State Public Service Commissions

State Engineering Services Examinations

Previous years’ Conventional Solved Papers

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- RPSC
- OPSC
- MPSC
- GPSC
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MADE EASY Publications
For each and every aspirant in the preparation for any competitive exam, consolidating what is learnt and to get the flavour & feel of actual exam are among the top list of desiderata. With the students’ expression of interest for a book to prepare effectively for State Level exams, this conventional book is debut of MADE EASY in exclusive State Level Services study material which will definitely fulfil all the requirements of aspirants.

This book covers conventional questions from various papers of 11 different PSCs across the country (BPSC, RPSC, OPSC, MPSC, GPSC, MPPSC, JPSC, UKPSC, KPSC, Kerala PSC and HPSC); which will certainly be a path for students to achieve their goal.

Reasonable efforts are been taken to make sure that answers are framed and transcribed accurately. With key formulae, relevant theory and graphical/pictorial representations this book will not only give questions of various PSCs over the years but also will equip students with concepts, knowledge and understanding of the subject. I hope this book will prove to be an efficient tool to prepare for subjective exams of different PSCs. This book will also come handy for ESE Mains and SSC conventional exams.

It is impossible to acknowledge all the individuals who helped us, but would like to sincerely thank all authors, editors and reviewers for putting their painstaking efforts to publish this book.
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1. (a) Two-plate load tests with square plates were performed on a soil deposit. For 30 mm settlement, the loads obtained are as follows:

<table>
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<th>Width of square plate (in mm)</th>
<th>Load (in kN)</th>
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<tr>
<td>300</td>
<td>38.2</td>
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<tr>
<td>600</td>
<td>118.5</td>
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Determine the width of square footing which would carry a net load of 1500 kN for a limiting settlement of 30 mm.

[10 Marks]

SOL: Given,

<table>
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<th>Load (in kN)</th>
</tr>
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<tbody>
<tr>
<td>300</td>
<td>38.2</td>
</tr>
<tr>
<td>600</td>
<td>118.5</td>
</tr>
</tbody>
</table>

Settlement recorded = 30 mm

As per Housel's equation,

We know that

\[ Q = A_p m + P_p n \]

where,

\[ Q = \text{Load applied on given plate} \]

\[ A_p = \text{Contact area of plate} \]

\[ P_p = \text{Perimeter of plate} \]

\[ m = \text{a constant corresponding to the bearing pressure} \]

\[ n = \text{another constant corresponding to perimeter shear} \]

Hence, for the two plate load test, we may write

\[ Q_1 = A_{p1} m + P_{p1} n \]

\[ A_{p1} = (0.3)^2 m^2 = 0.09 m^2, \quad P_{p1} = 4 \times 0.3 = 1.2 m \]

\[ 38.2 = 0.09 m + 1.2 n \] \hspace{1cm} \ldots(1)

\[ A_{p2} = (0.6)^2 m^2 = 0.36 m^2, \quad P_{p2} = 4 \times 0.6 = 2.4 m \]

\[ 118.5 = 0.36 m + 2.4 n \] \hspace{1cm} \ldots(2)

Operating equation (2) – equation (1) \times (4)

\[ 118.5 - 4 \times (38.2) = 2.4 n - 4.8 n \]

\[ -34.3 = -2.4 n \]

\[ n = 14.29 \]

Put the value of \( n \) in eq. (1)

\[ 38.2 = 0.09 \times m + 1.2 \times 14.29 \]

\[ m = 233.91 \]

For prototype foundation, we may write

\[ Q_r = 233.91 A_r + 14.29 P_r \]
Let size of footing, be \((B \times B)\),
\[
A_f = B^2, \quad P_f = 4B, \quad Q_f = 1500 \text{ kN}
\]
\[
1500 = 233.91B^2 + 14.29 \times 4B
\]
\[
233.41B^2 + 57.16B - 1500 = 0
\]
\[
B = 2.41 \text{ m} \approx 2.4 \text{ m}
\]
\[
\therefore \quad \text{The size of footing} = 2.4 \text{ m} \times 2.4 \text{ m}
\]

1. (b) For the simply-supported beam shown in figure 1, determine the deflection at centre point C and slopes at ends A and B. \([E = 2.05 \times 10^5 \text{ N/mm}^2 \text{ and } I = 80 \times 10^6 \text{ mm}^4]\)

![Figure 1](image)

**Sol:** Given, SSB

\[
E = 2.05 \times 10^5 \text{ N/mm}^2
\]
\[
I = 80 \times 10^6 \text{ mm}^4
\]

![Figure 1](image)

To find deflection at centre point C, and slope at point A and B.
Let us find it by using moment area method.

