\)
2. \(\neg (P \land Q)\)
3. \((P \land Q) \lor (P \land \neg Q) \lor (\neg P \land \neg Q)\)
4. \((P \lor Q) \lor (P \land \neg Q) \lor (\neg P \land Q)\)
(a) Only 1 and 2 (b) Only 1, 2 and 3 (c) Only 1, 2 and 4 (d) All of these

[2008 : 2 Marks]

1.33 Which one of the following is the most appropriate logical formula to represent the statement:
“Gold and silver ornaments are precious”
The following notations are used:
\(G(x): x\) is a gold ornament
\(S(x): x\) is a silver ornament
\(P(x): x\) is precious
(a) \(\forall x (P(x) \rightarrow (G(x) \land S(x)))\)
(b) \(\forall x(G(x) \land (S(x) \rightarrow P(x)))\)
(c) \(\exists x ((G(x) \land S(x)) \rightarrow P(x))\)
(d) \(\forall x ((G(x) \lor S(x)) \rightarrow P(x))\)

[2009 : 2 Marks]

1.34 The binary operation \(\square\) is defined as follows:

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>P (\square) Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

Which one of the following is equivalent to \(P \lor Q\)?
(a) \(\neg Q \or \neg P\)
(b) \(P \lor \neg Q\)
(c) \(\neg P \or Q\)
(d) \(\neg P \or \neg Q\)

[2009 : 2 Marks]

1.35 Consider the following well-formed formulae:
I. \(\neg \forall x (P(x))\)
II. \(\neg \exists x (P(x))\)
III. \(\neg \exists x (\neg P(x))\)
IV. \(\exists x (\neg P(x))\)
Which of the above are equivalent?
(a) I and III (b) I and IV (c) II and III (d) II and IV

[2009 : 2 Marks]

1.36 Suppose the predicate \(F(x, y, t)\) is used to represent the statement that person \(x\) can fool person \(y\) at time \(t\). Which one of the statements below expresses best the meaning of the formula \(\forall x \exists y \exists t (\neg F(x, y, t))\)?
(a) Everyone can fool some person at some time
(b) No one can fool everyone all the time
(c) Everyone cannot fool some person all the time
(d) No one can fool some person at some time

[2010 : 2 Marks]

1.37 Which one of the following options is CORRECT given three positive integers \(x, y\) and \(z\), and a predicate
\(P(x) = \neg (x = 1) \land \forall y (\exists z (x = y \times z))\)
\(\Rightarrow (y = x) \lor (y = 1)\)
(a) \(P(x)\) being true means that \(x\) is a prime number
(b) \(P(x)\) being true means that \(x\) is a number other than 1
(c) \(P(x)\) is always true irrespective of the value of \(x\)
(d) \(P(x)\) being true means that \(x\) has exactly two factors other than 1 and \(x\)

[2011 : 2 Marks]

1.38 Consider the following logical inferences.
\(I_1: \) If it rains then the cricket match will not be played.
The cricket match was played.
Inference: There was no rain.
\(I_2: \) If it rains then the cricket match will not be played.
It did not rain.
Inference: The cricket match was played.
Which of the following is TRUE?
(a) Both \(I_1\) and \(I_2\) are correct inferences
(b) \(I_1\) is correct but \(I_2\) is not a correct inference
(c) \(I_1\) is not correct but \(I_2\) is a correct inference
(d) Both \(I_1\) and \(I_2\) are not correct inferences

[2012 : 1 Mark]
1.39 What is the correct translation of the following statement into mathematical logic?

“Some real numbers are rational”
(a) \( \exists x \) (real \( x \) \& rational \( x \))
(b) \( \forall x \) (real \( x \) \implies rational \( x \))
(c) \( \exists x \) (real \( x \) \& rational \( x \))
(d) \( \exists x \) (rational \( x \) \implies real \( x \))

[2012 : 1 Mark]

1.40 What is the logical translation of the following statements?

“None of my friends are perfect”
(a) \( \exists x \) (\( F(x) \wedge \neg P(x) \))
(b) \( \forall x \) (\( \neg F(x) \wedge P(x) \))
(c) \( \exists x \) (\( \neg F(x) \wedge \neg P(x) \))
(d) \( \neg \exists x \) (\( F(x) \wedge P(x) \))

[2013 : 2 Marks]

1.41 Which one of the following is NOT logically equivalent to \( \neg \exists x (\forall y (\alpha) \land \forall z (\beta)) \)?
(a) \( \forall z (\exists z (\neg \beta) \rightarrow \forall y (\alpha)) \)
(b) \( \forall x (\exists y (\beta) \rightarrow \exists z (\neg \alpha)) \)
(c) \( \forall x (\forall y (\alpha) \land \exists z (\neg \beta)) \)
(d) \( \forall x (\exists y (\neg \alpha) \lor \exists z (\neg \beta)) \)

[2013 : 2 Marks]

