ESE 2019
UPSC ENGINEERING SERVICES EXAMINATION
Main Examination

Civil Engineering
Conventional Solved Questions

Paper-I

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State Engineering Services Examinations

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During the last few decades of engineering academics, India has witnessed geometric growth in engineering graduates. It is noticeable that the level of engineering knowledge has degraded gradually, while on the other hand competition has increased in each competitive examination including GATE and UPSC examinations. Under such scenario higher level efforts are required to take an edge over other competitors.

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The efforts have been made to provide close and illustrative solutions in lucid style to facilitate understanding and quick tricks are introduced to save time.

Following tips during the study may increase efficiency and may help in order to achieve success.

- Thorough coverage of syllabus of all subjects
- Adopting right source of knowledge, i.e. standard reading text materials
- Develop speed and accuracy in solving questions
- Balanced preparation of Paper-I and Paper-II subjects with focus on key subjects
- Practice online and offline modes of tests
- Appear on self assessment tests
- Good examination management
- Maintain self motivation
- Avoid jumbo and vague approach, which is time consuming in solving the questions
- Good planning and time management of daily routine
- Group study and discussions on a regular basis
- Extra emphasis on solving the questions
- Self introspection to find your weaknesses and strengths
- Analyze the exam pattern to understand the level of questions
- Apply shortcuts and learn standard results and formulae to save time

B. Singh (Ex. IES)
CMD, MADE EASY Group
## ESE 2019 : Main Examination
### Civil Engineering: Paper-I

Conventional Solved Questions of UPSC Engineering Services Examination

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Revised Syllabus of ESE: Stone, Lime, Glass, Plastics, Steel, FRP, Ceramics, Aluminum, Fly Ash, Basic Admixtures, Timber, Bricks and Aggregates: Classification, properties and selection criteria;

Cement: Types, Composition, Properties, Uses, Specifications and various Tests; Lime & Cement Mortars and Concrete: Properties and various Tests; Design of Concrete Mixes: Proportioning of aggregates and methods of mix design.

1. Cement

Q.1 What are the initial and final setting times of cement? How are they experimentally determined? Briefly explain the roles of gypsum and calcium chloride in cement.

[10 marks : 1995]

Solution:
Cement when mixed with water forms paste which gradually becomes less plastic and finally a hard mass is obtained. In this process of setting a stage, is reached when the cement paste is sufficiently rigid to withstand a definite amount of pressure. The time to reach this stage is termed as setting time. The setting time is divided into two parts, namely, the initial and the final setting times. The time elapsed from mixing of water in cement to the time at which cement paste loses its plasticity. The time taken to reach the stage when the paste becomes a hard mass is known as the final setting time.

Initial and final setting times are determined with the help of Vicat apparatus. A cement paste of standard consistency is filled into the Vicat mould in specified manner within 3-5 minutes. The needle of the apparatus is lowered gently and brought in contact with the surface of the test block. It is quickly released and allowed to penetrate into the test block. In the beginning, the needle completely pierce through the test block, but after some time when the paste starts losing its plasticity, the needle may penetrate only to a depth of 33-35 mm from the top. The period elapsing between the time when water is added to the cement and the time at which the needle penetrates the test block to a depth equal to 33-35 mm from the top is taken as the initial setting time.

For determining final setting time the needle of the Vicat apparatus is replaced by a circular attachment. The cement is considered as finally set when, upon lowering the attachment gently over the surface of the test block, the centre needle makes an impression while the circular cutting edge of the attachment fails to do so. In other words, the paste has attained such hardness that the centre needle does not pierce through the paste more than 0.5 mm.

The reaction of pure tricalcium aluminate (C₃A) with water is very fast and this may lead to flash set of cement. To prevent this flash set, gypsum is added at the time of grinding the cement clinker. The quantity of gypsum added has a bearing on the quantity of C₃A present. Calcium chloride is an accelerator which is added to cement in winter concreting which incidently works as antifreeze also. Calcium chloride increase the rate of hydration, thus can be used where the temperature is very low.
Q.2  Explain the purpose of conducting soundness test of cement. Describe the apparatus and method of test with the help of neat sketches. What are the permissible limits of observations in the test?  
[10 marks : 1996]

Or

Describe the procedure to list the soundness of cement. Name the constituents causing soundness.  
[10 marks : 2010]

Solution:

It is very important that the cement after setting shall not undergo any appreciable change of volume. Certain cements have been found to undergo a large expansion after setting causing disruption of the set and hardened mass. This will cause serious difficulties for the durability of the structures when such cement is used. The testing of soundness of cement to ensure that the cement does not show any appreciable subsequent expansion is of prime importance. Unsoundness in cement is due to excess of lime, excess of magnesia or excessive proportion of sulphates.

Soundness test can be carried out with the help of Le Chatelier apparatus. It consists of a small split cylinder of spring brass or other suitable metal. It is 30 mm in diameter and 30 mm high. On either side of the split are attached two indicator arms 165 mm long with pointed ends.

Cement is gauged with 0.78 times the water required for standard consistency in a standard manner and filled into the mould kept on a glass plate. The mould is covered on the top with another glass plate. The whole assembly is immersed in water at a temperature of 27°C-32°C and kept there for 24 hours. The distance between the indicator points is measured and the mould is again submerged in water. The water is heated and brought to boiling point in about 25-30 minutes and kept in the same condition for 3 hours. Now, the mould is removed from water and allowed to cool. The distance between the indicator points is measured again. The difference between these two measurements represents the expansion of cement. This must not exceed 10 mm for ordinary, rapid hardening and low heat portland cement. If in case the expansion is more than 10 mm as tested above, the cement is said to be unsound.

The most important limitation of the Le Chatelier test is that it detects unsoundness due to free lime only. This method of testing does not indicate the presence and after effect of the excess of magnesia. The sketches of Le Chatelier apparatus are shown:

Q.3  Name the four important constituents of cement and state the role of each in achieving its properties.  
[10 marks : 1998]

Solution:

The four important constituents of cement are:

(i) Lime (CaO) – 60 to 67%  
(ii) Silica (SiO₂) – 17 to 25%  
(iii) Alumina (Al₂O₃) – 3 to 8%  
(iv) Iron oxide (Fe₂O₃) – 0.5 to 6%

All these oxides interact with one another in the kiln at high temperature to form more complex compounds. The relative proportions of these oxides compositions are responsible for influencing the various properties of cement in addition to rate of cooling and fineness of grinding. The complex compounds which are formed due to the combination of these oxides are called Bogue's compounds and four of them are usually regarded as major compounds. They are tricalcium silicate (C₃S), dicalcium silicate (C₂S), tricalcium aluminate (C₃A) and tetra calcium aluminoferrite (C₄AF).

The two silicates namely C₃S and C₂S which together constitute about 70 to 80 per cent of the cement control the most of the strength giving properties. Upon hydration, both C₃S and C₂S give the same
product called calcium silicate hydrate \((C_2S_2H_3)\) and calcium hydroxide. \(C_3S\) giving a faster rate of reaction accompanied by a greater heat evolution develops early strength. On the other hand, \(C_2S\) hydrates and hardens slowly and provides the ultimate strength. But the hydration of \(C_3S\) liberates nearly three times are much calcium hydroxide as compared to \(C_2S\). That’s why \(C_2S\) provides more resistance to chemical attack.

The compound tricalcium aluminate \((C_3A)\) is characteristically fast reacting with water and may lead to an immediate stiffening of paste, and this process is termed as flash set. The role of gypsum added in the manufacture of cement is to prevent such a fast reaction. The hydrated aluminates do not contribute anything to the strength of concrete. On the other hand, their presence is harmful to the durability of concrete particularly where the concrete is likely to be attacked by sulphates. As it hydrates fast it may contribute a little to the early strength.

On hydration, \(C_4AF\) is believed to form a system of the form \(CaO\cdot Fe_2O_3\cdot H_2O\). A hydrated calcium ferrite of the form \(C_4FH_6\) is comparatively more stable. This hydrated product also does not contribute anything to the strength. The hydrates of \(C_4AF\) show a comparatively higher resistance to the attack of sulphates than the hydrates of calcium aluminate.

**Q.4** Describe the hydration of portland cement and outline the ways in which the Vicat apparatus and the Le-Chatelier apparatus can be used to assess the properties of fresh and hardened pastes.  
[15 marks : 1999]

**Solution:**

The chemical reactions that takes place between cement and water is known as **hydration of cement**. On account of hydration certain products are formed. These products are important because they have cementing or adhesive value. The quality, quantity, continuity, stability and the rate of formation of the hydration products are important.