![M/EI diagram](image)

At C, slope will be zero,
\[\theta_C = 0\]
i.e.
So, considering part AC only,
For slope at A,
\[
\text{Change of slope from } A \text{ to } C = \text{Area of } M/EI \text{ diagram between } A \text{ and } C
\]
\[
\theta_C - \theta_A = \frac{1}{2} \times \frac{L}{2} \times \frac{PL}{4EI}
\]
\[ \theta_A = \frac{-PL^2}{16EI} \]

By symmetry

\[ \theta_B = \frac{PL^2}{16EI} \]

Deflection at C,
\[ \Delta_C = I_{AC} = \text{moment of area of } M/EI \text{ diagram between } A \text{ and } C \text{ taken about } A. \]

\[ \Delta_C = \frac{1}{2} \times \frac{L}{2} \times \frac{PL}{4EI} \times \frac{2(L/2)}{3} = \frac{PL^3}{48EI} \]

\[ \therefore \quad \text{Deflection at } C = \frac{PL^3}{48EI} \quad (\downarrow \text{ ward}) \]

Given,
\[ P = 50 \text{ kN} \]
\[ E = 2.05 \times 10^5 \text{ N/mm}^2 \]
\[ I = 80 \times 10^6 \text{ mm}^4 \]
\[ EI = 1.64 \times 10^{13} \text{ Nmm}^2 = 1.64 \times \frac{10^{13} \times 10^{-6}}{10^3} \text{ kNm}^2 = 1.64 \times 10^4 \text{ kNm}^2 \]

Now,
\[ \theta_A = \frac{PL^2}{16EI} = \frac{50 \text{ kN} \times (8)^2 \text{ m}^2}{16 \times 1.64 \times 10^4 \text{ kNm}^2} = 0.01219 \text{ radian} \]
\[ \theta_B = 0.01219 \text{ radian} \]
\[ \Delta_C = \frac{PL^3}{48EI} = \frac{50 \text{ kN} \times (8)^2 \text{ m}^2}{48 \times 1.64 \times 10^4 \text{ kNm}^2} = 0.03252 \text{ m} \]
\[ \Delta_C = 0.03252 \text{ m} \downarrow \text{ Ward} \]

**1. (c)** A post-tensioned beam, 200 mm × 300 mm, prestressed with tendon 520 mm² area stretched to a stress of 1000 N/mm². The tendon passes through a hole 50 mm wide and 75 mm deep left in the beam, having the centre of the hole at 75 mm from the bottom. The loss at the time of prestressing is 5%. Find the stresses in concrete immediately after prestressing.

**Sol:**

Given:
Post tensioned beam (c/s) – (200 mm × 300 mm)
Cross section area of tendon = 520 mm$^2$
Initial stress = 1000 N/mm$^2$

Initial prestressing force,

\[ P = 520 \times \frac{1000}{1000} \text{kN} = 520 \text{kN} \]

Loss of prestress = 5%, \( k = \left(1 - \frac{P \%}{100}\right) = \left(1 - \frac{5}{100}\right) = 0.95 \)

Eccentricity of prestressing force = 75 mm
Stresses in concrete just after (immediately after) prestressing, i.e. At transfer stage,
No losses is considered here,
At ends,

\[ f_{\epsilon b} = \frac{P}{A} + \frac{P_0}{I} y_{\epsilon b} \]

\[ f_{\epsilon b} = \frac{P}{A} - \frac{P_0}{I} y_f \]

\[ f_i = \frac{520 \times 10^3}{(200 \times 300)} - \frac{520 \times 10^3 \times 75}{200 \times (300)^3} \times 150 \times \frac{12}{12} \]

\[ f_i = -4.33 \text{ N/mm}^2 \]

\[ f_b = \frac{P}{A} + \frac{P_0}{I} y_b \]

\[ f_b = \frac{520 \times 10^3}{200 \times 300} + \frac{520 \times 10^3 \times 75}{200 \times (300)^3} \times 150 \times \frac{12}{12} \]

\[ f_b = 21.67 \text{ N/mm}^2 \]

**SECTION - A**

2. (a) Analyze the frame shown in fig. 2 and draw the BM and SF diagram. \([EI\text{ is constant for all the members}].\)
Let us do it, by using slope deflection method, unknown joint displacements, 
\((\theta_B, \theta_C)\)

If we use modified slope deflection equation for span BC, then there is no need to consider \(\theta_C\), hence unknown reduces to only \(\theta_B\).