1.42 Consider the statement:

“Not all glitters is gold”
Predicate glitters\( x \) is true if \( x \) glitters and
predicate gold\( x \) is true if \( x \) is gold. Which one of the following logical formulae represents the above statement?
(a) \( \forall x \) : glitters\( x \) \implies \neg gold\( x \)
(b) \( \forall x \) : gold\( x \) \implies glitters\( x \)
(c) \( \exists x \) : gold\( x \) \& \neg glitters\( x \)
(d) \( \exists x \) : glitters\( x \) \& \neg gold\( x \)

[2014 (Set-1) : 1 Mark]

1.43 Which one of the following propositional logic formulas is TRUE when exactly two of \( p, q, \) and \( r \) are TRUE?
(a) \( (p \leftrightarrow q) \wedge r \) \lor (\( p \land q \land \neg r \))
(b) \( (p \leftrightarrow q) \wedge r \) \lor (\( p \land q \land r \))
(c) \( (p \rightarrow q) \wedge r \) \lor (\( p \land q \land \neg r \))
(d) \( (p \leftrightarrow q) \wedge r \) \lor (\( p \land q \land r \))

[2014 (Set-1) : 2 Marks]

1.44 Which one of the following Boolean expressions is NOT a tautology?
(a) \( (a \rightarrow b) \land (b \rightarrow c) \rightarrow (a \rightarrow c) \)
(b) \( (a \leftrightarrow c) \rightarrow (\neg b \rightarrow (a \land c)) \)
(c) \( (a \land b \land c) \rightarrow (c \land a) \)
(d) \( a \rightarrow (b \rightarrow a) \)

[2014 (Set-2) : 2 Marks]

1.45 Consider the following statements:
P: Good mobile phones are not cheap
Q: Cheap mobile phones are not good
L: P implies Q
M: Q implies P
N: P is equivalent to Q
Which one of the following about L, M, and N is CORRECT?
(a) Only L is TRUE (b) Only M is TRUE (c) Only N is TRUE (d) L, M and N are TRUE.

[2014 (Set-3) : 1 Mark]

1.46 The CORRECT formula for the sentence, “not all rainy days are cold” is
(a) \( \forall x (\neg (\text{Rainy}(x) \lor \neg \text{Cold}(x))) \)
(b) \( \forall x (\neg \text{Rainy}(x) \rightarrow \text{Cold}(x)) \)
(c) \( \exists x (\neg \text{Rainy}(x) \rightarrow \text{Cold}(x)) \)
(d) \( \exists x (\text{Rainy}(x) \lor \neg \text{Cold}(x)) \)

[2014 (Set-3) : 2 Marks]

1.47 Which one of the following is Not equivalent to \( p \leftrightarrow q \)?
(a) \( (\neg p \lor q) \land (p \lor \neg q) \)
(b) \( (\neg p \lor q) \land (q \rightarrow p) \)
(c) \( (\neg p \land q) \lor (p \land \neg q) \)
(d) \( (\neg p \land \neg q) \lor (p \land q) \)

[2015 (Set-1) : 1 Mark]

1.48 The binary operator \( \neq \) is defined by the following truth table.

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( p \neq q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
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<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Which one of the following is true about the binary operator \( \neq \)?
(a) Both commutative and associative  
(b) Commutative but not associative  
(c) Not commutative but associative  
(d) Neither commutative nor associative  

[2015 (Set-1) : 2 Marks]

1.49 Consider the following two statements:  

$S_1$: If a candidate is known to be corrupt, then he will not be elected.  

$S_2$: If a candidate is kind, he will be elected.  

Which one of the following statements follows from $S_1$ and $S_2$ as per sound inference rules of logic?  

(a) If a person is known to be corrupt, he is kind  
(b) If a person is not known to be corrupt, he is not kind  
(c) If a person is kind, he is not known to be corrupt  
(d) If a person is not known, he is not known to be corrupt  

[2015 (Set-2) : 1 Mark]

1.50 Which one of the following well formed formulae is a tautology?  

(a) $\forall x \exists y \ R(x, y) \iff \exists y \forall x \ R(x, y)$  
(b) $(\forall x [\exists y \ R(x, y) \rightarrow S(x, y)]) \rightarrow \forall x \exists y S(x, y)$  
(c) $[\forall x \exists y (P(x, y) \rightarrow R(x, y))]$  
(d) $\forall x \forall y P(x, y) \rightarrow \forall x \forall y P(y, x)$  

[2015 (Set-2) : 2 Marks]

1.51 In a room there are only two types of people, namely Type 1 and Type 2. Type 1 people always tell the truth and Type 2 people always lie. You give a fair coin to a person in that room, without knowing which type he is from and tell him to toss it and hide the result from you till you ask for it. Upon asking, the person replies the following:  

"The result of the toss is head if and only if I am telling the truth."  

Which of the following options is correct?  

(a) The result is head  
(b) The result is tail  
(c) If the person is of Type 2, then the result is tail  
(d) If the person is of Type 1, then the result is tail  

[2015 (Set-3) : 1 Mark]

1.52 Let $p, q, r, s$ represent the following propositions.  

\begin{align*}  
p : & \ x \in \{8, 9, 10, 11, 12\} 
q : & \ x \text{ is a composite number} 
r : & \ x \text{ is a perfect square} 
s : & \ x \text{ is a prime number} 
\end{align*}

The integer $x \geq 2$ which satisfies $\neg ((p \Rightarrow q) \land (r \lor \neg s))$ is ______.  