Anhydrous cement compounds when mixed with water, react with each other to form hydrated compounds of very low solubility. The hydration of cement can be visualized in two ways. The first is “through solution” mechanism. In this the cement compounds dissolve to produce a super saturated solution from which different hydrated products get precipitated. The second possibility is that water attacks cement compounds in the “solid state” converting the compounds into hydrated products starting from the surface and proceeding to the interior of the compounds with time. It is probable that both “through solution” and “solid state” types of mechanism may occur during the course of reactions between cement and water. The former mechanisms may predominate in the early stages of hydration in view of large quantities of water being available and the latter mechanism may operate during the later stages of hydration.

The reaction of cement with water is exothermic. The reaction liberates a considerable quantity of heat. This liberation of heat is called heat of hydration. The hydration process is not an instantaneous one. The reaction is faster in the early period and continues indefinitely at a decreasing rate. Complete hydration can not be obtained under a period of one year or more unless the cement is very finely ground and reground with excess of water to expose fresh surfaces at intervals. During the course of reaction of \(C_3S\) and \(C_2S\) with water, calcium silicate hydrate \((C-S-H)\) and calcium hydroxide \(Ca(OH)_2\) are formed. Calcium silicate hydrate is the essence that determines the properties of concrete. It makes up 50-60 per cent of the volume of solids in a completely hydrated cement paste. On the other hand, calcium hydroxide is a compound which is responsible for the lack of durability. The calcium hydroxide also reacts with sulphates presents in soils or water to form calcium sulphate which reacts further with \(C_3A\) and cause deterioration of concrete which is known as sulphate attack. The only advantage of \(Ca(OH)_2\) is that, being alkaline in nature, it maintain pH value around 13 in the concrete which resists the corrosion of reinforcements.

The hydration of aluminates \((C_3A)\) results in a calcium aluminate system \(CaO-Al_2O_3-H_2O\). This compound do not contribute anything to the strength of concrete. On the other hand their presence is harmful to the
durability of the concrete particularly where the concrete is likely to be attacked by sulphates. As it hydrates fast it may contribute a little to early strength.

On hydration, \( C_3AF \) is believed to form a system of the form \( \text{CaO-Fe}_2\text{O}_3\text{H}_2\text{O} \). The hydrates of \( C_3AF \) also do not contribute anything to the strength but they show a comparatively higher resistance to the attack of sulphates than the hydrates of calcium aluminate.

**Vicat apparatus** is used for determining the **normal consistency and setting time for cement**. A known weight of cement is taken and a paste is prepared with a weighed quantity of water (24\% by weight of cement) for the first trial. The Paste is then filled in a mould and the plunger of the apparatus is brought down to touch the surface of the paste in test block and quickly released allowing it to sink into the paste by its own weight. Similarly, trials are conducted with higher and higher water cement ratios till such time the plunger penetrates for a depth of 33-35 mm from the top. That particular percentage of water is known as the percentage of water required to produce a cement paste of standard consistency.

For setting times the plunger is replaced by a needle (for initial setting time) or a circular attachment (for final setting time).

**Le Chatelier apparatus** can be used to determine **soundness in cement**. Unsoundness in cement is due to excess of lime, magnesia or sulphates. Cement is gauged with 0.78 times the water required for standard consistency in a standard manner and filled into the mould kept on a glass plate. The mould is covered with the top with another glass plate. The whole assembly is immersed in water at a temperature of 27°C - 32°C and kept there for 24 hours. Now the distance is measured between the indicator points. The mould is again submerged in water and water is heated to brought to boiling point in about 25-30 minutes and it is kept boiling for three hours. The mould is now removed from water and allowed to cool. The distance between indicator points is measured again. The difference between these two measurements represent the expansion of cement. This must not exceed 10 mm. The Le Chatelier test detects the unsoundness due to free lime only.

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**Q.5** Explain how sulphate resisting cement and rapid hardening portland cement differ from OPC and specific circumstance in which these cements would be used.

**Solution:**

**Sulphate Resisting Cement**: Ordinary portland cement is vulnerable to sulphate attack. Sulphate attack is greatly accelerated if accompanied by alternate wetting and drying which normally takes place in marine structures in the zone of tidal variations.

To prevent the sulphate attack, the use of cement with low \( C_3A \) content is found to be effective. Such a cement with low \( C_3A \) content and comparatively low \( C_4AF \) content is known as sulphate resisting cement. In other words, this cement has a higher silicate content than OPC.

It is not often possible (feasible) to reduce the \( \text{Al}_2\text{O}_3 \) content of the raw materials. So \( \text{Fe}_2\text{O}_3 \) may be added to the mix so that \( C_4AF \) content increase at the expense of \( C_3A \). Many of the physical properties of sulphate resisting cement are similar to ordinary portland cement.

**The use of sulphate resistant cement is recommended under the following conditions:**

(i) Concrete to be used in marine conditions.

(ii) Concrete to be used in foundation and basement, where soil is infested with sulphates.

(iii) Concrete used for fabrication of pipes which are likely to be marshy region or sulphate bearing soils.

(iv) Concrete to be used in the construction of sewage treatment works.
Rapid Hardening Cement: This cement is similar to OPC but with higher C₃S content and finer grinding. It gains strength more quickly than OPC, though the final strength is only slightly higher. The one day strength of this cement is equal to three day strength of OPC with the same water-cement ratio. This cement is used where a rapid strength development is required. The rapid gain of strength is accompanied by a higher rate of heat development during the hydration of cement. It is about 10 per cent costlier than OPC.

The use of rapid hardening cement is recommended in the following situations:
(i) In prefabricated concrete construction.
(ii) Where formwork is required to be removed early for reuse elsewhere.
(iii) Road repair works.
(iv) In cold weather concrete where the rapid rate of development of strength reduces the vulnerability of concrete to the frost damage.

Q.6 Name the principal compounds in portland cement, their relative rates of reaction with water and their approximate proportions. [10 marks : 1999]

Or
List of the products of hydration and their influence on the properties of cement. [10 marks : 2001]

Or
Which are the four important compounds formed during the setting action of cement (four principal minerals in ordinary portland cement)? Mention their relative proportions expressed as percentages and also functions of these compounds. [10 marks : 2007]

Solution:

The principal compounds in portland cement are known as Bogue’s compounds. They are as follows:
(i) Tricalcium silicate (C₃S) or Alite
(ii) Dicalcium silicate (C₂S) or Belite
(iii) Tricalcium aluminate (C₃A) or Celite
(iv) Tetracalcium aluminoferrite (C₄AF) or Felite

The reaction of water with C₃A is very fast and in the process flash setting i.e. stiffening without strength development can occur because the C-A-H phase prevents the hydration of C₃S and C₂S. To prevent this flash set, gypsum is added at the time of grinding the cement clinker. The hydrated aluminates do not contribute anything to the strength of concrete. On the other hand, their presence is harmful to the durability of concrete particularly, where the concrete is likely to be attacked by sulphates. As it hydrates fast, it may contribute a little to the early strength.

\[ \text{C}_3\text{A} + \text{H}_2\text{O} \rightarrow \text{C}_3\text{AH}_6 \]

On hydration, C₄AF is believed to form a system of the form C-F-H. A hydrated calcium ferrite of this form is comparatively more stable. This hydrated product also does not contribute anything to the strength. However, the hydrates of C₄AF show a comparatively higher resistance to the attack of sulphates than the hydrates of calcium aluminate.

\[ \text{C}_3\text{AF} + \text{H}_2\text{O} \rightarrow \text{C}_3\text{AH}_6 + \text{CFH} \]

When C₃S and C₂S reacts with water, calcium silicate hydrate (C-S-H) and calcium hydroxide are formed. Calcium silicate hydrates are the most important products. It is the essence that determines the good properties of concrete.
\[ \text{C}_3\text{S} + \text{H}_2\text{O} \rightarrow \text{C}_3\text{S}_2\text{H}_3 + 3\text{Ca(OH)}_2 \]
\[ \text{C}_2\text{S} + \text{H}_2\text{O} \rightarrow \text{C}_3\text{S}_2\text{H}_3 + \text{Ca(OH)}_2 \]

It can be seen that \text{C}_3\text{S} produces a comparatively lesser quantity of calcium silicate hydrates and more quantity of \text{Ca(OH)}_2 than that formed in the hydration of \text{C}_2\text{S}. \text{C}_2\text{S} hydrates rather slowly than \text{C}_3\text{S}.

The relative rates of hydration of Bogue’s compounds can be shown with the help of a graph as shown.

The approximate proportions of Bogue’s compound are:

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<td>30-50</td>
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<tr>
<td>\text{C}_2\text{S}</td>
<td>20-45</td>
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<tr>
<td>\text{C}_2\text{A}</td>
<td>8-12</td>
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<td>\text{C}_4\text{AF}</td>
<td>6-10</td>
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Q.7 Explain how do the portland pozzolana cement and super sulphate cement differ from OPC. Under what specific circumstances these cements would be used?