Hence, joint equilibrium equations required is:

\[ M_{BA} + M_{BC} + M_{BD} = 0 \] \(\ldots(1)\)

Fixed end moments:

**Span AB**

\[ M_{FAB} = \frac{20 \times (4)^2}{12} = 26.67 \text{ kNm} \]
\[ M_{FBA} = 26.67 \text{ kNm} \]

**Span BC**

\[ M_{FBC} = \frac{40 \times 4}{8} = 20 \text{ kNm} \]
\[ M_{FCB} = 20 \text{ kNm} \]
\[ M_{FBC} = -20 - 10 = -30 \text{ kNm} \]

**Span BD**

\[ M_{FBD} = \frac{20 \times 4}{8} = 10 \text{ kNm} \]
\[ M_{FDB} = 10 \text{ kNm} \]
\[ M_{FBD} = 10 \text{kNm} \]
\[ M_{FDB} = -10 \text{kNm} \]

Now, using slope deflection eq.

Span \( AB \),
\[ M_{AB} = M_{FAB} + \frac{2EI}{L} \left( 2\theta_A + \theta_B - \frac{3\Delta}{L} \right) \{ \theta_A = 0, \Delta = 0 \} \]
\[ = -26.67 + \frac{2EI}{4} \theta_B = -26.67 + 0.5EI\theta_B \]
\[ \quad \text{(A)} \]
\[ M_{BA} = M_{FBA} + \frac{2EI}{L} \left( 2\theta_B + \theta_A - \frac{3\Delta}{L} \right) \]
\[ = 26.67 + \frac{2EI}{L} 2\theta_B = 26.67 + EI\theta_B \]
\[ \quad \text{(B)} \]

Span \( BC \),
\[ M_{BC} = M_{FBC} + \frac{3EI}{L} \left( \theta_B - \frac{\Delta}{L} \right) \]
\[ = -30 + \frac{3EI}{4} \theta_B \]
\[ \quad \text{(C)} \]

Span \( BD \),
\[ M_{BD} = M_{FBD} + \frac{2EI}{L} \left( 2\theta_B + \theta_D - \frac{3\Delta}{L} \right) \{ \theta_D = 0, \Delta = 0 \} \]
\[ = 10 + \frac{2EI}{4} 2\theta_B = 10 + EI\theta_B \]
\[ \quad \text{(D)} \]
\[ M_{DB} = M_{FDB} + \frac{2EI}{L} \left( 2\theta_D + \theta_B - \frac{3\Delta}{L} \right) \]
\[ = -10 + \frac{2EI}{4} 2\theta_B = -10 + 0.5EI\theta_B \]
\[ \quad \text{(E)} \]

Now, using eq. (1)
\[ M_{BA} + M_{BC} + M_{BD} = 0 \]
\[ 26.67 + EI\theta_B + \left( -30 + \frac{3EI}{4} \theta_B + 10 + EI\theta_B \right) = 0 \]
\[ 2.75EI\theta_B + 6.67 = 0 \]
\[ EI\theta_B = \frac{-6.67}{2.75} = -2.425 \]

Now,
\[ M_{AB} = -26.67 + 0.5 \times (-2.425) = -27.88 \text{kNm} \]
\[ M_{BA} = 26.67 - 2.425 = 24.245 \text{kNm} \]
\[ M_{BC} = -30 + 0.75 \times (-2.425) = -31.818 \text{kNm} \approx -32 \text{kNm} \]
\[ M_{BD} = 10 - 2.425 = 7.575 \]
\[ M_{DB} = -10 + 0.5 \times (-2.425) = -11.212 \text{kNm} \]

\[ S_F \]
\[ SF_{xx} = 0 \]
\[ 40.908 - 20x = 0 \]
\[ x = \frac{40.908}{20} = 2.045 \text{ m} \]

\[ M_{\text{max}} = -27.88 + 40.908 \times 2.045 - \frac{20 \times (2.045)^2}{2} = 13.956 \text{ kNm} \]

\[ \frac{20}{2} + \frac{11.212 - 7.575}{4} = 10.909 \text{ kN} \]

End moment diagram:

\[
27.88
\]

\[
11.212
\]

\[
\frac{20}{2} + \frac{11.212 - 7.575}{4} = 10.909 \text{ kN} \]

Free moment diagram:

Span AB:

\[ \frac{20 \times (4)^2}{8} = 40 \text{ kNm} \]

Span BC:

\[ \frac{40 \times 4}{4} = 40 \text{ kNm} \]

\[ \frac{20 \times 4}{4} = 20 \text{ kNm} \]
2. (b) A uniformly distributed load of 20 kN/m covering a length of 3 m crosses a girder of span 10 m. Find the maximum shear force and bending moment at a section 4 m from left-hand support. 