[2016 (Set-1) : 1 Mark]

1.53 Consider the following expressions:  

(i) False  
(ii) $Q$  
(iii) True  
(iv) $P \lor Q$  
(v) $\neg Q \lor P$  

The number of expressions given above that are logically implied by $P \land (P \Rightarrow Q)$ is ______.  

[2016 (Set-2) : 1 Mark]

1.54 Which one of the following well-formed formulae in predicate calculus is NOT valid?  

(a) $(\forall x p(x) \Rightarrow \forall x q(x)) \Rightarrow (\exists x p(x) \lor \forall x q(x))$  
(b) $(\exists x p(x) \lor \exists x q(x)) \Rightarrow \exists x (p(x) \lor q(x))$  
(c) $\exists x (p(x) \land q(x)) \Rightarrow (\exists x p(x) \land \exists x q(x))$  
(d) $\forall x (p(x) \lor q(x)) \Rightarrow (\forall x p(x) \lor \forall x q(x))$  

[2016 (Set-2) : 2 Marks]

1.55 Consider the first-order logic sentence $F : \forall x(\exists y R(x, y))$. Assuming non-empty logical domains, which of the sentences below are implied by $F$?  

I. $\exists y(\exists x R(x, y))$  
II. $\exists y(\forall x R(x, y))$  
III. $\forall y(\exists x R(x, y))$  
IV. $\neg \exists x(\forall y \neg R(x, y))$  

(a) IV only  
(b) I and IV only  
(c) II only  
(d) II and III only  

[2017 (Set-1) : 1 Mark]

1.56 The statement $\neg p \Rightarrow \neg q$ is logically equivalent to which of the statements below?  

I. $p \Rightarrow q$  
II. $q \Rightarrow p$  
III. $(\neg q) \lor p$  
IV. $(\neg p) \lor q$  

(a) I only  
(b) I and IV only  
(c) II only  
(d) II and III only  

[2017 (Set-1) : 1 Mark]
1.57 Let $p$, $q$ and $r$ be propositions and the expression $(p \rightarrow q) \rightarrow r$ be a contradiction. Then, the expression $(r \rightarrow p) \rightarrow q$ is
   (a) A tautology
   (b) A contradiction
   (c) Always TRUE when $p$ is FALSE
   (d) Always TRUE when $q$ is TRUE

[2017 (Set-1) : 2 Marks]

1.58 Let $p$, $q$, $r$ denote the statements “It is raining”, “It is cold”, and “It is pleasant”, respectively. Then the statement “It is not raining and it is pleasant, and it is not pleasant only if it is raining and it is cold” is represented by
   (a) $(\neg p \land r) \land (\neg r \rightarrow (p \land q))$
   (b) $(\neg p \land r) \land (p \land q) \rightarrow \neg r$
   (c) $(\neg p \land r) \lor ((p \land q) \rightarrow \neg r)$
   (d) $(\neg p \land r) \lor (r \rightarrow (p \land q))$

[2017 (Set-2) : 1 Mark]

1.59 Consider the first order predicate formula $\varphi$:
   $$\forall x \exists y \exists z \left( x \Rightarrow ((x = y) \lor (z = 1)) \Rightarrow \exists w \ (w > x) \land (\forall z \ w \Rightarrow ((w = z) \lor (z = 1))) \right)$$
   Here $a \mid b$ denotes that $a$ divides $b$, where $a$ and $b$ are integers. Consider the following sets:
   $S_1 : \{1, 2, 3, ..., 100\}$
   $S_2 :$ Set of all positive integers
   $S_3 :$ Set of all integers
   Which of the above sets satisfy $\varphi$?
   (a) $S_1$ and $S_3$  (b) $S_2$ and $S_3$
   (c) $S_1$, $S_2$ and $S_3$  (d) $S_1$ and $S_2$

[2019 : 2 Marks]
Answers | Mathematical Logic
--- | ---
1.1 (a)  | 1.2 (a)  | 1.3 (b)  | 1.4 (b)  | 1.5 (d)  | 1.6 (d)  | 1.7 (d)  | 1.8 (a)  | 1.9 (a)  | 1.10 (a)  
1.11 (a)  | 1.12 (d)  | 1.13 (d)  | 1.14 (b)  | 1.15 (d)  | 1.16 (c)  | 1.17 (c)  | 1.18 (a)  | 1.19 (b)  
1.20 (b)  | 1.21 (d)  | 1.22 (b)  | 1.23 (d)  | 1.24 (d)  | 1.25 (d)  | 1.26 (d)  | 1.27 (a)  | 1.28 (a)  
1.29 (a)  | 1.30 (c)  | 1.31 (d)  | 1.32 (b)  | 1.33 (d)  | 1.34 (b)  | 1.35 (b)  | 1.36 (b)  | 1.37 (a)  
1.38 (b)  | 1.39 (c)  | 1.40 (d)  | 1.41 (a)  | 1.42 (d)  | 1.43 (b)  | 1.44 (b)  | 1.45 (d)  | 1.46 (d)  
1.47 (c)  | 1.48 (a)  | 1.49 (c)  | 1.50 (c)  | 1.51 (a)  | 1.54 (d)  | 1.55 (b)  | 1.56 (d)  | 1.57 (d)  
1.58 (a)  | 1.59. (b)  

Explanations | Mathematical Logic
--- | ---

### 1.1 (a)

Option (a) is well known valid formula (tautology) because it is a rule of inference called hypothetical syllogism.

