SSC-JE

Staff Selection Commission

Junior Engineer

Civil Engineering

Conventional Solved Questions

Previous Years Solved Papers
(2004–2021)

Also useful for State Service Examinations
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SSC-Junior Engineer : Civil Engineering Previous Year Conventional Solved Papers

First Edition: 2016

Second Edition: 2018

Third Edition: 2019

Fourth Edition: 2020

Fifth Edition: 2021

Sixth Edition: 2022

Preface


B. Singh (Ex. IES)

Staff Selection Commission-Junior Engineer has always been preferred by Engineers due to job stability. SSC-Junior Engineer examination is conducted every year. MADE EASY team has deeply analyzed the previous exam papers and observed that a good percentage of questions are repetitive in nature, therefore it is advisable to solve previous years papers before a candidate takes the exam.

The SSC JE exam is conducted in three stages as shown in table given below.

Papers	Subject	Maximum Marks	Duration
Stage 1: Paper-I : Objective type	(i) General Intelligence & Reasoning (ii) General Awareness (iii) General Engineering : Civil	50 Marks 50 Marks 100 Marks	2 hours
Stage 2: Paper-II Conventional Type	General Engineering : Civil	300 Marks	2 hours
Note: In Paper-I, every question carry one mark and there is negative marking of ¼ marks for every wrong answer. Candidates shortlisted in Stage 1 are called for Stage 2. On the basis of combined score in Stage 1 and Stage 2, final merit list gets prepared.			

In this edition, the book has been thoroughly revised and Reasoning-Aptitude section is also added. MADE EASY has taken due care to provide complete solution with accuracy. Apart from Staff Selection Commission-Junior Engineer, this book is also useful for Public Sector Examinations and other competitive examinations for engineering graduates.

I have true desire to serve student community by providing good source of study and quality guidance. I hope this book will prove as an important tool to succeed in SSC-JE and other competitive exams. Any suggestion from the readers for improvement of this book is most welcome.

With Best Wishes

B. Singh

CMD, MADE EASY

Syllabus of Engineering Subjects

(For Conventional Type Papers)

Civil Engineering

Building Materials : Physical and Chemical properties, Classification, Standard Tests, Uses and manufacture/ quarrying of materials e.g. building stones, silicate based materials, Cement (Portland), Asbestos products, Timber and Wood based Products, Laminates, bituminous materials, Paints, Varnishes.

Estimating, Costing and Valuation : Estimate, Glossary of technical terms, Analysis of rates, Methods and unit of measurement, Items of work – Earthwork, Brick work (Modular & Traditional bricks), RCC work, Shuttering, Timber work, Painting, Flooring, Plastering. Boundary wall, Brick building, Water Tank, Septic tank, Bar bending schedule. Centre line method, Mid-section formula, Trapezodial formula, Simpson's rule. Cost estimate of Septic tank, flexible pavements, Tube well, isolated and combined footings, Steel Truss, Piles and pile-caps. Valuation – Value and cost, scrap value, salvage value, assessed value, sinking fund, depreciation and obsolescence, methods of valuation.

Surveying: Principles of surveying, measurement of distance, chain surveying, working of prismatic compass, compass traversing, bearings, local attraction, plane table surveying, theodolite traversing, adjustment of theodolite, Levelling, Definition of terms used in levelling, contouring, curvature and refraction corrections, temporary and permanent adjustments of dumpy level, methods of contouring, uses of contour map, tacheometric survey, curve setting, earth work calculation, advanced surveying equipment.

Soil Mechanics: Origin of soil, phase diagram, Definitions- void ratio, porosity, degree of saturation, water content, specific gravity of soil grains, unit weights, density index and interrelationship of different parameters, Grain size distribution curves and their uses. Index properties of soils, Atterberg's limits, ISI soil classification and plasticity chart. Permeability of soil, coefficient of permeability, determination of coefficient of permeability, Unconfined and confined aquifers, effective stress, quick sand, consolidation of soils, Principles of consolidation, degree of consolidation, pre-consolidation pressure, normally consolidated soil, e-log p curve, computation of ultimate settlement. Shear strength of soils, direct shear test, Vane shear test, Triaxial test. Soil compaction, Laboratory compaction test, Maximum dry density and optimum moisture content, earth pressure theories, active and passive earth pressures, Bearing capacity of soils, plate load test, standard penetration test.

Hydraulics: Fluid properties, hydrostatics, measurements of flow, Bernoulli's theorem and its application, flow through pipes, flow in open channels, weirs, flumes, spillways, pumps and turbines.

Irrigation Engineering : Definition, Necessity, Benefits, III effects of irrigation, types and methods of irrigation. Hydrology – Measurement of rainfall, run off coefficient, rain gauge, losses from precipitation – evaporation, infiltration, etc. Water requirement of crops, duty, delta and base period, Kharif and Rabi Crops, Command area, Time factor, Crop ratio, Overlap allowance, Irrigation efficiencies. Different type of canals, types of canal irrigation, loss of water in canals. Canal lining – types and advantages. Shallow and deep to wells, yield from a well. Weir and barrage, Failure of weirs and permeable foundation, Slit and Scour, Kennedy's theory of critical velocity. Lacey's theory of uniform flow. Definition of flood, causes and effects, methods of flood control, water logging, preventive measures. Land reclamation, Characteristics of affecting fertility of soils, purposes, methods, description of land and reclamation processes. Major irrigation projects in India.