[10 Marks]

Sol:
for maximum +ve SF at C,
Tall of udl is placed just to the right of C.
Maximum +ve SF = Load intensity × area under udl
\[ = 20 \times \left( \frac{1}{2} \left( \frac{6}{10} + \frac{3}{10} \right) \right) \times 3 = 27 \text{ kN} \]

For maximum –ve SF at C,
Head of udl is placed just to the left of C
maximum –ve SF = Loading intensity × area under udl
\[ = -20 \left( \frac{1}{2} \left( \frac{4}{10} + \frac{1}{10} \right) \times 3 \right) = -15 \text{ kN} \]

For maximum B.M. at C, udl should be placed so that the section divides the udl in the same ratio as it divides the span.

\[ \frac{\alpha}{3 - \alpha} = \frac{4}{6} \]
\[ 6\alpha = 12 - 4\alpha \]
\[ 10\alpha = 12 \]
\[ \alpha = 1.2 \text{ m} \]

Maximum B.M. at C,
\[ M^\text{max} = \text{Loading intensity} \times \text{Area under udl}. \]
\[ = 20 \times \left[ \frac{1}{2} \left( \frac{2.8 \times 6}{10} + \frac{4 \times 6}{10} \right) \times 1.2 \right] + 20 \left[ \frac{1}{2} \left( \frac{4 \times 6}{10} + \frac{4.2 \times 4}{10} \right) \times 1.8 \right] \]
\[ = 122.4 \text{ kNm} \]

3. (a) Design a lap joint to connect two plates of size 250 mm × 12 mm to mobilize full-plate tensile strength. The permissible tensile stress in plate is 225 N/mm² and permissible shear stress in weld is 180 N/mm². The size of weld is limited to 8 mm. Draw the details.

[10 Marks]

Sol:
Given,
size of plates = (250 mm × 12 mm)
Permissible tensile stress of plate = 225 N/mm²
Permissible shear stress of weld = 180 N/mm²
size of weld = 8 mm

Assuming welding is done on all sides
Let \( l \) be the length of each overlap,
Tensile strength of plate for full strength

\[ T_{dg} = (\text{Permissible tensile stress}) \times \text{Area of plate} \]

\[ \begin{align*}
T_{dg} &= 225 \times (250 \times 12) \times 10^{-3} = 675 \text{ kN}
\end{align*} \]

For full mobilization of strength,

strength of weld = \( T_{dg} \)

(Permissible shear stress) \( \times (L_{\text{eff}} \times t_i) = 675 \times 10^3 \text{ N} \)

\( t_i = \text{throat thickness} = 0.7S \)

\[ \begin{align*}
l &= 0.7 \times 8 = 5.6 \text{ mm}
\end{align*} \]

Now,

\[ \begin{align*}
180 \times (2l + 2 \times 250) \times 5.6 = 675 \times 10^3
\end{align*} \]

\[ \begin{align*}
l &= 84.82 \approx 85 \text{ mm}
\end{align*} \]

Overlap, \( l = 85 \text{ mm} \) \( (4t \text{ or 40 mm})_{\text{max}} \)

\[ \begin{align*}
&= (4 \times 12 \text{ or 40 mm})_{\text{max}} = 48 \text{ mm (OK)}
\end{align*} \]

Thus, provide 8 mm size fillet weld of length \( (2 \times 85 + 500) \)

\[ \begin{align*}
&= 670 \text{ mm all round the joint to develop the full strength.}
\end{align*} \]

3. (b) Find the maximum load \( (P) \) shown in fig. 3 for 20 mm diameter bolts connecting 10 mm thick bracket plate for nonslip joint having bolt capacity of 52.5 kN.

**Sol:**

\[ \begin{align*}
&\text{All dimension are in 'mm'
}\end{align*} \]
Given, diameter of bolt = 20 mm
Thickness of bracket plate = 10 mm
Bolt capacity or strength of bolt, \( V_{ab} = 52.5 \text{ kN} \)
Let \( P_u \) be the ultimate load (in kN)
Bolt ‘3’ is critical bolt,

![Diagram of bolt and load](attachment://diagram.png)

Torsional moment on the weld group

\[
T = -P_u \cos60° \times 0.100 + P_u \sin60° \times 0.400 = 0.2964 \times P_u \text{ kNm}
\]

Now, \( F_{D1} = \frac{P_u \cos60°}{5} = 0.1 \times P_u \)

\( F_{D2} = \frac{P_u \sin60°}{5} = 0.173 \times P_u \)

\( r_1 = \sqrt{(50)^2 + (50)^2} = 50\sqrt{2} \text{ mm} \)

\[
\Sigma r_1^2 = 4 \left( \frac{50\sqrt{2}}{2} \right)^2 = 20000 \text{ mm}^2
\]

Direct shear force in the bolt

\[
F_{D1} = 0.1P_u \text{ kN}; \quad F_{D2} = 0.173P_u \text{ kN}
\]

\[
F_x = F_{D2} + F_1 \sin45°
\]

\[
= 0.173P_u + 1.047P_u \times \frac{1}{\sqrt{2}} = 0.914P_u
\]

\[
F_y = F_{D1} + F_1 \cos45°
\]

\[
= 0.1P_u + 1.047P_u \times \frac{1}{\sqrt{2}} = 0.84P_u
\]