By applying boolean algebra and simplifying, we can show that (b), (c) and (d) are invalid.

For example,

Choice (b) \( \equiv (P \Rightarrow Q) \Rightarrow (\neg P \Rightarrow \neg Q) \)

\[ \equiv (P \Rightarrow Q) \Rightarrow (P' \Rightarrow Q') \]

\[ \equiv (P' + Q) \Rightarrow (P + Q') \]

\[ \equiv (P' + Q)' + (P + Q') \]

\[ \equiv PQ' + P + Q' \]

\[ \equiv P + Q' \]

\[ \neq 1 \]

So, invalid.

### 1.2 (a)

According to distributive properties

\[ \forall x \left( P(x) \land Q(x) \right) \leftrightarrow \forall x P(x) \land \forall x Q(x) \]

\[ \left( \forall x P(x) \lor \forall x Q(x) \right) \rightarrow \forall x \left( P(x) \lor Q(x) \right) \]

\[ \exists x \left( P(x) \lor Q(x) \right) \leftrightarrow \exists x P(x) \lor \exists x Q(x) \]

\[ \exists x \left( P(x) \land Q(x) \right) \rightarrow \exists x P(x) \land \exists x Q(x) \]

So option (a) is valid.

### 1.3 (b)

(a) \( (a \lor b) \rightarrow (b \land c) \)

\[ = (a' + b') + bc \]

\[ = a'b' + bc \]

Therefore, \((a \lor b) \rightarrow (b \land c)\) is contingency and not tautology.

(b) \( (a \land b) \rightarrow (b \lor c) \)

\[ = ab \rightarrow b + c \]

\[ = (ab') + b + c \]

\[ = a' + b' + b + c \]

### 1.4 (b)

The proposition \( p \land (\neg p \lor q) \)

\[ = p \lor q \]

\[ = p \land q \]

### 1.5 Sol.

**TRUE**

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( p \leftrightarrow q )</th>
<th>( p \rightarrow q )</th>
<th>( (p \leftrightarrow q) \rightarrow (p \rightarrow q) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
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<td>F</td>
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<td>T</td>
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</tbody>
</table>

From the truth table, \( p \leftrightarrow q \rightarrow (p \rightarrow q) \) is not tautology, hence it is true that \( p \leftrightarrow q \) doesn't imply \( p \rightarrow q \).
### 1.6 (d)

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( \neg p \lor q )</th>
</tr>
</thead>
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<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Now since \( \neg p \rightarrow q \) is given true, we reduce the truth table as follows:

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( \neg p \rightarrow q )</th>
</tr>
</thead>
<tbody>
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<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

In the reduced truth table we need to find the truth value of \( \neg p \lor (p \rightarrow q) \equiv p' + (p \rightarrow q) \equiv p' + p + q \equiv p' + q \)

The truth value of \( p' + q \) in the reduced truth table is given below:

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( p' + q )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Since in the reduced truth table also, the given expression is sometimes true and sometimes false, therefore the truth value of proposition \( \neg p \lor (p \rightarrow q) \) cannot be determined.

### 1.7 (d)

(a) \((x \rightarrow y) \land (y \rightarrow x) \rightarrow y\)

\[= \neg((\neg x \land y) \land y) \lor y\]

\[= \neg(\neg x \land y) \lor (y \land y) \lor y\]

\[= (\neg y \land y) \lor (y \land y) \lor y\]

\[= (\neg y \lor y) \lor y\]

\[= T \lor y\]

\[= T\]

(b) \(((x \rightarrow y) \land \neg (y \rightarrow x)) \rightarrow x\)

\[= \neg((\neg x \land y) \land \neg y) \lor x\]

\[= ((x 

\[= (x \land y) \land (x \land \neg y) \lor x\]

\[= (x 

\[= \neg(x \land y) \land x\]

\[= \neg x \lor x\]

\[= T\]

(c) \((x \rightarrow (y \lor x))\)

\[= (\neg x \lor (x \land y))\]

\[= (x \land y)\]

\[= (x \lor y)\]

\[= (x \lor y)\]

\[= (x \lor y)\]

\[= (x \lor y)\]

\[= (x \lor y)\]

\[= (x \lor y)\]