Transportation Engineering : Highway Engineering – cross sectional elements, geometric design, types of pavements, pavement materials – aggregates and bitumen, different tests, Design of flexible and rigid pavements – Water Bound Macadam (WBM) and Wet Mix Macadam (WMM), Gravel Road, Bituminous construction, Rigid pavement joint, pavement maintenance, Highway drainage. Railway Engineering – Components of permanent way – sleepers, ballast, fixtures and fastening, track geometry, points and crossings, track junction, stations and yards. Traffic Engineering – Different traffic survey, speed-flow-density and their interrelationships, intersections and interchanges, traffic signals, traffic operation, traffic signs and markings, road safety.

Environmental Engineering: Quality of water, source of water supply, purification of water, distribution of water, need of sanitation, sewerage systems, circular sewer, oval sewer, sewer appurtenances, sewage treatments. Surface water drainage. Solid waste management – types, effects, engineered management system. Air pollution – pollutants, causes, effects, control. Noise pollution – causes, health effects, control.

Structural Engineering

Theory of structures: Elasticity constants, types of beams - determinate and indeterminate, bending moment and shear force diagrams of simply supported, cantilever and over hanging beams. Moment of area and moment of inertia for rectangular & circular sections, bending moment and shear stress for tee, channel and compound sections, chimneys, dams and retaining walls, eccentric loads, slope deflection of simply supported and cantilever beams, critical load and columns, Torsion of circular section.

Concrete Technology: Properties, Advantages and uses of concrete, cement aggregates, importance of water quality, water cement ratio, workability, mix design, storage, batching, mixing, placement, compaction, finishing and curing of concrete, quality control of concrete, hot weather and cold weather concreting, repair and maintenance of concrete structures.

RCC Design: RCC beams-flexural strength, shear strength, bond strength, design of singly reinforced and doubly reinforced beams, cantilever beams. T-beams, lintels. One way and two way slabs, isolated footings. Reinforced brick works, columns, staircases, retaining walls, water tanks (RCC design questions may be based on both Limit State and Working Stress methods).

Steel Design: Steel design and construction of steel columns, beams roof trusses plate girders.

■■■■

Contents

SSC-JE

Civil Engineering

Conventional Solved Papers (Paper-II)

Unit I Theory of Structures

Section-A: Strength of Materials

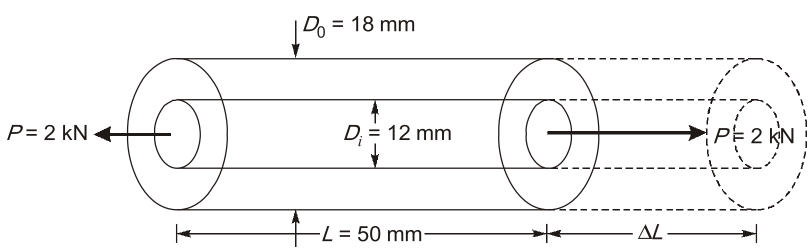
1. Simple Stress-Strain and Elastic Constants

1.1 The tensile test has been performed on a 50 mm long steel tube having 18 mm external diameter and 12 mm internal diameter. The axial load of 2 kN produced a stretch of 3.36×10^{-3} mm and a lateral contraction of the outer diameter of 3.62×10^{-4} mm. Calculate Young's modulus, Poisson's ratio and bulk modulus for the material

[SSC JE - 2004 : 3 × 5 = 15 Marks]

Solution:

Given, for a tensile test conducted on steel tube:



$$\text{Longitudinal stretch } (\Delta L) = 3.36 \times 10^{-3} \text{ mm}$$

$$\text{Lateral contraction } (\Delta D) = -3.62 \times 10^{-4} \text{ mm}$$

$$\epsilon_L = \left(\frac{\Delta L}{L}\right) = \frac{3.36\times10^{-3}}{50} = 6.72\times10^{-5}$$

$$\text{Lateral strain} = +\frac{\Delta D}{D} = \frac{3.62\times10^{-4}}{18} = 2.011\times10^{-5}$$

(i) Calculation for Young's modulus:

Now, from Hooke's, law we know that

$$\frac{\text{Stress}}{\text{Strain}} = \text{Modulus of elasticity}$$

$$\Rightarrow \quad \text{Modulus of elasticity, } (E) = \frac{P/A}{\epsilon_L} = \frac{\dfrac{(2\times10^3)}{\frac{\pi}{4}(18^2-12^2)}}{6.72\times10^{-5}}$$

$$= \frac{2\times10^3}{\frac{\pi}{4}\cdot(18^2-12^2)\times6.72\times10^{-5}} = 210522.4115 \text{ N/mm}^2$$

$$E = 210.522 \times 10^3 \text{ N/mm}^2$$

(ii) Calculation of Poisson's ratio (μ)

$$(\mu) = -\frac{\text{lateral strain}}{\text{longitudinal strain}}$$

$$= -\frac{(-2.011\times10^{-5})}{6.72\times10^{-5}} = 0.299 \simeq 0.3$$

(iii) Calculation of bulk modulus of the material

We know that, $E = 3K(1-2\mu)$

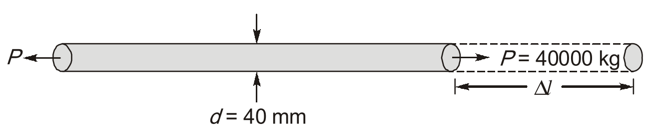
$$K = \frac{E}{3(1-2\mu)} = \frac{210.522\times10^3}{3(1-(2\times0.3))}$$

$$= 175435 \text{ N/mm}^2 = 175.435\times10^3 \text{ N/mm}^2$$

1.2 A bar 40 mm in diameter is subjected to a tensile force of 40000 kg. The extension of bar measured over a gauge length of 200 mm was 0.318 mm. The decrease in diameter was found to be 0.02 mm. Calculate the values of Young's modulus of elasticity and modulus of rigidity of the material.