Resultant stress on the weld

\[
F_r = \sqrt{F_x^2 + F_y^2} = \sqrt{(0.914P_u)^2 + (0.84P_u)^2} = 1.241P_u
\]
for safety

\[ F_r \leq V_{ob} \]

\[ 1.241 P_u \leq 52.5 \]

\[ P_u \leq 42.3 \text{ kN} \]

\[ P_u = 42.3 \text{ kN} \]

Maximum load that can be carried by the plate = 42.3 kN

**Section - B**

4. (a) Design a singly-reinforced rectangular beam of clear span 7.5 m, simply supported on walls of 350 mm width in flexure. The beam is subjected to 35 kN/m including its self-weight. Assume the width of the beam as 350 mm. [Use M-25 and Fe-415.]

[10 Marks]

**Sol:**

Design of singly reinforced rectangular beam:

Given,

- SSB of clear span = 7.5 m
- Supports wall width = 350 mm
- Width of beam = 350 mm
- Load on the beam = 35 kN/m (including self weight)

M25, Fe415

Effective span of beam

\[ l_{eff} = (7.5 + 0.35) = 7.85 \text{ m} \]

Factored load on the beam

\[ w_u = 1.5 \times 35 = 52.5 \text{ kN/m} \]

Factored bending moment,

\[ M_u = \frac{w_u l_{eff}^2}{8} = \frac{52.5 \times (7.85)^2}{8} = 404.4 \text{ kN m} \]

Depth required for balanced section,

\[ d = \sqrt{\frac{(BM_u)}{Q.d.}} \]

for Fe415, \( Q = 0.138 f_{ck} \)

\[ d = \sqrt{\frac{404.4 \times 10^6}{0.138 \times 25 \times 350}} = 578.71 \text{ mm} \]

Provide, \( d = 650 \text{ mm} \)

Effective cover = 50 mm

Overall depth = 700 mm

\[ b = 350 \text{ mm} \]
Since, provided depth is higher than that required for balanced section, hence section must be under reinforced.

\[
A_{sf} = \frac{0.5f_{ck}bd}{f_y} \left(1 - \sqrt{1 - \frac{4.6BM_y}{f_{ck}bd^2}}\right)
\]

\[
= \frac{0.5 \times 25 \times 350 \times 650}{415} \left(1 - \sqrt{1 - \frac{4.6 \times 404.4 \times 10^6}{25 \times 350 \times 650^2}}\right) = 2022.52 \text{ mm}^2
\]

\[
A_{sf}^{\text{min}} = \left(\frac{0.85}{f_y}\right) \times bd = \frac{0.85}{415} \times 350 \times 650 = 465.96 \text{ mm}^2
\]

\[
A_{sf}^{\text{max}} = \frac{4}{100} \times bD = \frac{4}{100} \times 350 \times 700 = 9800 \text{ mm}^2
\]

Provide 4-28 mm φ bar = \[4 \times \frac{\pi}{4} (28)^2 = 2463 \text{ mm}^2\]

\[
A_{sf}^{\lim} = (0.048 f_{ck} \%) \times bd
\]

\[
= \frac{0.048 \times 25}{100} \times 350 \times 650 = 2730 \text{ mm}^2
\]

\[
A_{sf}^{\text{min}} < A_{sf}^{\text{provided}} < A_{sf}^{\lim} < A_{sf}^{\text{max}} \quad (\text{OK})
\]

**Design details:**

\[
\text{4-28φ}
\]

**4. (b)** Design a reinforced concrete footing of uniform thickness for reinforced concrete column of 550 mm × 550 mm which is subjected to axial load of 2100 kN at service state. Allowable bearing capacity is 125 kN/m². Design by limit-state method and draw the details.

[10 Marks]

**Sol:** Design of RC footing:

**Size of foundation:** While deciding the size of footing are work with service load.

\[\Rightarrow \text{ load from column } = 2100 \text{ kN}\]

Safe bearing capacity of soil, SBC = 125 kN/m²

Assuming self weight of footing and backfill as 10% of working load.

\[
\text{Area required for footing} = \left(\frac{P + 0.1P}{SBC}\right) = \frac{2100 + 0.1 \times 2100}{125} = 18.48 \text{ m}^2
\]

Adopting square footing, \(B = \sqrt{18.48} = 4.29 \text{ m} \approx (4.5 \text{ m})\)
Adopt footing size of \((4.5 \, m \times 4.5 \, m)\)

\[
\text{final area } = (4.5 \times 4.5) = 20.25 \, m^2
\]

Net soil pressure, \(\omega_0 = \frac{P + 0.1P_{\text{provided}}}{A_{\text{provided}}} = \frac{2100(1+0.1)}{(4.5 \times 4.5)} = 114.074 \, kN/m^2 < (SBC) \, (O.K.)
\]

For designing the foundation slab, we will work with the factored load.