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Now convert the answers one-by-one into boolean form only choice (a) converts to \(X' + Y + Z\) as can be seen below:

\[
\begin{align*}
(\forall x) [\alpha \Rightarrow \beta] & \Rightarrow (\forall x) [\alpha] \Rightarrow (\exists x) [\beta]
\end{align*}
\]

So the notation for the given statement is

\[(\exists x) (\text{boy}(x) \land (\forall y) (\text{girl}(y) \rightarrow \text{taller}(x, y)))\]

1.16 (c)

Choice (c) is

\[-(\forall x)(\exists y)((a(x, y) \land b(x, y)) \rightarrow c(x, y))\]

\[
\begin{align*}
&= -(\forall x)(\exists y)(a \land b \rightarrow c) \\
&= -(\forall x)(\exists y)((ab') + c) \\
&= \exists x \forall y[(ab') + c] \\
&= \exists x \forall y[(abc')] \equiv \exists x \forall y[a \land b \land \neg c]
\end{align*}
\]

which is same as the given expression.

\[(\exists x)(\forall y)[(a(x, y) \land b(x, y)) \land \neg c(x, y)]\]

1.17 (c)

\[P: [(\neg p \lor q) \land (r \rightarrow s) \land (p \lor r)] \rightarrow (\neg s \rightarrow q)\]

\[
\begin{align*}
&= [(p \rightarrow q) \land (r \rightarrow s) \land (p \lor r)] \rightarrow (q \lor s) \\
&= prq \rightarrow q \\
&= (prq)' + q \\
&= p' + q' + q \\
&= p' + r' + q' + q \\
&= 1
\end{align*}
\]

Therefore S is valid.

Q and R can be similarly simplified in boolean algebra to show that they are both not equivalent to 1.

So only P and S are valid.

1.18 (a)

\[(P \rightarrow (Q \lor R)) \rightarrow ((P \land Q) \rightarrow R)\]

\[
\begin{align*}
&= (P \rightarrow Q + R) \rightarrow (PQ) \rightarrow R \\
&= [P' + Q + R] \rightarrow [(PQ)' + R] \\
&= [P' + Q + R] \rightarrow [P' + Q' + R] \\
&= (P' + Q + R)' \lor P' + Q' + R \\
&= P'Q' + P'Q' + R \\
&= Q' + Q'P + Q' + R \\
&= Q' + P' + R \text{ (by absorption law)}
\end{align*}
\]

Which is a contingency (i.e. satisfiable but not valid).

1.19 (b)

\[X: (P \lor Q) \rightarrow R\]

\[Y: (P \lor R) \lor (Q \lor R)\]

\[X: P + Q \rightarrow R \equiv (P + Q)' + R \equiv P'Q' + R\]

\[Y: (P' + R) + (Q' + R) = P' + Q' + R\]
Clearly $X \not\equiv Y$

Consider $X \rightarrow Y$

\[\equiv (P'Q' + R) \rightarrow (P' + Q' + R)\]
\[\equiv (P'Q' + R') + P' + Q' + R\]
\[\equiv (P'Q') \cdot R' + P' + Q' + R\]
\[\equiv P'R' + Q'R' + P' + Q' + R\]
\[\equiv (P + R)(R' + R) + (Q + Q') \times (R' + Q') + P'\]
\[\equiv (P + R) + (R' + Q') + P'\]
\[\equiv P + P' + R + R' + Q'\]
\[\equiv 1 + 1 + Q'\]
\[\equiv 1\]

\[\therefore X \rightarrow Y \text{ is a tautology.}\]

1.20 (b)

Every teacher is liked by some student: then the logical expression is $\forall x [\text{teacher}(x) \rightarrow \exists(y) [\text{student}(y) \land \text{likes}(y, x)]]$

Where likes $(y, x)$ means $y$ likes $x$, such that $y$ represents the student and $x$ represents the teacher.

1.21 (b)

Consider choice (b)

$(\forall x (P(x) \Rightarrow Q(x))) \Rightarrow ((\forall x P(x)) \Rightarrow (\forall x Q(x)))$

Let the LHS of this implication be true

This means that

$P_1 \rightarrow Q_1$

$P_2 \rightarrow Q_2$

\[\vdots\]

$P_n \rightarrow Q_n$

Now we need to check if the RHS is also true.

The RHS is $(\forall x P(x)) \Rightarrow (\forall x Q(x))$

To check this let us take the LHS of this as true i.e. take $\forall x P(x)$ to be true. This means that $(P_1, P_2, \ldots, P_n)$ is taken to be true. Now $P_1$ along with $P_1 \rightarrow Q_1$ will imply that $Q_1$ is true. Similarly $P_2$ along with $P_2 \rightarrow Q_2$ will imply that $Q_2$ is true. And so on...

Therefore $(Q_1, Q_2, \ldots, Q_n)$ all true.

i.e. $\forall x Q(x)$ is true. Therefore the statement (b) is a valid predicate statement.

1.22 (b)

Since a relation may or may not be symmetric, the given predicate is satisfiable but not valid. So (a) is clearly false.

Whenever a predicate is satisfiable its negation also is satisfiable. So option (b) is the correct answer.