[SSC JE - 2007 : 15 Marks]
[SSC JE - 2009 : 10 Marks]

Solution:



Pull, $P = 40000 \text{ kg} = 400 \text{ kN}$

Gauge length, $l = 200 \text{ mm}$

Extension, $\Delta l = 0.318 \text{ mm}$

Longitudinal Strain, $\epsilon = \frac{\Delta l}{l} = \frac{0.318}{200} = 1.59\times10^{-3}$

Cross-sectional area of bar, $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.04)^2 = 1.2566\times10^{-3} \text{ m}^2$

$\therefore$ Stress, $\sigma = \frac{P}{A} = \frac{400\times10^3}{1.2566\times10^{-3}} \text{ N/m}^2 = 318.319\times10^6 \text{ N/m}^2$

$\therefore$ Modulus of elasticity, $E = \frac{\sigma}{\epsilon} = \frac{318.319\times10^6}{1.59\times10^{-3}} \text{ N/m}^2 = 200.2\times10^9 \text{ N/m}^2 \simeq 2\times10^5 \text{ N/mm}^2$

Given; Decrease in diameter, $\Delta d = -0.02 \text{ mm}$

$\therefore$ Lateral strain, $\frac{\Delta d}{d} = \frac{-0.02}{40} = -5\times10^{-4}$

$\therefore$ Poisson's ratio, $\mu = -\frac{\Delta d/d}{\epsilon} = -\frac{5\times10^{-4}}{1.59\times10^{-3}} = 3.145\times10^{-1} \simeq 0.315$

$\therefore$ Modulus of rigidity, $G = \frac{mE}{2(m+1)}$ where $\mu = \frac{1}{m} = 0.315$

$$= \frac{E}{2\left(1+\frac{1}{m}\right)} = \frac{2\times10^5}{2(1.315)}$$

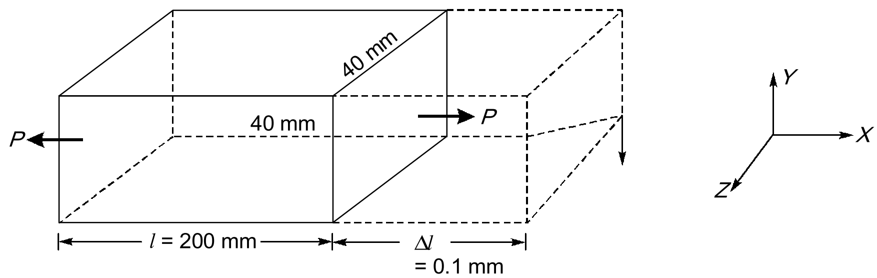
$$= 0.7605 \times 10^5 \text{ N/mm}^2 = 7.6 \times 10^4 \text{ N/mm}^2$$

1.3 When a bar of certain material 40 mm square is subjected to an axial pull of 1,60,000 N the extension on a gauge length of 200 mm is 0.1 mm and the decrease in each side of the square is 0.005 mm. Calculate Young's modulus, Poisson's ratio. Shear modulus and bulk modulus for this material.

[SSC JE - 2008 : 10 Marks]

Solution:

Given:



Axial load, $P = 160000$ N

C/s area, $A = 40 \times 40 = 1600 \text{ mm}^2$

$$\text{Longitudinal strain } (\varepsilon_x) = \frac{\Delta l}{l} = \frac{0.1}{200} = 5\times10^{-4}$$

Decrease in each side of square

$(\Delta a) = -0.005$ mm

$$\therefore \quad \text{Lateral strain } (\varepsilon_y = \varepsilon_z) = -\frac{0.005}{40} = -1.25\times10^{-4}$$

Now, calculation of Young's modulus (E):

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{P}{A\cdot\varepsilon_x} = \frac{160000}{1600\times5\times10^{-4}} = 200\times10^3 \text{ N/mm}^2$$

$$\text{Calculation of Poisson's ratio,} \quad \mu = -\frac{\text{lateral strain}}{\text{longitudinal strain}} = -\frac{(-1.25\times10^{-4})}{5\times10^{-4}} = \frac{1}{4} = 0.25$$

Calculation of shear modulus (G):

$$E = 2G(1+\mu)$$

$$G = \frac{E}{2(1+\mu)} = \frac{200\times10^3}{2(1+0.25)} = \frac{200\times10^3}{2.5} = 80\times10^3 \text{ N/mm}^2$$

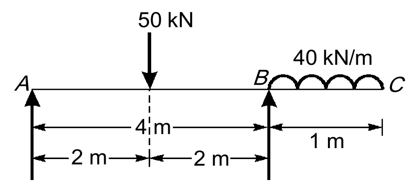
Calculation of bulk modulus of material (K):

$$E = 3K(1-2\mu)$$

$$K = \frac{E}{3(1-2\mu)} = \frac{200\times10^3}{3(1-(2\times0.25))} = 133.33 \times 10^3 \text{ N/mm}^2$$

2. Shear Force and Bending Moment Diagram

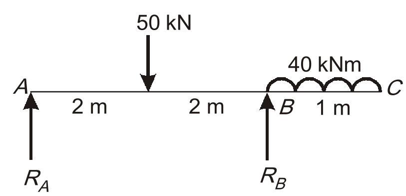
2.1 A horizontal beam ABC of total length 5 m is simply supported over the span *AB* (length, 4m) and overhand *BC* (length = 1 m) as shown in figure. The beam carries a concentrated load of 50 kN at mid point of span *AB* and uniformly distributed load of 40 kN per metre over the overhang portion *BC*. Draw the bending moment and shear force diagrams indicating values at significant point.