Factored net soil pressure, \(\omega_f = 1.5 \times 114.074 = 171.111 \, kN/m^2\)

**Calculation of depth of footing:** Depth of footing slab is generally governed by shear. However, will also check the depth obtained from shear considerable for safe in bending.

**Case 1:** For one way shear: Critical section will be at a distance \(d\) from face of column.

\[
\tau_v = \frac{w_u \left( \frac{l-a}{2} - d \right)}{1 \times d} < 0.28 \times 10^3
\]

Assuming \(\tau_c\) for minimum %age of steel (0.15%)

\[
\tau_c = 0.28 \, N/mm^2
\]

\[
\tau_c = 0.28 \times 10^3 \, kN/m^2
\]

Assuming

\[
k = 1
\]

For safety in shear,

\[
\tau_v < k\tau_c
\]

\[
\frac{171.111 \left( \frac{4.5 - 0.55}{2} - d \right)}{1 \times d} < 0.28 \times 10^3
\]

\[
d > 0.749 \, m
\]

**Case 2:** For two way shear,

Critical section for two way shear is at a distance \(d/2\) from the face of column.

\[
(\tau_v)_{\text{punching}} = \frac{w_u \left[ B^2 - (a + d)^2 \right]}{2 \left[ 2(a + d) \times d \right]} = \frac{171.111 \left[ 4.5^2 - (0.55 + d)^2 \right]}{4(0.55 + d) \times d}
\]
For safety,
\[(\tau)_{\text{punching}} < k_p \sqrt{f_{ck}} (0.25)\]
\[k_p = \min \left\{ \frac{0.5 + b/a}{1}, 1 \right\} = 1\]
\[\frac{171.11 \left(4.5^2 -(0.55+d)^2\right)}{4 \times (0.55+d) \times d} < 0.25 \sqrt{20 \times 10^3}\]
Assuming M20, Fe415
\[d > 0.618 \text{ m}\]

**Case 3:** For maximum bending moment:
Critical section for bending is at the face of column
\[BM_{\text{max}} = \frac{w_u (L-a)^2}{2} = \frac{171.11 \left(\frac{4.5 - 0.55}{2}\right)^2}{2} = 333.72 \text{ kNm}\]
Assume
\[BM_{\text{max}} = M_{u,\text{lim}}\]
\[333.72 \times 10^6 < 0.36 f_{ck} x_{u,\text{lim}} b(d - 0.42 x_{u,\text{lim}})\]
\[333.72 \times 10^6 < 0.36 \times 20 \times 0.48 \times d \times 1000 (d - 0.42 \times 0.48 \times d)\]
\[d > 347.77 \text{ mm}\]
\[> 0.347 \text{ m}\]

Thus, according to following three criteria, uniform depth of the footing should be provided = 760 mm with 40 mm effective cover.

Total depth of footing = 760 + 40 = 800 mm

\[R/F \to \text{Reinforcement provided}\]
\[A_{\text{st}} = \frac{0.5 f_{ck} b d f_y}{f_{ck} b d^2} \left[1 - \left[1 - \frac{4.6BM_u}{f_{ck} b d^2}\right]\right]
\[= \frac{0.5 \times 20 \times 1000 \times 760}{415} \left[1 - \frac{0.65 \times 333.72 \times 10^6}{20 \times 1000 \times 760^2}\right] = 1260.153 \text{ mm}^2\]

Number of 12 mm φ bar
\[n = \frac{1260.153}{\pi / 4 (12)^2} = 11.14 \approx 12 \text{ no.s}\]

Spacing = \[\frac{1000 \times \pi (12)^2}{1260.153} = 89.78 \text{ mm}\]
5. (a)  Find the plastic hinge length and draw the shape of elastic-plastic zone for a rectangular simply-supported beam of span \( L \) subjected to concentrated load at midspan.

[10 Marks]

**Sol:**

Plastic hinge length for a simply supported beam with concentrated load at mid point:

The value of moment at section adjacent to the yield zone for a certain length is more then the yield moment. This little length is called as hinge length.

This hinge length depends on the loading type and geometry of the cross-section.

Now, from similar triangle

\[
\frac{M_p}{L/2} = \frac{M_y}{\left(\frac{L-L_p}{2}\right)}
\]
\[
\frac{M_p}{M_y} = \frac{L}{L - L_p} \\
S = \frac{1}{(1 - L_p/L)} \\
1 - \frac{L_p}{L} = \frac{1}{S} \\
L_p = L(1 - 1/S)
\]

For rectangular section of beam, shape factor, \( S = 1.5 \)

\[
L_p = L \left(1 - \frac{1}{1.5}\right) \\
L_p = \frac{L}{3}
\]

Hence, plastic hinge length for a simply supported rectangular beam subjected to concentrated load at mid-span = \( L/3 \).