1.23 (d)

The given statement should be read as

“If an animal is a tiger or a lion, then (if the animal is hungry or threatened, then it will attack). Therefore the correct translation is

$\forall x [(\text{tiger}(x) \lor \text{lion}(x)) \rightarrow (\text{hungry}(x) \lor \text{threatened}(x)) \rightarrow \text{attacks}(x)]$}

which is choice (d).

1.24 (d)

$P_1 : ((A \land B) \rightarrow C) \equiv ((A \rightarrow C) \land (B \rightarrow C))$

LHS :

$(A \land B) \rightarrow C$

$\equiv AB \rightarrow C$

$\equiv (A B') + C$

$\equiv A' + B' + C$

RHS :

$(A \rightarrow C) \land (B \rightarrow C)$

$\equiv (A' + C) (B' + C)$

$\equiv A'B' + C$

Clearly, LHS $\neq$ RHS

$P_1$ is not a tautology

$P_2 : ((A \lor B) \rightarrow C) \equiv ((A \rightarrow C) \lor (B \rightarrow C))$

LHS $\equiv (A + B) \rightarrow C$

$\equiv (A + B') + C$

$\equiv A'B' + C$

RHS $\equiv (A \rightarrow C) \lor (B \rightarrow C)$

$\equiv (A' + C) + (B' + C)$

$\equiv A' + B' + C$

Clearly, LHS $\neq$ RHS $\Rightarrow P_2$ is also not a tautology.

Therefore, both $P_1$ and $P_2$ are not tautologies. Correct choice is (d).

1.25 (d)

By using min terms we can define

$A \odot B = AB + AB' + A'B'$

$= A + A'B'$

$= (A + A') \cdot (A + B') = A + B'$

(a) $\sim A \odot B = A' \odot B = A' + B'$

(b) $\sim (A \odot \sim B) = (A \odot B)' = (A + (B'))'$

$= (A + B)' = A'B'$

(c) $\sim (A \odot \sim B) = (A' \odot B)' = (A' + (B'))'$

$= (A' + B') = AB'$
(d) \(\sim (\sim A \odot B) = (A' \odot B)' = (A' + B)'
\)
\[ = A \cdot B = A \wedge B\]
\[ \therefore \text{ Only, choice (d) is } A \wedge B\]

**Note:** This problem can also be done by constructing truth table for each choice and comparing with truth table for \(A \wedge B\).

1.26 (d)

The statement “Not every graph is connected” is same as “There exists some graph which is not connected” which is same as \(\exists x (\text{graph}(x) \wedge \neg \text{connected}(x))\)

Which is choice (b)

By boolean algebra we can see that option (a) and (c) are same as (b). Only option (d) is not the same as (b).

Infact option (d) means that “all graphs are not connected”.

**Alternate solution:**

We can translate the given statement “NOT (every graph is connected)” as \(\neg (\forall x \text{ graph}(x) \rightarrow \text{connected}(x))\)

\[ \equiv \exists x (\neg \text{graph}(x) \lor \neg \text{connected}(x))\]

\[ \equiv \exists x (\neg \text{graph}(x) \lor \neg \text{connected}(x))\]

By boolean algebra we can see that option (a) and (c) are same as (b). Only option (d) is not the same as (b).

Infact option (d) means that “all graphs are not connected”.

1.27 (a)

In conjunctive normal form, for any particular assignment of truth values, all except one clause, will always evaluate to true. So, the proportion of clauses which evaluate to true to the total number of clauses is equal to \(\frac{2^n - 1}{2^n}\).

Now putting \(n = 1, 2, \ldots\), we get \(\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \ldots\)

All of these proportions are \(\geq \frac{1}{2}\) and so choice (a) atleast half of the clauses evaluate to true, is the correct answer.

1.28 (a)

Option (a) is a standard one way distributive property of predicates.

1.29 (a)

“For x which is an fsa, there exists a y which is a pda and which is equivalent to x.”

\((\forall x \text{fsa}(x) \Rightarrow (\exists y \text{pda}(y) \land \text{equivalent}(x, y)))\) is the logical representation.

1.30 (c)

Option (c) is \([\exists x, \alpha(x) \rightarrow \beta] \rightarrow [\forall x, \alpha(x) 
\rightarrow \beta]\)

Let us check the validity of this predicate.

Let the LHS of this predicate be true. This means that some \(\alpha \rightarrow \beta\).

Let \(\alpha_5 \rightarrow \beta\)

Now we will check if the RHS is true. The RHS is \([\forall x, \alpha(x) \rightarrow \beta]\) to check this implication let us take \(\forall x, \alpha(x)\) to be true.

This means that all the \(\alpha\) are true. It means that \(\alpha_5\) is also true.

But \(\alpha_5 \rightarrow \beta\). Therefore \(\beta\) is true.

So the RHS \([\forall x, \alpha(x) \rightarrow \beta]\) is true.

Whenever the LHS \([\exists x, \alpha(x) \rightarrow \beta]\) is true. So option (c) is valid.