[10 marks : 2000]

Solution:

**Portland Pozzolana Cement:** It is manufactured either by grinding together portland cement clinker and pozzolana with the addition of gypsum or calcium sulphate or by intimately and uniformly blending portland cement and fine pozzolana. PPC produces less heat of hydration and offers greater resistance to the attack of impurities in water than normal portland cement. However, it is important to appreciate that the addition of pozzolana does not contribute to strength at early ages. Strengths similar to those of OPC can be expected in general only at later ages. In PPC costlier clinker is replaced by cheaper pozzolanic material making it more economical than OPC. PPC consumes calcium hydroxide and does not produce calcium hydroxide as much as that of OPC. It generates lower heat of hydration than OPC and that too at a lower rate. As the fly ash is finer and of lower density, the bulk volume of 50 kg bag is slightly more than OPC. The long term strength of PPC beyond a couple of months is higher than OPC if enough moisture is available for continued pozzolanic action.

The PPC is particularly suitable under the following conditions:

(i) For hydraulic structures.
(ii) For mass concrete structure like dam, bridge piers and thick foundations.
(iii) For marine structures.
(iv) For sewage disposal works, sewers etc.

**Super Sulphate Cement:** It is manufactured by grinding together a mixture of 80-85 per cent granulated slag, 10-15 per cent hard gypsum and about 5 per cent portland cement clinker. It is rather more sensitive to deterioration during storage than portland cement. Super sulphated cement has a low heat of hydration than OPC. This cement has high sulphate resistance. This cement like high alumina cement, combines with more water on hydration than OPC.
This cement is recommended for the following:

(i) Due to high sulphate resistance, it is particularly used in foundations, where chemically aggressive conditions exist.
(ii) It can be used in marine works.
(iii) It is used in fabrication of reinforced concrete pipes which are likely to be buried in sulphate bearing soils.

Q.8 Explain the difference between various grades of OPC. [10 marks : 2002]

Solution:
The commonly used portland cement in India is branded as 33-grade (IS:269-1989), 43-grade (IS:8112-1989) and 53 grade (IS:12269-1987) having 28 days mean compressive strengths exceeding 33 MPa, 43 MPa and 53 MPa respectively. All the three grades are produced from same materials. The higher strengths are achieved by increasing C_3S content and also by finely grinding the clinker. The fineness of 53-grade OPC obtained by Blaine’s air permeability test is specified to be of the order of 350000 mm²/g. The initial and final setting times are same for all the three grades. The 33-grade cement has virtually disappeared and has been displaced by high strength 43-grade cement. The minimum compressive strengths of the 43-grade cement are 23 MPa and 33 MPa at the end of 3 and 7 days respectively. At higher water cement ratios, the concrete produced with high strength cement has about 80% higher strength and at lower water cement ratios, it has 40% higher strength than that of concrete using 33-grade cement. Greater fineness of 43 and 53 grade cements increase workability due to reduction of friction between aggregates. Moreover, due to shorter setting time and faster development of strength, the stripping time is shorter. Although cements of grade 43 and 53 are desirable for economical design of high grade concretes but they can also be used for lower grade concretes.

Q.9 Explain pozzolanic action. [10 marks : 2003]

Solution:
A pozzolana is a finely ground siliceous material which as such does not possess cementitious property in itself but reacts in the presence of water with calcium hydroxide at normal temperature to form compounds of low solubility having cementitious properties. The action is known as pozzolanic action.

The reaction can be shown as

$$\text{Pozzolana} + \text{Ca(OH)}_2 + \text{H}_2\text{O} \rightarrow \text{C-S-H (gel)}$$

This reaction is called pozzolanic reaction. The characteristic feature of pozzolanic reaction is initially slow, with the result that heat of hydration and strength development will accordingly be slow. The reaction involves the consumption of Ca(OH)₂ and not production of Ca(OH)₂. It may be noted that on hydration of C₃S and C₂S present in cement, Ca(OH)₂ is formed as one of the products of hydration. This compound has no cementitious value and it is soluble in water and may be leached out by the percolating water. It is pointed out that Ca(OH)₂, otherwise, a water soluble material is converted into insoluble cementitious material by reaction of pozzolanic materials.

The reduction of Ca(OH)₂ also improves the durability of cement paste by making the paste dense and impervious. Pozzolanic materials can be natural or artificial. Clay and shales, opaline cherts, diatomaceous earth and volcanic tuffs are natural pozzolanic materials. Fly ash, blast furnace slag, silica fume, rice husk ash are artificial pozzolanic materials.

The pozzolanic action also reduce the expansion caused by the alkali-aggregate reaction in concrete. Excessive expansion causes pattern cracking of concrete. This expansion can usually be controlled by using of pozzolana ranging from 2 to 35% by mass of cement depending upon the type of aggregate and alkali content of cement.
Revised Syllabus of ESE: Elastic constants, Stress, plane stress, Strains, plane strain, Mohr’s circle of stress and strain, Elastic theories of failure, Principal Stresses, Bending, Shear and Torsion.

1. Simple Stress-Strain & Elastic Constants

Q.1 A vertical tapered rod of length \(L\) has its diameter varying linearly from ‘\(d\)’ at lower end to \(D\) at the upper end which has fixed support. Young’s modulus of the material is \(E\). Show that the elongation of the rod at its lower end when subjected to a longitudinal force \(F\) is given by

\[ \delta = \frac{4FL}{\pi dDE} \]

[10 marks : 1996]

Solution:

Diameter of lower end = \(d\), diameter of upper end = \(D\), Length of tapering bar = \(L\)

Now, the diameter of the tapered bar at a distance ‘\(x\)’ from the smaller end may be given as

\[ D_x = d + \left( \frac{D - d}{L} \right) x \Rightarrow D_x = d + kx \text{ where } k = \frac{D - d}{L} \]

Let the width of strip at distance \(x\) from lower end is \(dx\). Then increase in length of the elemental strip \(x = \frac{Fdx}{A_x E}\) using \(\Delta = \frac{PL}{AE}\)

where, \(A_x\) is area of cross-section of elemental strip and \(E\) is modulus of elasticity.

\[ \Rightarrow \:\:\text{Total increase in length,} \quad \delta = \int_0^L \frac{Fdx}{A_x E} = \int_0^L \left( \frac{F}{\pi} \right) D_x^2 E \]

\[ = \int_0^L \frac{4Fdx}{\pi ED_x^2} = \frac{4F}{\pi E} \int_0^L (d + kx)^{-2} dx \]

\[ = \frac{4F}{\pi E} \left[ \frac{(d + kx)^{-1}}{-(-2+1)} \right]_0^L = -\frac{4F}{\pi E} \left[ \frac{1}{k(d + kx)} \right]_0^L = -\frac{4F}{\pi E} \left( \frac{-L}{dD} \right) \]

\[ \Rightarrow \quad \delta = \frac{4FL}{\pi E dD} \quad \text{(Ans.)} \]
Q.2  A cylindrical piece of steel of 80 mm diameter and 120 mm long is subjected to an axial compressive force of 50,000 kg. Calculate the change in volume of the piece if bulk modulus $K = 1.7 \times 10^8$ kg/cm$^2$ and Poisson’s ratio $\mu = 0.3$. 

Solution:

Bulk modulus $(K) = 1.7 \times 10^8$ kg/cm$^2$; Poisson’s ratio $(\mu) = 0.3$; Diameter of cylindrical piece $(d) = 80$ mm = 8 cm; Length of cylindrical piece $(L) = 120$ mm = 12 cm; Compressive force $(P) = 50,000$ kg

We know that,

$$E = 3K(1-2\mu)$$

$$E = 3 \times 1.7 \times 10^8 (1 - 2 \times 0.3)$$

$$E = 2.04 \times 10^8$$ kg/cm$^2$

and

$$A = \frac{\pi d^2}{4} = \frac{\pi}{4} \times 8^2 = 50.26 \text{ cm}^2$$

$$V = AL = 50.26 \times 12 = 603.12 \text{ cm}^3$$

stress,

$$\sigma = \frac{P}{A} = \frac{50,000}{50.26} = 994.83 \text{ kg/cm}^2$$

we know,

$$\varepsilon_x = \frac{\Delta l}{l} = -\frac{\sigma}{E} = -\frac{994.83}{2.04 \times 10^8} = -4.876 \times 10^{-4}$$

$$\varepsilon_y = \frac{\Delta d}{d} = -\frac{\mu (-\Delta l)}{E} = \frac{0.3 \times 994.83}{2.04 \times 10^8} = 1.463 \times 10^{-4}$$

we know that,

$$\varepsilon_V = \frac{\Delta V}{V} = \frac{\Delta l}{l} + \frac{2\Delta d}{d} = \varepsilon_x + 2\varepsilon_y$$

$$\frac{\Delta V}{V} = -4.876 \times 10^{-4} + 2 \times 1.463 \times 10^{-4}$$

$$\Delta V = (-4.876 + 2.926) \times 10^{-4} \times V$$

$$\Delta V = -1.950 \times 10^{-4} \times 603.12 = -0.1176 \text{ cm}^3$$

Negative sign indicate that there is a decrease in volume due to the axial compressive load.