5. (b) A uniform beam of plastic moment capacity \( M_p \) and span \( L \) is fixed at one end and simply supported at the other end is subjected to u.d.l. \( w \) per unit length. Find the location of intermediate hinge and ultimate load.

[Sol:

Given,

Let us find it by using kinematic method:

\( D_s = 1 \)

Number of plastic hinges required for collapse = \( D_s + 1 = 2 \)

Possible location of plastic hinges = \( A \), (in between \( AB \))

\( N = 2 \)

Number of independent mechanism = \( N - P_s = 2 - 1 = 1 \)

Mechanism 1:

Plastic hinges formed at \( A \) and in between \( A \) and \( B \) at a distance ‘\( x \)’ from \( B \),

\[
x\phi = (L - x)\theta
\]

\[
\phi = \left(\frac{L - x}{x}\right)\theta
\]

By the principle of virtual work

Work done by external force = Work done by internal force

\[
w_u\left[\frac{1}{2} \times L \times x\phi\right] = 2M_p\theta + M_p\phi
\]

\[
w_u\left[\frac{Lx}{2} \left(\frac{L-x}{x}\right)\theta\right] = 2M_p\theta + M_p \times \left(\frac{L-x}{x}\right)\theta
\]
\[ w_u \left[ \frac{L(L-x)}{2} \right] = 2M_p + M_p \frac{L-x}{x} \]
\[ w_u = \frac{M_p \left( \frac{2x+L-x}{x} \right)}{L(L-x)} = \frac{2M_p(L+x)}{x(L-x)L} \]

For \( w_u \) to be minimum

\[ \frac{dw_u}{dx} = 0 \]
\[ \frac{x(L-x)-(L+x)(L-2x)}{x(L-x)^2} = 0 \]
\[ xL - x^2 - (L^2 - 2xL + xL - 2x^2) = 0 \]
\[ xL - x^2 - L^2 + Lx + 2x^2 = 0 \]
\[ x^2 + 2Lx - L^2 = 0 \]
\[ x = \frac{-2L \pm \sqrt{4L^2 + 4L^2}}{2 \times 1} = \frac{-2L \pm 2\sqrt{2L}}{2} \]

Taking +ve sign,
\[ x = \left( \sqrt{2} - 1 \right)L \]
\[ x = 0.414L \]

Hence, plastic hinge will form at a distance 0.414L from the propped end.

Now, collapse/ultimate load, \( w_u = \frac{2M_p(L+0.414L)}{0.414L(L-0.414L)L} \]
\[ w_u = 11.656 \frac{M_p}{L^2} \]

---

**Section - C**

6. (a) From engineering point of view, explain the properties of clay minerals.

[10 Marks]

**Sol:** The clay minerals are hydrous aluminium silicate with other metallic ions in a sheet-like structure. They are very small in size, very flaky in shape and thus have considerable surface area. These clay minerals are evolved mainly from the chemical weathering of certain rock minerals.

**Clayey soils are made of three basic minerals:**

<table>
<thead>
<tr>
<th>Clay mineral</th>
<th>Grain size</th>
<th>Base exchange capacity</th>
<th>( I_p )</th>
<th>Dry strength</th>
<th>Active tip</th>
</tr>
</thead>
<tbody>
<tr>
<td>Montmorillonite</td>
<td>Min</td>
<td>Largest</td>
<td>Max.</td>
<td>Max.</td>
<td>Largest</td>
</tr>
<tr>
<td>Kaolinite</td>
<td>Max.</td>
<td>Least</td>
<td>Min</td>
<td>Min</td>
<td>Least</td>
</tr>
</tbody>
</table>

6. (b) In a triaxial test on a saturated clay, the sample was consolidated under a cell pressure of 160 kN/m\(^2\). After consolidation, the cell pressure was increased to 350 kN/m\(^2\) and the sample was failed under undrained condition. If the shear strength parameters of the soil are \( C' = 15.2 \) kN/m\(^2\), \( \phi' = 26^\circ \), \( B = 1.0 \) and \( A = 0.27 \), determine the effective major and minor principal stresses at the time of failure of the sample.