[SSC JE - 2004 : 15 Marks]

Solution:

Given:



Calculation of reaction:

$$\Sigma F_v = 0 \; ; \; R_A + R_B = 90 \qquad \text{...(i)}$$

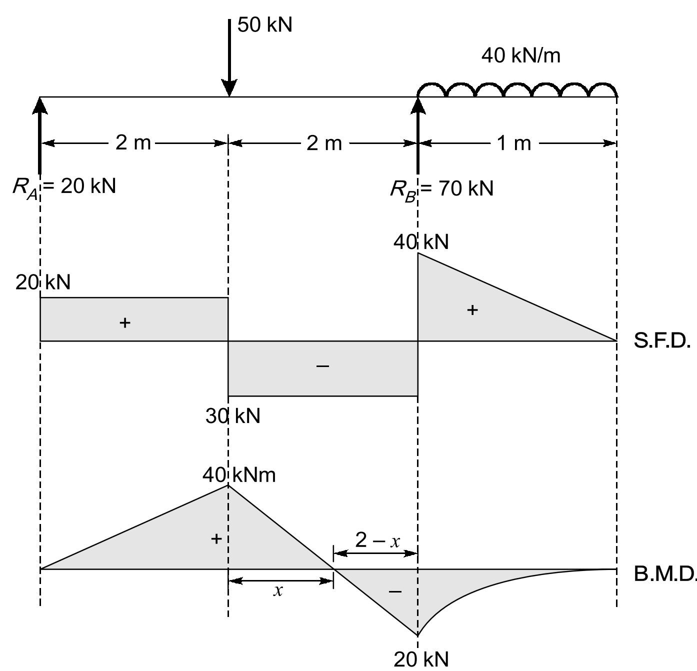
$$\Sigma M_A = 0$$

$$R_B \times 4 = 50 \times 2 + 40 \times 1 \times (0.5 + 4)$$

$$R_B = \frac{100+180}{4} = 70 \text{ kN}$$

$\therefore$ From equation (i) $\quad R_A = 90 - 70 = 20$ kN

Hence



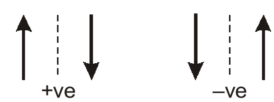
$$\frac{x}{40} = \frac{2x}{20}$$

$$x = 4 - 2x$$

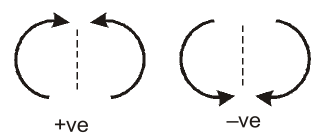
$$3x = 4$$

$$x = \frac{4}{3} \text{ m}$$

Two draw S.F.D. and B.M.D.
Sign convention:
Shear force:



Bending moment:



Method 1: To draw S.F.D and B.M.D
To draw S.F.D. We know that

$$w = \frac{dv}{dx} \begin{cases} w = \text{loading intensity} \\ v = \text{shear force} \end{cases}$$

⇒ Slope of shear force diagram is equal to its loading intensity.
At *A*: There is an upward load of 20 kN, hence shear force jump by 20 kN
A to *D*: There is no loading intensity, hence shear force is constant.
At *D*: There is downward concentrated load of 50 kN, hence shear force drop by 50 kN

∴ $V_D = 20 - 50 = -30$ kN

D to *B*: There is no loading intensity, hence shear force is constant.
At *B*: There is an upward point load of 70 kN, hence shear force jumps by 70 kN.

∴ $V_D = 70 - 30 = 40$ kN

B to *C*: Loading intensity is –ve and constant, hence slope of shear force diagram is –ve and constant

$$V_C - V_B = -40 \times 1$$

$$V_C - 40 = -40$$

⇒ $V_C = 0$

B.M.D. We know that: $$V = \frac{dM}{dx} \begin{cases} V = \text{Shear force} \\ M = \text{Bending moment} \end{cases}$$

⇒ Slope of bending moment diagram is equal to shear force.
At *A*: B.M is zero (Hinged end)
A to *B*: S.F is +ve and constant, hence slope of B.M. D is +ve and constant.

∴ $M_D - M_A = 20 \times 2$

$$M_D = 40 \text{ kNm}$$

D to *B*: S.F is –ve and constant, hence slope of B.M.D,. –ve and constant.

$$M_B - M_D = -60$$

$$M_B - 40 = -60$$

$$M_B = -20 \text{ kNm}$$

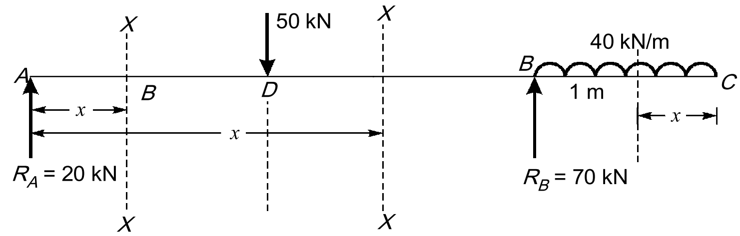
B to *C*: S.F is +ve and decreasing, hence slope of B.M.D is +ve and decreasing.