1.31 (d)

The given predicate is \([\forall x, \alpha \rightarrow (\exists y, \beta \rightarrow (\forall u, \exists v, \gamma))]\)

The negation of this predicate is \(\neg[\forall x, \alpha \rightarrow (\exists y, \beta \rightarrow \forall u, \exists v, \gamma)]\)

\(\neg[\forall x, \alpha \rightarrow (\forall y, \neg \beta \lor \forall u, \exists v, \gamma)]\)

\(\neg[\exists x, \neg \alpha \lor (\forall y, \neg \beta \land \forall u, \exists v, \neg \gamma)]\)

\([\forall x, \alpha \land (\exists y, \beta \land \exists u, \forall v, \neg \gamma)]\)

Which is option (d).

1.32 (b)

1. \(P \lor \sim Q = P + Q' \)
2. \(\sim(P \land Q) = (P' \land Q') = P + Q' \)
3. \((P \land Q) \lor (P \land \sim Q) = PQ + PQ' + P'Q\)
   \[= P(Q + Q') + P'Q\]
   \[= P + P'Q\]
   \[= (P + P')(P + Q')\]
   \[= P + Q'\]
4. \((P \land Q) \lor (P \land \sim Q) \lor (\sim P \land Q) = PQ + PQ' + P'Q\)
   \[= P(Q + Q') + P'Q\]
   \[= P + P'Q\]
   \[= (P + P')(P + Q) = P + Q\]

Clearly (i), (ii) and (iii) are equivalent. Correct choice is (b).
1.35 (d)

The correct translation of “Gold and silver ornaments are precious” is choice (d)
\[
\forall x ((G(x) \lor S(x)) \rightarrow P(x))
\]
which is read as “if an ornament is gold or silver, then it is precious”.
Now since a given ornament cannot be both gold and silver at the same time.
Choice (b) \( \forall x ((G(x) \land S(x)) \rightarrow P(x)) \) is incorrect.

1.34 (b)

The given table can be converted into boolean function by adding minterms corresponding to true rows.
\[
\begin{array}{|c|c|c|}
\hline
P & Q & P \square Q \\
\hline
T & T & T \\
T & F & T \\
F & T & F \\
F & F & T \\
\hline
\end{array}
\]
Since there is only one false in the above truth table, we can represent the function \( P \square Q \) more efficiently, in conjunctive normal form.
Translates \( P \square Q = P + Q' \) (the max-term corresponding to the third row, where the function is false).
Now, we can easily translate the choices into boolean algebra as follows:
Choice (a) \( \neg Q \square \neg P = Q' \square P' = Q' + P \)
Choice (b) \( P \square \neg Q = P \square Q' = P + Q \)
Choice (c) \( \neg P \square Q = P' \square Q = P' + Q' \)
Choice (d) \( \neg P \square \neg Q = P' \square Q' = P' + Q \)
As we can clearly see only choice (b) \( P \square Q \) is equivalent to \( P + Q \).

1.35 (b)

I \( \neg \forall x P(x) \equiv \exists x \neg P(x) \)
and IV \( \exists x \rightarrow P(x) \)
Clearly, choices I and IV are equivalent.
II \( \neg \exists x P(x) \equiv \forall x \neg P(x) \)
and III \( \neg \exists x \neg P(x) \equiv \forall x P(x) \)
Clearly II and III are not equivalent to each other or to I and IV.

1.36 (b)

\( \forall x \exists y \exists t \rightarrow F(x, y, t) \)
\( \equiv \neg (\exists x \forall y \forall t F(x, y, t)) \)
"it is not true that (someone can fool all people at all time)
= no one can fool everyone all the time"

1.37 (a)

If \( P(x) \) is true, then
\( x \neq 1 \) and also
\( x \) is broken into two factors, only if, one of the factors is \( x \) itself and the other factor is 1, which is exactly the definition of a prime number.
So \( P(x) \) is true means \( x \) is a prime number.

1.38 (b)

Let \( p \) : It rains
\( q \) : cricket match will not be played.
\( I_1 : p \Rightarrow q \)
\( \sim q \)
\( \therefore \sim p \)
Clearly \( I_1 \) is correct since it is in the form of Modus Tollens (rule of contrapositive)
\( I_2 : p \Rightarrow q \)
\( \sim p \)
\( \therefore \sim q \)
which corresponds \( [p \Rightarrow q \land \sim p] \Rightarrow \sim q \)
\( \equiv [(p' + q) p'] \Rightarrow q' \)
\( \equiv [p' + qp'] \Rightarrow q' \)
\( \equiv p' \Rightarrow q' \)
\( \equiv (p')' + q' = p + q' \)
which is not a tautology.
So \( I_2 \) is incorrect inference.