Q.3  Draw the diagram of normal forces, stresses and displacements along the length of the stepped bar ABC shown in figure below. Cross-sectional area of $AB = 100$ mm$^2$ and of $BC = 200$ mm$^2$, modulus of elasticity $= 200$ kN/mm$^2$.

[10 marks : 1998]
Solution:

Area of cross-section of $AB (A_{AB}) = 100 \text{ mm}^2$; Area of cross-section of $BC (A_{BC}) = 200 \text{ mm}^2$; Modulus of elasticity ($E$) = 200 kN/mm$^2$; Axial tensile load ($P$) = 50 kN; Length of portion $AB (L_{AB}) = 1 \text{ m}$; Length of portion $BC (L_{BC}) = 1 \text{ m}$

Let $R_A$ be the reaction at fixed end $A$

By drawing the FBD of the given figure we came to know that to maintain equilibrium

$$R_A - 50 = 0$$

$$R_A = 50 \text{ kN}$$

**Portion AB:**

$$R_A = P_{AB} = 50 \text{ kN} \quad \text{(Tensile)}$$

$$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{50 \times 10^3}{100} = 500 \text{ N/mm}^2 \quad \text{(Tensile)}$$

$$\Delta_{AB} = \frac{P_{AB} \cdot L_{AB}}{A_{AB} \cdot E} = \frac{50 \times 10^3 \times 1000}{100 \times 200 \times 10^3} = 2.5 \text{ mm} \quad \text{(Elongation)}$$

**Portion BC:**

$$P_{BC} = 50 \text{ kN} \quad \text{(Tensile)}$$

$$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{50 \times 10^3}{200} = 250 \text{ N/mm}^2 \quad \text{(Tensile)}$$

$$\Delta_{BC} = \frac{P_{BC} \cdot L_{BC}}{A_{BC} \cdot E} = \frac{50 \times 10^3 \times 1000}{200 \times 200 \times 10^3} = 1.25 \text{ mm} \quad \text{(Elongation)}$$

Assuming:

- Tensile force = Positive
- Tensile elongation = Positive

(i) Normal force diagram

(ii) Normal stress diagram

(iii) Elongation/Displacement diagram
3. Arches and Suspended Cables

Q.12 A symmetrical triangular shaped arch ACB is hinged at supports A and B. The two members of the arch are of the same cross-section. A vertical load ‘W’ is placed as shown in the figure. Joint C is rigid. Determine the components of reactions and draw bending moment diagram indicating maximum positive and negative magnitudes.

[20 marks : 2007]

Solution:

The figure below shows the triangular arch carrying the load ‘W’ at a distance \( \frac{L}{3} \) from end A. Let the vertical reactions at A and B be \( V_1 \) and \( V_2 \) respectively. Let \( H \) be the horizontal thrust at each support.

From the diagram above, we have, \( V_1 + V_2 = W \)

Taking moment about hinge A, we get

\[ V_2 \times L - W \times \frac{L}{3} = 0 \]

\[ \Rightarrow \quad V_2 = \frac{W}{3} \]

\[ \therefore \quad V_1 = W - \frac{W}{3} = \frac{2W}{3} \]

Now taking the same arch, but placing the load \( W \) at a distance \( \frac{L}{3} \) from B. In this case the vertical reactions will interchange i.e. the reaction at A and B will be respectively \( V_2 \) and \( V_1 \).

But the horizontal thrust will remain same.

If both the above arrangement are clubbed together, we get

For this load system, the horizontal thrust will be \( 2H \) at each support. The vertical reaction at each support will be \( V_1 + V_2 \)

But \( V_1 + V_2 = \frac{2W}{3} + \frac{W}{3} = W \)

At any section \( x \) from \( A \), \( \left( x < \frac{L}{3} \right) \) the beam moment at the section = \( M = Wx \)

But at any section \( x \) from \( A \), \( \left( \frac{L}{3} < x < \frac{L}{2} \right) \) the beam moment at the section

\[ = M = Wx - W \left( x - \frac{L}{3} \right) = Wx - Wx + \frac{WL}{3} = \frac{WL}{3} \text{ (constant)} \]

If \( x \) and \( y \) be the coordinates of a point on the arch, then we have

\[ \Rightarrow \quad \frac{y}{x} = \frac{L/4}{L/2} = \frac{L \times 2}{L \times 4} \]
\[ y = \frac{x}{2} \]

Now we know that the **horizontal thrust** is given by

\[
2H = \left[ \frac{\int y^2 \, dx}{\int y^2 \, dx} \right] = \frac{2 \left[ \int_{0}^{L/3} x^3 \, dx \times \frac{x}{2} \, dx + \int_{L/3}^{L/2} \frac{WL}{3} \cdot \frac{x}{2} \, dx \right]}{2 \int_{0}^{L/2} x^2 \, dx}
\]

\[
\Rightarrow 2H = \frac{\frac{W}{2} \int_{0}^{L/3} x^3 \, dx + \frac{WL}{6} \int_{L/3}^{L/2} x \, dx}{\frac{1}{4} \int_{0}^{L/2} x^2 \, dx} = \frac{\frac{W}{2} \left[ \frac{x^3}{3} \right]_{0}^{L/3} + \frac{WL}{6} \left[ \frac{x^2}{2} \right]_{L/3}^{L/2}}{\frac{1}{4} \left[ \frac{x^3}{3} \right]_{0}^{L/2}}
\]

\[
\Rightarrow 2H = \frac{\frac{W}{6} \times \frac{L^3}{27} + \frac{WL}{12} \times \left[ \frac{L^2}{4} - \frac{L^2}{9} \right]}{\frac{1}{12} \times \frac{L^3}{8}} = \frac{\frac{WL^3}{96} + \frac{5L^2}{36}}{\frac{L^3}{96}} = \frac{WL^3 + \frac{5WL^3}{162}}{L^3 / 96} = \frac{WL^3 + \frac{5WL^3}{432}}{L^3 / 96}
\]

\[
\Rightarrow 2H = \frac{8WL^3 + 15WL^3}{L^3 / 96} = \frac{23WL^3 \times 96}{L^3 \times 1296}
\]

\[
\Rightarrow H = \frac{23 \times 96 \times W}{1296 \times 2} = \frac{23}{27} W
\]

Assuming the inner face of the arch as reference face.

**Portion AD**

\[ M_s(x \text{ from } A) = V_s \times x - Hy \left( x < \frac{L}{3} \right) = \frac{2W}{3} \times \frac{L}{3} - \frac{23}{27} W \times \frac{x}{2} \]

\[
\Rightarrow M_D = \frac{2WL}{9} - \frac{23}{27} W \times \frac{L}{6} = \frac{13WL}{162}
\]

**Portion DC**

\[ M_s(x \text{ from } A) = V_s \times \frac{x}{3} - Hy - W \left( \frac{x - \frac{L}{3}}{3} < x < \frac{L}{2} \right) \]

\[
= \frac{2Wx}{3} - \frac{23}{27} W \times \frac{x}{2} - Wx + \frac{WL}{3}
\]

\[
= \frac{2Wx}{3} - \frac{23Wx}{54} - Wx + \frac{WL}{3}
\]

\[
= \frac{36Wx - 23Wx - 54Wx + 18WL}{54}
\]

\[
M_C = \frac{18WL - 4Wx \times \frac{L}{2}}{54} = \frac{5WL}{108}
\]
Portion BC

\[ M_c (x \text{ from B}) = V_2 x - Hy \left[ x < \frac{L}{2} \right] \]

\[ = \frac{W}{3} x - \frac{23Wx}{27 \times 2} \]

\[ = \frac{18Wx}{54} - \frac{23Wx}{54} = -\frac{5Wx}{54} \]

\[ M_c = -\frac{5W}{54} \times \frac{L}{2} = -\frac{5WL}{108} \]

Q.13 A cable suspends across a gap of 250 m and carries a UDL of 10 kN/m horizontally. Calculate the maximum tension if the maximum sag is 1/25. Also compute sag at 50 m from one end.