$$M_C - M_B = \frac{1}{2}\times 40\times 1$$

$$M_C + 20 = \frac{1}{2}\times 40\times 1$$

$$M_C = 0$$

Method 2 : Analytical method



For span *AD*: $[0 \le x \le 2\text{ m}]$ (section taken from *A*)

$$V_X = +R_A = 20\text{ kN} \quad \text{(constant)}$$

$\therefore$ $$V_A = V_D = 20\text{ kN}$$

$$M_X = R_A \times x = 20 \times x \text{ (Linear)}$$

At $x = 0$, $M_A = 0$

$x = 2$ m, $$M_D = 20 \times 2 = 40\text{ kNm}$$

For span *DB*: $[2\text{ m} \le x \le 4\text{ m}]$ (x taken from *A*)

$$V_X = +R_A - 50 = +20 - 50 = -30\text{ kN} \quad \text{(constant)}$$

$\Rightarrow$ $$V_D = V_B = -30\text{ kN}$$

$$M_X = R_A \times (x) - 50\,(x - 2)$$

$$= 20\,x - 50x + 100 = -30x + 100 \quad \text{(Linear)}$$

$\Rightarrow$ $$M_D \cdot (x = 2\text{ m}) = -30 \times 2 + 100 = 40\text{ kNm}$$

$$M_E\,(x = 4\text{ m}) = -30 \times 4 + 100 = -20\text{ kNm}$$

Since, bending moment values changes from +ve to negative, in span *DB*. Hence there must be a point where B.M is zero.

$\Rightarrow$ $$M_{XX} = 0$$

$$-30x + 100 = 0$$

$$x = \frac{100}{30} = 3.33\text{ m} \quad \text{(from } A\text{)}$$

For span BC: $(0 \le x \le 1\text{ m})$ (x from *C*)

$$V_{XX} = 40\,x \quad \text{(Linear)}$$

$\Rightarrow$ $$V_C = 0;\ V_B = 40\text{ kN}$$

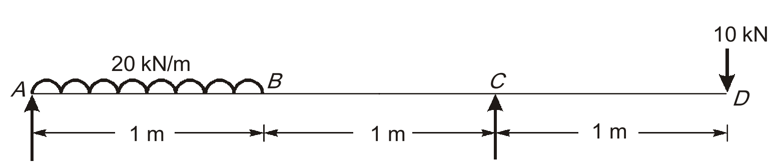
$$M_{XX} = -40x \times \frac{x}{2} = -20x^2$$

$\Rightarrow$ $$M_C = 0$$

$$M_B = -20 \times (1)^2 = -20\text{ kNm} \quad \text{(Parabolic)}$$

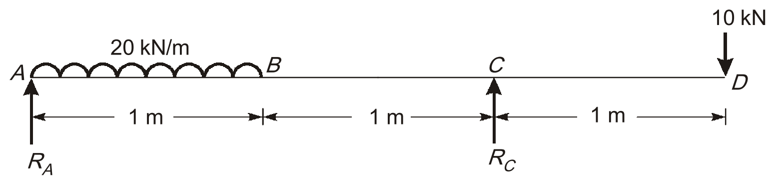
The B.M.D. and S.F.D. as shown above.

2.2 **Draw the shear force and bending moment diagrams for the beam carrying loads shown in figure and locate the point of contraflexure.**



[SSC JE - 2005 : 15 Marks]

Solution:



Calculation of reaction:

$\Sigma F_v = 0$ $\quad R_A + R_C = 30$ ...(i)

$$\Sigma F_v \curvearrowright = 0$$

$$-R_C \times 2 + 10 \times 3 + 20 \times 1 \times \frac{1}{2} = 0$$

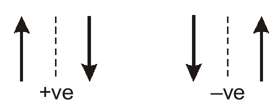
$$R_C = 20 \text{ kN}$$

$\therefore$ From equation (i) $\quad R_A = 10$ kN

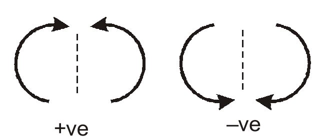
To draw S.F.D and B.M.D

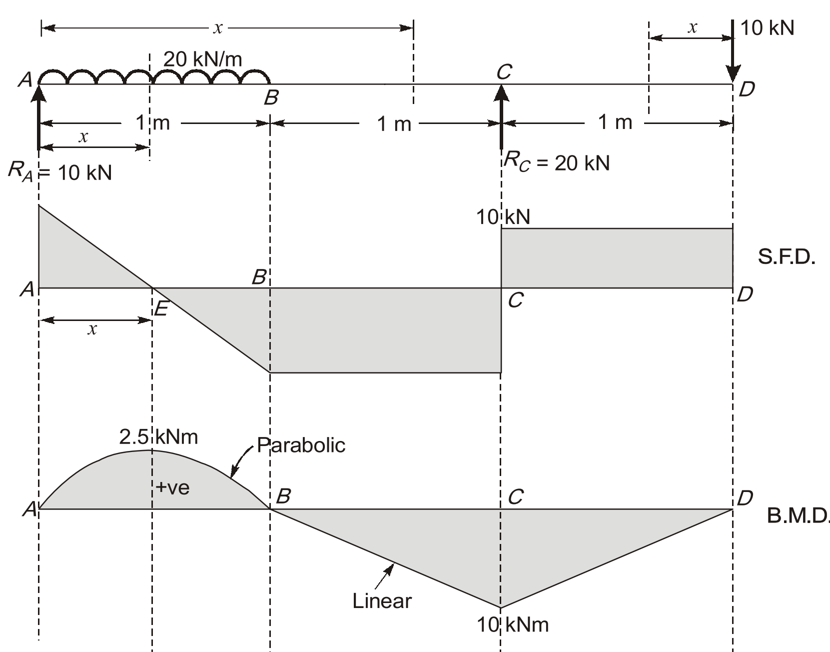
Sign convention:

For shear force



For bending moment





Method 1:

To draw S.F.D.