[10 Marks]
Sol:

Given, in a triaxial test,
On saturated clay,

Cell pressure, \( \sigma_3 = 160 \text{ kN/m}^2 \)

Now, cell pressure increased to

\[
\sigma_3' = 350 \text{ kN/m}^2 \\
\Delta \sigma_3 = 350 - 160 = 190 \text{ kN/m}^2
\]

Shear strength parameter

\[
c' = 15.2 \text{ kN/m}^2 \\
\phi' = 26^\circ
\]

Pore pressure parameter, \( B = 1.0 \)

\[
A = 0.27 \\
\theta = 45^\circ + \frac{\phi'}{2} \\
\theta = 45^\circ + \frac{26^\circ}{2} = 58^\circ
\]

At confining stage,

\[
B = 1
\]

\[
\frac{\Delta u_c}{\Delta \sigma_3} = 1 = \frac{\Delta u_c}{350 - 160}
\]

\( \Delta u_c = 190 \text{ kPa} \)

At 2nd stage,

\( A = \bar{A} = 0.27 \{ B = 1 \} \)

\[
\bar{A} = \frac{\Delta u_d}{\Delta \sigma_1 - \Delta \sigma_3} = \frac{\Delta u_d}{\Delta \sigma_3} = 0.27
\]

\( \Delta u_d = 0.27 \Delta \sigma_3 \)

\( \therefore \)

\( \sigma_1 = \sigma_3 + \Delta \sigma = 350 + \Delta \sigma_d \)

\( \Delta u = \Delta u_c + \Delta u_d \)

\( = 190 + 0.27 \Delta \sigma_d \)

Calculation of effective major and minor principal stresses:

\[
\sigma_1' = \left[ (\sigma_3 + \Delta \sigma_d) - \Delta u \right] = (350 + \Delta \sigma_d) - (190 + 0.27 \Delta \sigma_d)
\]

\[
= 160 + 0.73 \Delta \sigma_d
\]

\[
\sigma_3' = \sigma_3 - \Delta u = 350 - (190 + 0.27 \Delta \sigma_d)
\]

\[
= 160 - 0.27 \Delta \sigma_d
\]

\[
\tan \theta = \tan 58^\circ = 1.6
\]

\[
\tan^2 \theta = 2.56
\]

\[
160 + 0.73 \Delta \sigma_d = (160 - 0.27 \Delta \sigma_d) \times 2.56 + 2 \times 15.2 \times 1.6
\]

\( \Delta \sigma_d = 209.85 \text{ kPa} \)

\( \therefore \)

Effective major principal stress,

\( \sigma_1' = 160 + 0.73 \times 209.85 = 313.19 \text{ kPa} \)

Effective minor principal stress, \( \sigma_3' = 160 - 0.27 \times 209.85 = 103.34 \text{ kPa} \)

7(a) The bulk unit weight of a soil is 1900 kg/m³, the water content is 12.5% and the specific gravity of soil is 2.67. Find the dry unit weight, void ratio, porosity and degree of saturation.
Sol: Given,

Bulk unit weight of soil, \( \rho = 1900 \text{ kg/m}^3 \)

Water content, \( \omega = 12.5\% \)

Specific gravity of soil solid, \( G_s = 2.67 \)

Dry unit weight

\[
\rho_d = \frac{\rho}{1 + \omega} = \frac{1900}{1 + 0.125} = 1688.89 \text{ kg/m}^3
\]

Void ratio,

\[
e = \frac{G_s \rho_w}{\rho_d} - 1 = \frac{2.67 \times 1000}{1688.89} - 1 = 0.581
\]

Porosity,

\[
\eta = \frac{e}{1 + e} = \frac{0.581}{1 + 0.581} = 0.367
\]

Degree of saturate, \( e = \frac{\omega G_s}{S} \)

\[
S = \frac{0.125 \times 2.67}{0.581} = 0.5744
\]

S \approx 57.44\%

7. (b) A total load of 1200 kN is acting on a circular foundation of radius 2.0 m. Find the vertical stress intensity at a depth of 3.0 m and the depth at which the stress reduces to 10% of the applied stress.

[10 Marks]

Sol: Given,

Total load acting on circular foundation = 1200 kN

Radius = 2 m

\[
q = \frac{Q}{\pi (D^2)} = \frac{Q}{\pi R^2} = \frac{1200}{\pi \times (2)^2} = \frac{300}{\pi} = 95.49 \text{ kN/m}^2
\]

i) Vertical stress intensity at depth 3 m,

\[
\sigma_z = \sigma_3 = q \left[ \frac{1}{1 + \left( \frac{R}{Z} \right)^2} \right]^{3/2} = 95.49 \left[ 1 - \left( \frac{1}{1 + \left( \frac{2}{3} \right)^2} \right)^{3/2} \right]^{3/2}
\]

\( \sigma_3 = 40.48 \text{ kN/m}^2 \)

ii) Depth at which stress reduces to 10% of the applied stress i.e.

\[
\frac{\sigma_z}{10} = q \left[ \frac{1}{1 + \left( \frac{R}{Z} \right)^2} \right]^{3/2}
\]

\( Z = 1.048 \text{ m} \)