[10 marks : 2014]

Solution:

Since load is uniform along the horizontal span and supports are at the same level

\[ \therefore \quad \frac{d^2 y}{dx^2} = \frac{w}{H} \quad \ldots \text{(i)} \]

Integrating (i), we get

\[ \frac{dy}{dx} = \frac{w}{H} x + C_1 \quad \ldots \text{(ii)} \]

Again integrating (ii), we get

\[ y = \frac{w}{H} \times \frac{x^2}{2} + C_1 x + C_2 \]

Using boundary condition,
With origin at C, at \( x = 0 \), \( y = 0 \)
Hence from eq. (iii) \( C_2 = 0 \)
From eq. (ii) \( C_1 = 0 \)

at \( x = 0; \quad \frac{dy}{dx} = 0 \) (due to symmetry)

The equation of cable becomes,

\[ y = \frac{w}{2H} x^2 \quad \ldots \text{(iii)} \]

at \( x = \frac{L}{2}, y = h \)

\[ \therefore \quad h = \frac{w}{2H} \left( \frac{L}{2} \right)^2 \]

or

\[ H = \frac{wl^2}{8h} \]

Substituting value of \( H \) in eq. (iii), we get

\[ y = \frac{8hH}{l^2} \times \frac{1}{2H} x^2 = \frac{4hx^2}{l^2} \quad \ldots \text{(iv)} \]

When origin is shifted to A, in order to find the shape of the cable, moment about a point \( P \) with coordinate \( x \) and \( y \) as shown in figure will be
\[ M_p = V_A(x) - \frac{wx^2}{2} - Hy = 0 \]

\[
\Rightarrow \quad \frac{wL}{2}x - \frac{wx^2}{2} = \frac{wt^2}{8h}y
\]

On rearranging, we get
\[ y = \frac{4h}{L^2}x(L - x) \]

given,
\[ h = \frac{L}{25} = 10 \text{ m} \]
\[ L = 250 \text{ m} \]
\[ w = 10 \text{ kN/m} \]

(i) Maximum tension

By symmetry, \[ R_A = R_B = \frac{wL}{2} = \frac{250 \times 10}{2} = 1250 \text{ kN} \]

Horizontal thrust,
\[ H_A = H_B = H = \frac{wL^2}{8h} \]

\[ H = \frac{10 \times (250)^2}{8 \times 10} = 7812.5 \text{ kN} \]

Maximum tension occurs at higher support, here both supports are at same level

\[ T_{\text{max}} = \sqrt{R_A^2 + H_A^2} = \sqrt{(1250)^2 + (7812.5)^2} \]

\[ = 7911.86 \text{ kN} \]

(ii) Sag at 50 m from one end

Equation of cable,
\[ y = \frac{4h}{L^2}x(L - x) \]

at \( x = 50 \text{ m} \)
\[ y = \frac{4 \times 10}{(250)^2} \times 50 \times (250 - 50) \]

\[ \Rightarrow \quad y = 6.4 \text{ m} \]

Q.14 A 3-hinged parabolic arch of 16 m span has its abutments \( A \) and \( B \) at a depth of 4 m and 8 m respectively below the crown \( C \). It is loaded as shown in figure. Determine the horizontal thrust and the vertical reactions at the supports.
Solution:

Let $R_A$ and $R_B$ be the vertical reactions at the supports $A$ and $B$. Let $H$ be the equal horizontal reaction at both supports $A$ and $B$

$\Sigma F_y = 0; \quad R_A + R_B = 30 \times 16 + 300 = 780 \quad \ldots(i)$

Since there is a hinge at crown. Hence $BM$ at crown will be zero.
So taking moment about $C$ from left side of $C$, $M_C = 0$

$$R_A \times 6 - H \times 4 - \frac{w \times 6^2}{2} = 0$$

$$6R_A = 4H + \frac{30 \times 6^2}{2}$$

$$6R_A = 4H + 540 \quad \ldots(ii)$$

Taking moment about $C$ from right side of $C$, $M_C = 0$

$$-R_B \times 10 + H \times 8 + \frac{30 \times 10^2}{2} + 300 \times 3 = 0$$

$$-10R_B + 8H + 2400 = 0 \quad \ldots(iii)$$

From (i) and (ii), we get

$$6(780 - R_B) = 4H + 540$$

$$4860 - 6R_B = 4H + 540$$

$$4H + 6R_B = 4140 \quad \ldots(iv)$$

On solving eq. (iii) and (iv), we get

$$R_B = 485.45 \text{ kN}$$

$$H = 306.81 \text{ kN}$$

From eq. (i),

$$R_A = 780 - R_B = 294.55 \text{ kN}$$

Thus,

$$R_A = 294.55 \text{ kN}$$

$$R_B = 485.45 \text{ kN}$$

$$H = 306.81 \text{ kN}$$

Check:

$$\Sigma F_y = R_A + T_B - 30 \times 16 - 300$$

$$\Sigma F_y = 294.55 + 485.45 - 480 - 300$$

$$\Sigma F_y = 0 \quad \text{(OK)}$$

Q.15 A symmetrical three hinged parabolic arch has 40 m span and 5 m rise. A vertical downward load of 30 kN and a horizontal load of 20 kN (acting in the right hand side direction) act at one quarter span form left hand support. Determine reactions at the supports.

[5 marks : 2015]

Solution:

$$R_A + R_B = 30 \text{ kN} \quad \ldots(i)$$

$$H_A + 20 = H_B \quad \ldots(ii)$$

Equation of arch is,

$$y = \frac{4y_c}{L^2} (Lx - x^2)$$

$$\Rightarrow \quad y = \frac{4(5)}{40^2} (40x - x^2)$$

$$\Rightarrow \quad y = \frac{1}{80} (40x - x^2)$$
At \( P \),
\[ x = 10 \text{ m}, \]
\[ \therefore \quad y = \frac{1}{80}(40 \times 10 - 10^2) = 3.75 \text{ m} \]

Taking moments about \( B \),
\[ R_A (40) = 30 (30) - 20 (3.75) \]
\[ \Rightarrow \quad R_A = 20.625 \text{ kN} \quad (\uparrow) \]
\[ \therefore \quad R_B = 30 - R_A \]
\[ \Rightarrow \quad R_B = 9.375 \text{ kN} \quad (\uparrow) \]

Taking moments about \( C \) of the right hand portion of arch,
\[ R_B (20) = H_B (5) \]
\[ \Rightarrow \quad H_B = \frac{9.375 \times 20}{5} = 37.5 \text{ kN} \]
\[ \therefore \quad H_A = H_B - 20 = 17.5 \text{ kN} \]

**Q.16** A two hinged symmetrical parabolic arch of span 30 m and central rise 6 m carries a point of load of 40 kN at a distance of 5 m from the left support. Find the horizontal thrust at each support. Also find the maximum bending moment.

**Solution:**

Horizontal thrust \((H)\) = \[ \frac{\int M'y dx}{\int y^2 dx} \] where \( M' \) = Beam moment

Apply a fictitious load of 40 kN at 5 m from right support \( B \) to take advantage of symmetry.
\[ \therefore \quad \text{Horizontal thrust will become twice i.e. } 2H. \]
\[ \therefore \quad 2H = \frac{\int M'y dx}{\int y^2 dx} \]

and
\[ R_A = W = 40 \text{ kN due to symmetry} \]

**Equation of parabolic arch is**
\[ y = \frac{4y_c}{L^2}(Lx - x^2) = \frac{4 \times 6}{30^2}(30x - x^2) = \frac{2}{75}(30x - x^2) \]

Denominator = \[ \int y^2 dx = \int_0^{30} \left( \frac{2}{75} \right)^2 (30x - x^2)^2 dx \]

\[ = \left( \frac{2}{75} \right)^2 \int_0^{30} (900x^2 + x^4 - 60x^3) dx \]
\[ = \left( \frac{2}{75} \right)^2 \left[ 900 \frac{x^3}{3} + \frac{x^5}{5} - 60 \frac{x^4}{4} \right]_0^{30} \]
\[ = \left( \frac{2}{75} \right)^2 \left[ 300(30^3 - 0^3) + \frac{1}{5}(30^5 - 0^5) - 15(30^4 - 0^4) \right] = 576 \]

**Short tip: For parabolic arch**
\[ y = \frac{4y_c}{L^2}(Lx - x^2),\quad \int_0^{L} y^2 dx = \frac{8}{15} y_c^2L = 576 \]
Beam moment for \(0 < x < 5\)

\[ M' = R_A x \]

Beam moment for \(5 < x < 15\)

\[ M' = R_A x - W(x - 5) \]
\[ = Wx - W(x - 5) \]
\[ = Wx - Wx + W(5) \]
\[ M' = W(5) \]