We know that $$\omega = \frac{dv}{dx} \quad \begin{cases} \omega = \text{loading intensity} \\ v = \text{shear force} \end{cases}$$

$\Rightarrow$ Slope of shear force diagram = loading intensity

At *A*, there is an upward point load of 10 kN. Hence, shear force jumps up by 10 kN.

A to *B*: Loading intensity is negative and constant, hence slope of shear force diagram is negative and constant.

$$V_B - V_A = -20 \times 1$$
$$V_B - 10 = -20$$
$$V_B = -10 \text{ kN}$$

B to *C* : There is no loading intensity, hence shear force is constant.

i.e. $$V_B = V_C = -10 \text{ kN}$$

At *C* : There is an upward point load of 20 kN, hence shear force jump up by 20 kN.

$\therefore$ $$V_C = -10 + 20 = 10 \text{ kN}$$

C to *D* : There is no loading intensity, hence shear force is constant.

$$V_C = V_D = 10 \text{ kN}$$

(B.M.D.) We know that

$$V = \frac{dM}{dx} \quad \begin{cases} V = \text{shear force} \\ M = \text{Bending moment} \end{cases}$$

$\Rightarrow$ Slope of bending moment diagram is equal to shear force.

At *A* : $$BM = 0$$

A to *E*: SF is positive and decreasing, hence slope of B.M.D is positive and decreasing.

$$M_E - M_A = \frac{1}{2} \times 10 \times 0.5$$
$$M_E = 2.5 \text{ kNm}$$

E to *B*: SF is negative and increasing. Hence slope of B.M.D is negative and increasing.

$$M_B - M_E = -\frac{1}{2} \times 10 \times 0.5 = -2.5$$
$$M_B = -2.5 + 2.5 = 0$$

B to *C*: SF is negative and constant. Hence slope of B.M.D is negative and constant.

$\therefore$ $$M_C - M_B = -10 \times 1$$
$$M_C = -10 \text{ kNm}$$

C to *D*: SF is positive and constant, hence slope of B.M.D. is positive and constant.

$$M_D - M_C = 10 \times 1$$
$$M_D = -(-10) = 10$$
$$M_D = 0$$

Point of contraflexure is the point where bending moment changes sign.

i.e. At *B*: (x = 1 m from *A*)

Method 2 : Analytical method

Portion AB: $[0 \le x \le 1\text{ m}]$ x from A

$$V_{xx} = 10 - 20x \quad \text{(Linear)}$$

At $x = 0$; $V_A = 10$ kN

$x = 1$ m, $V_B = 10 - 20 \times 1 = -10$ kN

Since, shear force changes from positive to negative. Hence in span AB there must be a point where S.F. = 0

$$\Rightarrow \quad -20x + 10 = 0$$

$$x = 0.5 \text{ m (from } A)$$

and at this point B.M. will be maximum.

$$M_{xx} = 10x - 20x \times \frac{x}{2} = 10x - 10x^2 \quad \text{(parabolic)}$$

At $x = 0$, $M_A = 0$

$x = 1$ m, $M_B = 10 \times 1 - 10 \times 1 = 0$

At $x = 0.5$ m

$$\therefore \quad BM_{\max} = 10 \times \frac{1}{2} - 10 \times \left(\frac{1}{2}\right)^2$$

$$= 5 - 2.5 = 2.5 \text{ kNm}$$

Portion BC: $[1\text{ m} \le x \le 2\text{ m}]$ x from A

$$\therefore \quad V_{xx} = 10 - 20 = -10 \text{ kN (constant)}$$

$$\Rightarrow \quad V_B = V_C = -10 \text{ kN}$$

$$M_{xx} = 10x - 20 \times 1 \times (x - 0.5)$$

$$= 10x - 20x + 10$$

$$= -10x + 10 \text{ (Linear)}$$

$$M_B(x = 1\text{ m}) = 0$$

$$M_C(x = 2\text{ m}) = -10 \times 2 + 10 = -10 \text{ kNm}$$

Portion CD: $[0 \le x \le 1\text{ m}]$ x from A

$$\therefore \quad V_{xx} = 10 \text{ kN (constant)}$$

$$\Rightarrow \quad V_D = V_C = 10 \text{ kN}$$

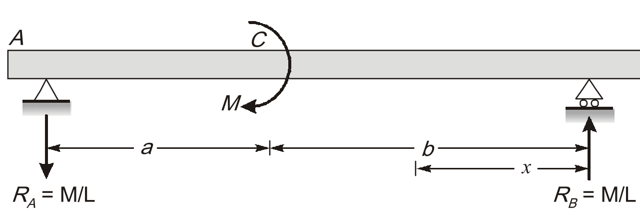
$$M_{xx} = -10x \text{ (Linear)}$$

At $x = 0$; $M_D = 0$

$x = 1$ m, $M_C = -10$ kNm

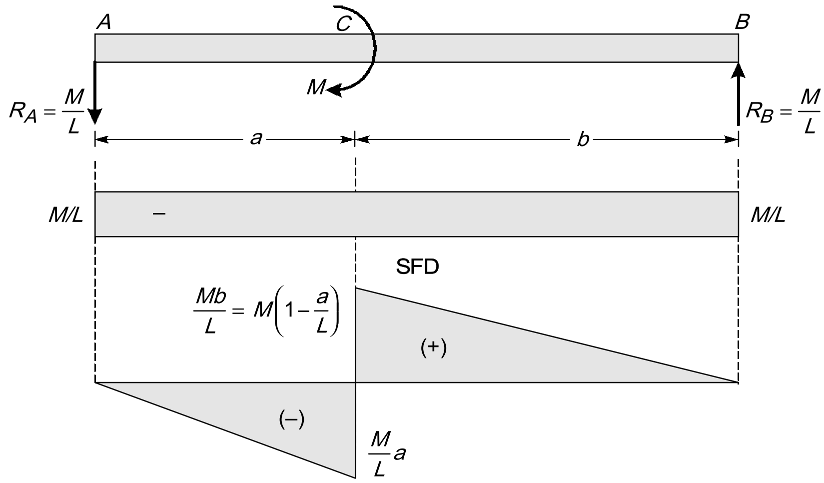
The B.M.D. and S.F.D. of the beam shown above.