1.39 (c)

Some real numbers are rational
\( \equiv \exists x [\text{real}(x) \land \text{rational}(x)] \)

1.40 (d)

None of my friends are perfect i.e., all of my friends are not perfect
\( \forall x ((P(x) \rightarrow \neg P(x)) \)
\( \forall x (\neg F(x) \lor \neg P(x)) \)
\( \neg \exists x (F(x) \land P(x)) \)
Alternatively
\( \exists x (F(x) \land P(x)) \) gives
there exist some of my friends who are perfect.
\( \neg \exists x (F(x) \land P(x)) \)
there does not exits any friend who is perfect i.e.,
none of my friends are perfect.
So (d) is correct option.
1.41 (a)
Let, \( \forall y(\alpha) = P, \forall z(\beta) = Q \)
Then, \( \exists y(\neg \alpha) = \neg P \) and \( \exists z(\neg \beta) = \neg Q \)
Given, \( \neg \exists x (\forall y(\alpha) \land \forall z(\beta)) \)
\[ = \neg \exists x (P \land Q) \]
\[ = \neg \forall x (\neg P \lor \neg Q) \]
\[ = \forall x (P \rightarrow \neg Q) \]
(a) \( \forall x (\exists z(\neg \beta) \rightarrow \forall y(\alpha)) = \forall x (\neg Q \rightarrow P) \)
(b) \( \forall x (\forall z(\beta) \rightarrow \exists y(\neg \alpha)) = \forall x (Q \rightarrow \neg P) \)
\[ = \forall x (P \rightarrow \neg Q) \]
(c) \( \forall x (\forall y(\alpha) \rightarrow \exists z(\neg \beta)) = \forall x (P \rightarrow \neg Q) \)
(d) \( \forall x (\exists y(\neg \alpha) \lor \exists z(\neg \beta)) = \forall x (\neg P \lor \neg Q) \)
\[ = \forall x (P \rightarrow \neg Q) \]
\[ \therefore \text{Only (a) is not logically equivalent to} \]
\[ \forall x (P \rightarrow \neg Q). \]

1.42 (d)
(a) \( \forall x \text{ glitters}(x) \Rightarrow \neg \text{gold}(x) \)
All glitters are not gold
(b) \( \forall x \text{ gold}(x) \Rightarrow \text{glitters}(x) \)
All golds are glitters
(c) \( \exists x \text{ gold}(x) \land \neg \text{glitters}(x) \)
There exist gold which is not glitter i.e. not all golds are glitters.
(d) \( \exists x \text{glitters}(x) \land \neg \text{gold}(x) \)
Not all that glitters is gold i.e., there exist some which glitters and which is not gold.

1.43 (b)
Option (b) is \( (\neg (p \leftrightarrow q) \land r) \lor (p \land q \land \neg r) \)
\[ = (p \leftrightarrow q)(r) + pq'r \]
\[ = (pq' + p'q)r + pq'r \]
\[ = pq'r + p'qr + pq'r \]
\[ = pq'r + pq'r + p'qr \]
This is exactly the min-term form of a logical formula which is true when exactly two variables are true (only \( p, q \) true or only \( p, r \) true or only \( q, r \) true).

1.44 (b)
\[ (a \leftrightarrow c) \rightarrow (\neg b \rightarrow (a \land c)) \]
\[ = (a \leftrightarrow c)' + (b' \rightarrow ac) \]
\[ = (a \land c)' + (b' \rightarrow ac) \]
\[ = ac' + a'c + b + ac \]
\[ = a(c' + c) + a'c + b \]
\[ = a + a'c + b = a + c + b \]
which is not a tautology.

1.45 (d)
\( g : \) mobile is good
\( c : \) mobile is cheap
\( P : \) Good mobile phones are not cheap
\( \rightarrow c' \equiv (g' + c') \)
\( Q : \) Cheap mobile phones are not good
\( c \rightarrow g' \equiv (c' + g') \)
\[ \therefore \text{Both } P \text{ and } Q \text{ are equivalent.} \]
\( L : P \rightarrow Q \)
\( M : Q \rightarrow P \)
\( N : P \equiv Q \)
Since both \( P \) and \( Q \) are equivalent, all three of \( L, M, N \) are true.
So correct option is (d).

1.46 (d)
Not (all rainy days are cold):
\[ \sim (\forall d \text{ (Rainy } (d) \rightarrow \text{ Cold } (d))) \]
\[ \equiv \sim (\forall d (\sim \text{ Rainy } (d) \lor \text{ Cold } (d))) \]
\[ \equiv \exists d \text{ (Rainy } (d) \land \sim \text{ Cold } (d)) \]
Alternate Method:
Not all rainy days are cold is same as some rainy days are not cold which is same as
\[ \exists d \text{ (Rainy } (d) \land \sim \text{ Cold } (d)) \]

1.47 (e)
Here, option (a) and (b) can be reduced to \( (p \rightarrow q) \land (q \rightarrow p) \) and hence \( p \equiv q \)
Option (d) is \( p'q' + pq = p \leftrightarrow q \).
Option (c) is \( p'q + pq' = p \oplus q \) which is not equivalent to \( p \leftrightarrow q \).

1.48 (a)
The given truth table corresponds to \( p'q + pq' \)
\[ = p \oplus q. \]
\( \oplus \) is known to be both commutative and associative.