Numerator

\[
= \left[ M' y dx \right] = 2 \left[ \int_{0}^{5} R_A \cdot xy dx + \int_{5}^{15} W(5) y dx \right]
\]
\[
= 2 \left[ \int_{0}^{5} W \cdot x \cdot \frac{4y_c}{L^2} (Lx - x^2) dx + \int_{5}^{15} 5W \cdot \frac{4y_c}{L^2} (Lx - x^2) dx \right]
\]
\[
= 2 \left[ \frac{4Wy_c}{L^2} \left( \frac{L}{3} (5^3) - \frac{1}{4} (5^4) \right) + \frac{20Wy_c}{L^2} \left( \frac{L}{2} (15^2 - 5^2) - \frac{1}{3} (15^3 - 5^3) \right) \right]
\]
\[
= 2 \left[ \frac{4 \times 40 \times 6}{30^2} \left( \frac{30 \times 5^3}{3} - \frac{5^4}{4} \right) + \frac{20 \times 40 \times 6}{30^2} \left( \frac{30}{2} (200) - \frac{1}{3} (3250) \right) \right]
\]
\[
= 2[1166.67 + 10222.22] = 22777.78
\]

\[ 2H = \int_{y^2 dx}^{M' y dx} = \frac{22777.78}{576} = 39.545 \]

\[ H = 19.772 \text{ kN} \]

\[ \therefore \text{ Horizontal thrust at supports } = 19.772 \text{ kN} \]

**Short tip for check:**

\[ H = \frac{5W}{8y_c L^3} (L - a)(L^2 + aL - a^2) \]

\[ = \frac{5 \times 40 \times 5}{8 \times 6 \times 30^3} (30 - 5)(30^2 + 5 \times 30 - 5^2) = 19.772 \text{ kN} \]

Taking BM about \(A\)

\[ R_B = (30) - 40 \times 5 = 0 \]
\[ R_B = 6.67 \text{ kN} \]

\[ M_{ve} = R_B \cdot x - H_y \] where \(x\) is measured from \(B\)

\[ = 6.67x - 19.772 \times \frac{2}{75} (30x - x^2) \]

For \(M_{ve}\) to be maximum, \(\frac{dM_{ve}}{dx} = 0\)

\[ \Rightarrow 6.67 - 19.772 \times \frac{2}{75} (30 - 2x) = 0 \]

\[ \Rightarrow 12.65 = 30 - 2x \]
\[ \Rightarrow x = 8.675 \text{ m from } B \]

\[ \therefore M_{ve \text{ max}} = 6.67(8.675) - 19.772 \times \frac{2}{75} (30 \times 8.675 - 8.675^2) = -39.68 \text{ Nm} \]
Q.10 (i) List out five merits and three demerits of welded joints.

(ii) Determine the size of the fillet weld required to join a bracket plate with the flange of a column section shown in the figure below. Permissible stress in weld = 108 MPa.

\[4 + 4 = 8 \text{ marks : 2011}\]

Solution:

(i) **Advantages of welded joints:**
   (i) As no holes are required for welding, the structural members are more effective in taking load.
   (ii) The overall weight of structural steel required is reduced by the use of welded joints.
   (iii) Welded joints are often economical as less labour and material are required for a joint.
   (iv) The speed of fabrication is higher with the welding process.
   (v) Any shape of joint can be made with ease.
   (vi) Complete rigid joints can be provided with the welding process.

(ii) **Disadvantages of welded joints:**
   (i) Skilled labour and electricity are required for welding.
   (ii) Internal stresses and warping are produced due to uneven heating and cooling.
   (iii) Welded joints are more brittle and therefore their fatigue strength is less than the members joined.
   (iv) Defects like internal air pockets, slag inclusion and incomplete penetration are difficult to detect.

Let, thickness of weld throat = \( t \) mm

Vertical shear stress at weld,

\[
p_s = \frac{W}{2 \times d \times t} = \frac{60 \times 10^3}{2 \times 250 \times t} = \frac{60 \times 10^3}{500 \times t} = \frac{120}{t} \text{ MPa}
\]

Horizontal shear stress due to bending at extreme fibre,

\[
p_b = \frac{W \times e \times d}{\left( \frac{t \times d^2}{12} \right) \times 2} = \frac{6 \times W \times e}{2 \times t \times d^2}
\]

\[
= \frac{6 \times 60 \times 10^3 \times 150}{2 \times t \times 250^2} = \frac{432}{t} \text{ MPa}
\]

Resultant stress at extreme fibre,

\[
p_r = \sqrt{p_s^2 + p_b^2} = \sqrt{\left( \frac{120}{t} \right)^2 + \left( \frac{432}{t} \right)^2}
\]

\[
= \frac{448.357}{t} \text{ MPa} = \frac{448.4}{t} \text{ MPa}
\]

Since,

\[
p_r = \text{ Permissible stress}
\]

\[
\therefore \quad \frac{448.4}{t} = 108
\]

or

\[
t = 4.15 \text{ mm}
\]

Therefore, size of weld

\[
s = \frac{t}{0.7} = 5.9 \text{ mm} = 6 \text{ mm (say)}
\]
Q.11 A plate is connected to the flange of an ISMB as shown. The factored load is 100 kN. Find the size of the weld. Assume shop weld and ultimate strength of weld as 410 MPa.

Solution:
Let \( \bar{x} \) = distance of centroid of weld group from left edge.

\[ s = \text{size of weld} \]
\[ t = \text{effective throat thickness of weld} \]

\[ \therefore \quad t = 0.7 \, s \]
\[ \bar{x} = \frac{(2 \times 200 \times t)100 + (300 \times t)0}{(2 \times 200 \times t) + (300 \times t)} \]
\[ \bar{x} = \frac{(400 \times 100)100 + (700 \times 100)t}{700t} = 57.143 \text{ mm from left edge} \]
\[ \tan \theta_1 = \frac{150}{200 - \bar{x}} = \frac{150}{200 - 57.143} = 1.05 \]
\[ \theta_1 = 46.397^\circ \]

\[ I_x = \frac{t \times 300^3}{12} + 2 \left( \frac{200^3}{12} + 200t \times (150)^2 \right) \]
\[ = (2.25 \times 10^6) + 33.33 \, \text{ft}^3 + (9t \times 10^6) \approx 11.25 \times 10^6 \, \text{mm}^4 \text{[Neglecting 33.33 \, \text{ft}]} \]
\[ I_{xy} = \left\{ \frac{300t^3}{12} + 300t(57.143)^2 \right\} + 2 \left\{ \frac{t \times 200^3}{12} + 200t(100 - 57.143)^2 \right\} \]
\[
= \left\{ 25t^3 + 0.9796t \times 10^6 \right\} + \left\{ 1.333t \times 10^6 + 0.7347t \times 10^6 \right\}
\]
\[
= 3.0473 \times 10^6 \text{ mm}^4
\]

\[\text{Neglecting 25 } \ell^3\]

\[\therefore \text{ Polar MOI } = J = I_{xx} + I_{yy} = (11.25t + 3.0473t) \times 10^6 = 14.2973 \times 10^6 \text{ mm}^4\]

**Calculating 'e' of load P**

![Diagram of triangle ABC with labeled sides and angles]

\[
\frac{242.86}{x_1} = \cos 30^\circ
\]

\[
\Rightarrow x_1 = 280.43 \text{ mm}
\]

\[
\Rightarrow \frac{y}{242.86} = \tan 30^\circ
\]

\[
\Rightarrow y = 140.215 \text{ mm}
\]

\[
(150 - y) = 9.78 \text{ mm}
\]

In \(\triangle ABC\),

\[
\frac{x_2}{9.78} = \sin 30^\circ
\]

\[
x_2 = 4.89 \text{ mm}
\]

\[
e = x_1 + x_2 = 280.43 + 4.89 = 285.32 \text{ mm}
\]

Direct shear stress, \(\tau_{\text{vfr}} = \frac{142.857}{t} \text{ N/mm}^2\)

\[
\theta = 46.397^\circ - 30^\circ = 16.397^\circ
\]

Where \(\theta = 16.397^\circ\) is angle between direct shear stress and torsional shear stress.

Shear stress due to twisting/torsion = \(\frac{Pe}{J}\)

\[
\tau_{\text{vfr}} = \frac{(100 \times 10^3)(285.32)(207.143)}{14.2973t \times 10^6} = \frac{413.379}{t}
\]

Resultant shear stress \(= \sqrt{\tau_{\text{vfr}}^2 + \tau_{\text{vfr}}^2 + 2\tau_{\text{vfr}} \cdot \tau_{\text{vfr}} \cdot \cos \theta}\)

\[
f_R = \frac{551.9}{t} \text{ N/mm}^2
\]

Ultimate strength of weld = \(f_u = 410 \text{ N/mm}^2\)

\(f_{\text{wd}}\) Design shear strength of weld = \(\left(\frac{f_u}{\sqrt{3}}\right) \frac{1}{\gamma_{mW}}\)

\[
\gamma_{mW} = 1.25
\]

\[
f_{\text{wd}} = 189.37 \text{ N/mm}^2\]
Hence,
\[ f_R \leq f_{wd} \]
\[ \Rightarrow \frac{551.9}{t} \leq 189.37 \]
\[ t \geq 2.914 \text{ mm} \]
\[ s \geq 2.914 \times \sqrt{2} = 4.12 \text{ mm} = 5 \text{ mm} \]

Q.12 (i) Name 10 commonly used hot rolled structural section for steel construction.
(ii) Give 6 reasons for failure of riveted joints.