2.3 Draw SF and BM diagrams for the beam with applied moments as shown in figure.



[SSC JE - 2007 : 15 Marks]

Solution:



BMD

$\Sigma M_B = 0$

$\Rightarrow \quad R_A(L) + M = 0$

$\Rightarrow \quad R_A = -\frac{M}{L}$ i.e., R_A is acting downwards

$\Sigma F_y = 0$

$\Rightarrow \quad R_A + R_B = 0$

$\Rightarrow \quad R_B = -R_A = \frac{M}{L}(\uparrow)$

Portion AC: $0 \le x \le a$

$$\text{SF} = -\frac{M}{L}$$

$$\text{BM} = -\frac{M}{L}x$$

At $x = a$, $\quad \text{BM} = -\frac{M}{L}a$

Portion CB: $a \le x \le L$

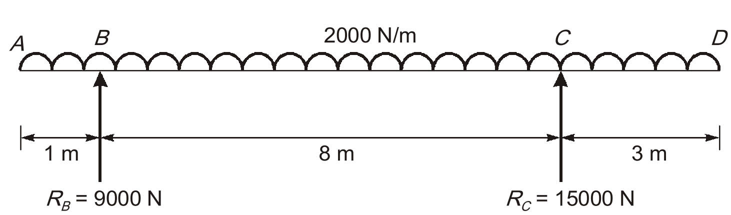
$$\text{SF} = -\frac{M}{L}$$

$$\text{BM} = -\frac{M}{L}x + M = M\left(1-\frac{x}{L}\right)$$

At $x = a$, $\quad \text{BM} = M\left(1-\frac{a}{L}\right)$

At $x = L$, $\quad \text{BM} = 0$

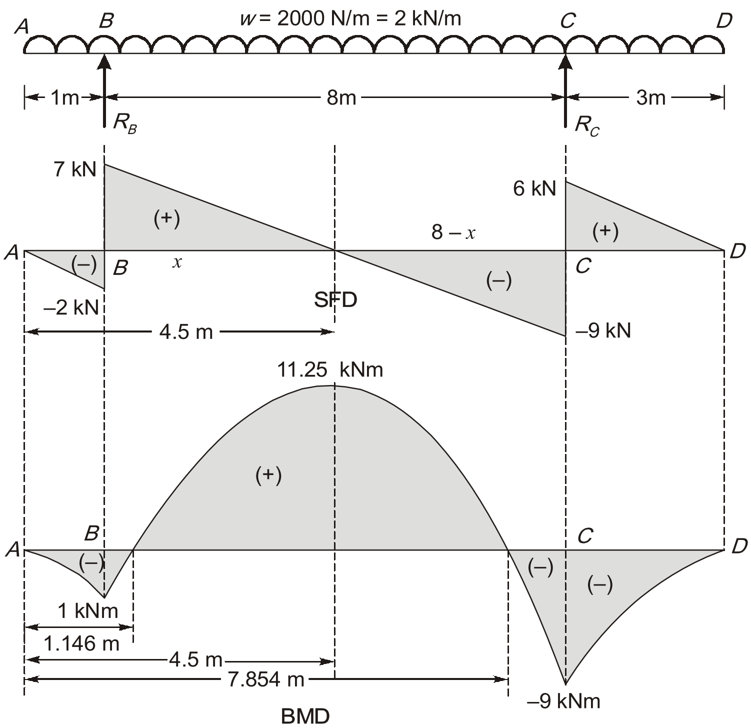
2.4 Draw SF and BM diagrams for the beam having overhangs on both sides and loaded as shown in figure.



[SSC JE - 2008 : 20 Marks]

Solution:

Method I:



$$\frac{x}{7} = \frac{8-x}{9}$$

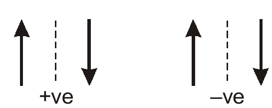
$$9x = 56 - 7x$$

$$16x = 56$$

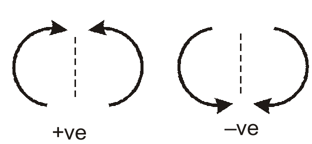
$$x = 3.5 \text{ m}$$

Sign convention:

For shear force



For bending moment



$\Sigma F_y = 0$

$$\Rightarrow \quad R_B + R_C = 2\,(12) = 24 \text{ kN} \qquad \text{...(i)}$$

$\Sigma M_C = 0$

$$\Rightarrow \quad R_B(8) + 2 \times 3 \times \frac{3}{2} = 2 \times 9 \times \frac{9}{2}$$

$$\Rightarrow \quad R_B = 9 \text{ kN}$$

$$\therefore \quad R_C = 24 - R_B = 24 - 9 = 15 \text{ kN}$$

Portion AB: $0 \le x \le 1$ m

$$\text{SF} = -2x \qquad \text{(Linear variation)}$$

$$\text{BM} = -2x\frac{x}{2} = -x^2 \qquad \text{(Parabolic variation)}$$

Portion BC: $1 \text{ m} \le x \le 9$ m

$$\text{SF} = -2x + R_B = 9 - 2x \qquad \text{(Linear variation)}$$

$$\text{BM} = -2x\frac{x}{2} + R_B(x-1)$$

$$= -2x\frac{x}{2} + R_B(x-1)$$

$$= 9x - 9 - x^2 \qquad \text{(Parabolic variation)}$$

At $x = 1$ m, SF = 7 kN

BM = –1 kNm

At $x = 9$ m, SF = –9 kN

BM = –9 kNm

when SF = 0

$\Rightarrow$ $9 - 2x = 0$

$\Rightarrow$ $x = 4.5$ m (from A)

$\therefore$ BM is maximum at $x = 4.5$ m

$\therefore$ $BM_{max} = 9(4.5) - 9 - (4.5)^2 = 11.25$ kNm

Portion CD: $9x \le x \le 12$ m

$$SF = -2x + R_B + R_C$$

$$= -2x + 24 \qquad \text{(Linear variation)}$$

$$BM = -2\frac{(12-x)^2}{2} = -(12-x)^2 \qquad \text{(Parabolic variation)}$$

At $x = 9$ m, SF = –2(9) + 24 = 6 kN

BM = $-(12-9)^2 = -9$ kNm

At $x = 12$ m, SF = –2(12) + 24 = 0

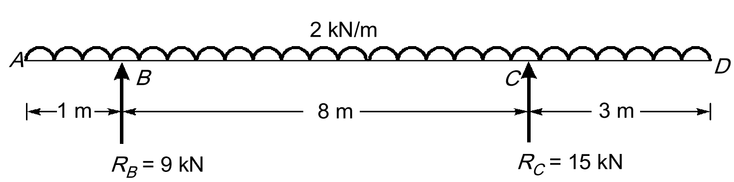
BM = –(12 – 12) = 0

Point of contraflexure where BM changes sign i.e., BM = 0

$$9x - 9 - x^2 = 0 \Rightarrow x^2 - 9x + 9 = 0$$

$\therefore$ $x = 1.146$ m, 7.854 m

Method 2:



To draw S.F.D., we know that

$$w = \frac{dv}{dx} \quad (w = \text{loading intensity, } v = \text{shear force})$$

At A: $V_A = 0$

A to B: Loading intensity is negative and constant, hence slope of shear force diagram (S.F.D.) is negative and constant.

$$V_B - V_A = -2 \times 1$$

$$V_B = -2 \text{ kN}$$

At B: There is an upward point load of 9 kN, hence shear force jump up by 9 kN.

$\therefore$ $V_B = -2 + 9 = 7$ kN

B to C: Loading intensity is negative and constant, hence slope of S.F.D is negative and constant.

$$V_C - V_B = -2 \times 8$$

$$V_C - 7 = -16$$

$$V_C = -9 \text{ kN}$$

Since value of S.F changes from positive to negative. In span BC there must be a point where S.F = 0, and at that point B.M is maximum.

At C: There is an upward point load of 15 kN, hence shear force jumps up by 15 kN

$$\therefore \quad V_C = -9 + 15 = 6 \text{ kN}$$

C to D: Loading intensity is negative and constant, hence slope of S.F.D. is negative and constant.

$$V_D - V_C = -3 \times 2$$
$$V_D - 6 = -6$$
$$V_D = 0$$

To draw B.M.D.

We know that $$V = \frac{dM}{dx} \quad (M = \text{Bending moment},\ V = \text{shear force})$$

$\Rightarrow$ Slope of bending moment diagram is equal to shear force.

At A: $$BM = 0$$

A to B: S.F is negative and increasing, hence slope of B.M.D is negative and increasing.

$$M_B - M_A = \frac{1}{2}\times(-2)\times 1$$
$$M_B = -1 \text{ kNm}$$

B to E: S.F is positive and decreasing, hence slope of BMD is positive and decreasing.

$$M_E - M_B = \frac{1}{2}\times 7\times 3.5$$
$$M_E + 1 = 12.25$$
$$M_E = 11.25 \text{ kNm}$$
$$\therefore \quad BM^{max} = 11.25 \text{ kNm}$$

E to C: Shear force is negative and increasing, hence slope of B.M.D is negative and increasing.

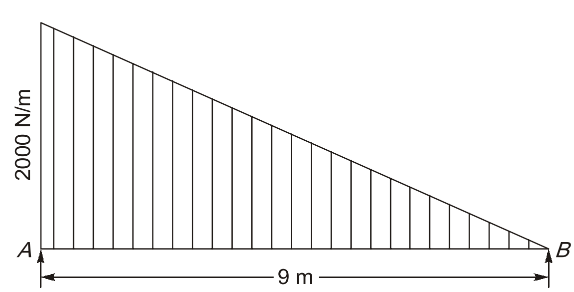
$$M_C - M_E = \frac{-1}{2}\times 9\times(8-3.5)$$
$$M_C - 11.25 = \frac{-1}{2}\times 9\times 4.5$$
$$M_C = -9 \text{ kNm}$$

C to D: Shear force is positive and decreasing, hence slope of B.M.D. is positive and decreasing.

$$M_D - M_C = +\frac{1}{2}\times 6\times 3$$
$$M_D - (-9) = 9$$
$$M_D = 0$$

The corresponding S.F.D. and B.M.D is shown above.

2.5 Draw SF and BM diagrams for beam loaded with varying load as shown in figure.



[SSC JE - 2009 : 10 Marks]