[2 + 2 = 4 marks : 2013]

Solution:
(i) The largest category of standard shapes of steel sections that is available for steel construction are those which are produced by hot rolling.

Commonly used hot rolled sections are:

<table>
<thead>
<tr>
<th>• Rolled steel I-section</th>
<th>• Rolled steel channel sections</th>
<th>• Rolled steel T-section</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="I-section" /></td>
<td><img src="image" alt="Channel section" /></td>
<td><img src="image" alt="T-section" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>• Rolled steel angle sections</th>
<th>• Rolled steel tube sections</th>
<th>• Rolled steel bars</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Angle section" /></td>
<td><img src="image" alt="Tube section" /></td>
<td><img src="image" alt="Bars" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>• Rolled steel flats</th>
<th>• Rolled steel plates</th>
<th>• Rolled steel sheets</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Flats" /></td>
<td><img src="image" alt="Plates" /></td>
<td><img src="image" alt="Sheets" /></td>
</tr>
</tbody>
</table>

(ii) Failure of riveted joints.
The six (06) modes of failure of riveted joints are as follows:

1. **Shear failure of rivets:** When shear stress in rivet exceeds the permissible shear stress of rivet.
   ![Shear failure of rivets](image)

2. **Shear failure of plate:** When rivets are placed very near to the edge of plate, then this failure takes place. This type of failure can be avoided by providing adequate edge distance.
   ![Shear failure of plate](image)
### 3. Tearing/tension failure of plates

Here in this failure, at the location of net cross-section, tensile stress in plate exceeds the permissible limit.

![Diagram of tension failure](image)

### 4. Bearing failure of plate

When bearing stress in plate exceeds the permissible limit, then crushing of plates takes place.

![Diagram of bearing failure](image)

### 5. Splitting of plate

When rivets are placed at very small edge distance then the plate tends to split or shear out.

![Diagram of splitting failure](image)

### 6. Bearing failure of rivets

When plates are stronger than rivets in bearing then there is a possibility that rivets may get crushed around the half circumference when bearing stress of rivet exceeds the allowable limit.

![Diagram of bearing failure of rivets](image)

---

Q.13 What are the various limit states of design for a steel structure as per IS : 800-2007?  

[5 marks : 2014]

**Solution:**

Limit state are the states beyond which the structure no longer satisfies the specified performance requirement. **As per IS 800: 2007, the limit states are generally grouped under.**

1. **Limit State of Strength:** Limit state of strength are associated with failure of structure under the worst combination of loading including appropriate partial factor of safety. The limit state of strength include
   (a) Loss of stability/equilibrium of structure (including the effect of sway) or any of parts including supports and foundation.
   (b) Strength limit (general yielding, formation of mechanism, rupture of structure or any of its parts of components)
   (c) Fatigue and brittle failure.

2. **Limit state of serviceability:** There are limit states beyond which specified service criteria are no longer met. These include
   (a) Deformation and deflections
   (b) Vibrations in the structure or any of its component causing discomfort to people or damages to the structure
   (c) Ponding of structures
   (d) Corrosion and durability
   (e) Repairable damage due to fatigue
Revised Syllabus of ESE: Construction-Planning, Equipment, Site investigation and Management including Estimation with latest project management tools and network analysis for different Types of works; Analysis of Rates of various types of works; Tendering Process and Contract Management, Quality Control, Productivity, Operation Cost; Land acquisition; Labour safety and welfare.

1. Project Management and Network Techniques

Q.1 Explain various steps involved in the development of networks. [10 marks : 2006]

Solution:

Various steps involved in the development of networks are as follows:

(i) **Objective**: During the planning of a project, the first and foremost step is to define the project and to decide the way in which it is to be carried out. The task to be undertaken requires to be set down as specific, definite, complete and well defined verbal statement. Specific verbal statement means the specific description of particular dimensions, type of materials, plants, etc. necessary for the project. Objective specifies the task to be undertaken and policy of its execution. This specification defines the project and determines the way in which it is to be carried out.

(ii) **Plan breakdown**: After establishing objective of the task, the planner has to adopt either forward planning or backward planning (or mixed planning) to achieve the goal. This backward to forward thinking will give a list of activities or jobs to be performed to achieve the task and also stages in the project execution.

(iii) **Sequencing**: In the second step we have obtained a general list of various activities and events necessary for the completion of the project. This general list is to be reviewed so that in each of the main group, those with definite similarities can be put in suitable subgroups.

(iv) **Location of nodes**: Now the events listed above are required to be located on paper so that a visual effect of movement along a time scale is obtained. Events should be located in such a way that they represent initial picture of the relation amongst them. This relationship results from the proposed use of manpower, money, material and other resources during a particular period of time.

(v) **Drawing Arrows**: Events having close and direct relationship are joined to each other by arrows representing activity to be performed for passing from one stage of the project to the other. These activities should fall in logical sequence.

(vi) **Checking**: At this stage, the diagram is checked with respect to content, sequence and sense and degree of detail.

It is essential to check the diagram for events and activities in respect of logic and accuracy. Particular attention should be paid to multiple events, i.e., those events at which more than one arrows enters and/or more than one arrows leave, since it is at this point that errors are most likely to occur. The checking ensures that the network correctly represents the sequence.
It should be ensured that network does not contain loops or cycles. If located these should be removed. Also, it should be checked whether there is any event (other than first) which has only outgoing arrows, or whether there is any event (other than last one) which has only incoming arrows. Such situation, if found, should be rectified. There should be no dead ends left.

An arrow should always represent singular situation but an event may represent commencement of more than one operations. In respect of sufficient detail, a ratio, known, as \( E/A \) ratio given as

\[
\frac{E}{A} = \text{Total number of events}
\]

\[
\text{Total number of activities}
\]

In a good network, its value should lie between 1 to 1.6.

(vii) Redraw: The errors found in the previous steps are removed and the diagram is redrawn by introducing unique dummies, if necessary.

(viii) Number: After drawn the final network, the events are numbered using Fulkerson’s rule.

Q.2 Draw a network for the following situations:

(i) A is prerequisite on D
(ii) B is prerequisite on D and E
(iii) C is prerequisite on E
(iv) D is prerequisite on F
(v) E is prerequisite on F

[10 marks : 2016]

Solution:

Network diagram will be as shown:

![Network Diagram]

Q.3 Briefly answer the following:

(i) How can an existing bar chart be modified to depict the project progress made?
(ii) Differentiate between 'Forward Planning' and 'Backward Planning' for network construction.

[10 marks : 2017]

Solution:

(i) A bar chart doesn’t show the progress of work and hence it cannot be used as a control device. Controlling is essential for rescheduling the remaining activities. However, an existing bar chart can be modified to depict the progress made. This can be done by showing the progress of each of each activity, by hatched lines along the corresponding bar of the activity. Generally, hatching is done in half the width of the bar.

![Bar Chart with Hatching]
(ii) ‘Forward Planning’: In this method, the planner starts from the initial event and builds up the events and activities logically and sequentially until the end event is reached. In this method, while considering activity, a planner asks himself the following questions:

What event comes next?
What are dependent events?
What events can take place concurrently?

**Backward planning**: In this method, the planner starts with the end event and arranges the events and activities until the initial event is reached. Keeping the goal in view, the planner asks himself if we want to achieve this, what events or activities should have taken place.

**Q.4** What is a work breakdown structure in Construction Project Management? Define and explain in brief. Further, how Work Breakdown Structure is classified into different levels for making the job convenient? Explain with an example.

[12 marks : 2018]

**Solution:**

(i) **Work breakdown structure in construction project management:**

A work breakdown structure is a key project deliverable that organizes the team’s work into manageable sections. The project management body of knowledge (PMBOK) defines the work breakdown structure as a ‘deliverable oriented hierarchical decomposition of work to be executed by the project team’. Work breakdown structure visually defines the scope into manageable chunks that a project team can understand, as each level of work breakdown structure provides further definition and detail. Figure below explains how work breakdown structure is classified into different levels for making the job convenient.

![Diagram of Work Breakdown Structure for Construction of a House](image-url)
Construction of a House

Level 1 Deliverables = 100%
Level 2 = 100%
Level 3 Work Package = 100%

1. Internal
   Work : 45.60%
   Budget : $86,000.00

   1.1 Electrical
      Work : 11.80%
      Budget : $25,000.00
      1.1.1 Rough-in Electrical
         Work : 2.80%
         Budget : $5,000.00
      1.1.2 Install and Terminate
         Work : 1.90%
         Budget : $5,000.00
      1.1.3 HVAC equipment
         Work : 7.10%
         Budget : $15,000.00

   1.2 Plumbing
      Work : 33.80%
      Budget : $61,000.00
      1.2.1 Rough-in Plumbing
         Work : 11.30%
         Budget : $22,000.00
      1.2.2 Set Plumbing Fixtures and Trim
         Work : 13.20%
         Budget : $31,000.00
      1.2.3 Test and Clean
         Work : 9.30%
         Budget : $8,000.00

2. Foundation
   Work : 24.00%
   Budget : $46,000.00

   2.1 Excavate
      Work : 18.20%
      Budget : $37,000.00
      2.1.1 Pour Concrete
         Work : 7.90%
         Budget : $30,000.00
      2.1.2 Cure & Strip Forms
         Work : 10.30%
         Budget : $7,000.00

   2.2 Steel Erection
      Work : 5.80%
      Budget : $9,000.00
      2.2.1 Steel Columns
         Work : 2.80%
         Budget : $5,000.00
      2.2.2 Beams
         Work : 1.90%
         Budget : $2,000.00
      2.2.3 Joist
         Work : 1.10%
         Budget : $2,000.00

3. External
   Work : 30.40%
   Budget : $63,500.00

   3.1 Masonry Work
      Work : 16.20%
      Budget : $62,000.00
      3.1.1 Lay Masonry
         Work : 9.00%
         Budget : $35,000.00
      3.1.2 Install Roof Drains
         Work : 3.10%
         Budget : $2,000.00
      3.1.3 Install Tile in Toilet Rooms
         Work : 1.30%
         Budget : $10,000.00
      3.1.4 Roofing
         Work : 2.80%
         Budget : $15,000.00

   3.2 Building Finishes
      Work : 14.20%
      Budget : $21,500.00
      3.2.1 Paint Walls
         Work : 4.00%
         Budget : $8,000.00
      3.2.2 Ceiling Tile
         Work : 3.60%
         Budget : $4,000.00
      3.2.3 Hang Wallpaper
         Work : 2.30%
         Budget : $1,500.00
      3.2.4 Carpet
         Work : 1.80%
         Budget : $6,000.00
      3.2.5 Hardware
         Work : 2.50%
         Budget : $2,000.00
Hence, various levels of WBS can show the various details at a glance:
1. What the various elements of project are,
2. How the necessary work is distributed between the elements of the project.
3. How the cost or budget is distributed between the elements of the project.
4. How the larger elements of the project are sub-divided into smaller ones.
WBS is a systematic approach to scooping the project work in which logical hierarchical pattern is devised in a Top-Down Approach.

2. PERT and CPM

Q.5 The interdependence of a job consisting of seven activities viz. A to G is given as:

<table>
<thead>
<tr>
<th>Activity</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predecessor activity</td>
<td>-</td>
<td>-</td>
<td>A</td>
<td>B</td>
<td>A</td>
<td>B</td>
<td>C,D</td>
</tr>
<tr>
<td>Succeeding activity</td>
<td>C,E</td>
<td>D,F</td>
<td>G</td>
<td>G</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

The time estimates in days for each activity are:

<table>
<thead>
<tr>
<th>Activity</th>
<th>Time Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6 – 9 – 18</td>
</tr>
<tr>
<td>B</td>
<td>5 – 8 – 17</td>
</tr>
<tr>
<td>C</td>
<td>4 – 7 – 22</td>
</tr>
<tr>
<td>D</td>
<td>4 – 7 – 16</td>
</tr>
<tr>
<td>E</td>
<td>4 – 7 – 10</td>
</tr>
<tr>
<td>F</td>
<td>2 – 5 – 8</td>
</tr>
<tr>
<td>G</td>
<td>4 – 10 – 22</td>
</tr>
</tbody>
</table>

\[
Z(\pm) \quad 0.8 \quad 0.9 \quad 1.0 \quad 1.1 \quad 1.2 \\
% probability \quad 78.81 \quad 81.59 \quad 84.13 \quad 86.43 \quad 88.49
\]

Draw the network and determine the probability of completing the job in 35 days. [20 marks : 1995]

Solution:
The network diagram is shown below:

![Network Diagram]

Now we know that

\[
t_e = \frac{t_o + 4t_m + t_p}{6}
\]

where \( t_e \) is expected time, \( t_o \) is optimistic time, \( t_p \) is pessimistic time and \( t_m \) is most likely time of completion of an activity.
The calculations for $t_p$ are tabulated below:

<table>
<thead>
<tr>
<th>Activity</th>
<th>$t_o$</th>
<th>$t_m$</th>
<th>$t_p$</th>
<th>$t_e$</th>
<th>$t_o + 4t_m + t_p$</th>
<th>$t_o - t_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6</td>
<td>9</td>
<td>18</td>
<td>10</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>8</td>
<td>17</td>
<td>9</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>4</td>
<td>7</td>
<td>22</td>
<td>9</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>D</td>
<td>4</td>
<td>7</td>
<td>16</td>
<td>8</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>E</td>
<td>4</td>
<td>7</td>
<td>10</td>
<td>7</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>F</td>
<td>2</td>
<td>5</td>
<td>8</td>
<td>5</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>G</td>
<td>4</td>
<td>10</td>
<td>22</td>
<td>11</td>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

where

$T_E$ (successor event) = $T_E$ (predecessor event) + $t_s$ (activity)

$T_L$ (predecessor event) = $T_L$ (successor event) - $t_s$ (activity)

The slack for any event is given by

$S = T_L - T_E$

The critical path will flow the events with zero slack.

Thus the critical path is 1 → 2 → 4 → 5

\[
\sigma = \sqrt{\sigma_A^2 + \sigma_C^2 + \sigma_G^2} = \sqrt{2^2 + 3^2 + 3^2} = 4.69 \text{ days}
\]

Now we know that

\[
Z = \frac{T_S - T_E}{\sigma}
\]

where $T_S$ = Scheduled completion time of project = 35 days

\[
Z = \frac{35 - 30}{4.69} = 1.07
\]

% $p$ at $Z = 1.0$ is 84.13

% $p$ at $Z = 1.1$ is 86.43

% probability at $Z = 1.07$ will be given by

\[
% p = 84.13 + \frac{86.43 - 84.13}{1.1 - 1.0} (1.07 - 1.0) = 85.74\%
\]

Hence, the probability of completing the project in 35 days is 85.74%.

Q.6 Define slack. What does negative slack indicate in PERT network analysis? [5 marks : 1996]

Solution:

Slack may simply be defined as the difference between the latest allowable time and the earliest expected time of an event. The difference between the two times indicates the range between which the occurrence time of an event can vary.
\[ S = T_L - T_E \]

where \( S \) is the slack for any event, \( T_E \) is the earliest expected time of an event and \( T_L \) is the latest allowable time of an event.

Negative slack is obtained when the scheduled completion time, \( T_S \) (and hence \( T_L \)) is less than the earliest expected time of completion (\( T_E \)). It is an indication of a behind of schedule condition (lack of resources).

**Q.7** The network shown in the figure below has the estimated duration for each activity marked. Determine the total float for each activity and establish the critical path.

![Network Diagram](image)

**Solution:**

The earliest expected time of each event is calculated and shown in the network below:

![Network Diagram](image)

The calculations for total float are shown below:

\[
EST = t_i^E;\ EFT = EST + t_i^f\]

\[
LST = LFT - t_i^E;\ LFT = T_L^i\]

**Total Float,**

\[ F_T = LST - EST \text{ or } LFT - EFT \]

<table>
<thead>
<tr>
<th>Activity</th>
<th>Duration</th>
<th>( EST )</th>
<th>( EFT )</th>
<th>( LST )</th>
<th>( LFT )</th>
<th>( F_T )</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>0</td>
<td>3</td>
<td>8</td>
<td>11</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>0</td>
<td>4</td>
<td>6</td>
<td>10</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>14</td>
<td>0</td>
<td>14</td>
<td>0</td>
<td>14</td>
<td>0</td>
<td>Critical</td>
</tr>
<tr>
<td>D</td>
<td>3</td>
<td>4</td>
<td>7</td>
<td>11</td>
<td>14</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>5</td>
<td>3</td>
<td>8</td>
<td>11</td>
<td>16</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>6</td>
<td>4</td>
<td>10</td>
<td>10</td>
<td>16</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>4</td>
<td>4</td>
<td>8</td>
<td>11</td>
<td>15</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>1</td>
<td>14</td>
<td>15</td>
<td>14</td>
<td>15</td>
<td>0</td>
<td>Critical</td>
</tr>
<tr>
<td>I</td>
<td>1</td>
<td>15</td>
<td>16</td>
<td>15</td>
<td>16</td>
<td>0</td>
<td>Critical</td>
</tr>
</tbody>
</table>

Hence the critical activities are \( C, H \) and \( I \) and events joining these activities will be the critical events. Thus the critical path will be 1-4-5-6.