GATE 2020

Civil Engineering

✔ Fully solved with explanations  ✔ Topicwise presentation
✔ Analysis of previous papers  ✔ Thoroughly revised & updated

B. Singh (Ex. IES)
CMD, MADE EASY Group

MADE EASY Publications
Preface

Over the period of time the GATE examination has become more challenging due to increasing number of candidates. Though every candidate has ability to succeed but competitive environment, in-depth knowledge, quality guidance and good source of study is required to achieve high level goals.

The new edition of *GATE 2020 Solved Papers : Civil Engineering* has been fully revised, updated and edited. The whole book has been divided into topicwise sections.

At the beginning of each subject, analysis of previous papers are given to improve the understanding of subject.

I have true desire to serve student community by way of providing good source of study and quality guidance. I hope this book will be proved an important tool to succeed in GATE examination. Any suggestions from the readers for the improvement of this book are most welcome.

B. Singh (Ex. IES)
Chairman and Managing Director
MADE EASY Group
<table>
<thead>
<tr>
<th>Sl.</th>
<th>Unit</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Solid Mechanics</td>
<td>1-63</td>
</tr>
<tr>
<td>2</td>
<td>Structural Analysis</td>
<td>64-117</td>
</tr>
<tr>
<td>3</td>
<td>RCC Structures and Prestressed Concrete</td>
<td>118-161</td>
</tr>
<tr>
<td>4</td>
<td>Design of Steel Structures</td>
<td>162-205</td>
</tr>
<tr>
<td>5</td>
<td>Geotechnical Engineering</td>
<td>206-307</td>
</tr>
<tr>
<td>6</td>
<td>Fluid Mechanics &amp; Fluid Machines</td>
<td>308-386</td>
</tr>
<tr>
<td>7</td>
<td>Environmental Engineering</td>
<td>387-460</td>
</tr>
<tr>
<td>8</td>
<td>Irrigation Engineering</td>
<td>461-480</td>
</tr>
<tr>
<td>9</td>
<td>Engineering Hydrology</td>
<td>481-506</td>
</tr>
<tr>
<td>10</td>
<td>Transportation Engineering</td>
<td>507-568</td>
</tr>
<tr>
<td>11</td>
<td>Geometrics Engineering</td>
<td>569-595</td>
</tr>
<tr>
<td>13</td>
<td>Engineering Mathematics</td>
<td>609-681</td>
</tr>
<tr>
<td>14</td>
<td>General Aptitude</td>
<td>682-716</td>
</tr>
</tbody>
</table>
# Solid Mechanics

## Contents

<table>
<thead>
<tr>
<th>Sl.</th>
<th>Topic</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Properties of Metals, Stress &amp; Strain</td>
<td>3</td>
</tr>
<tr>
<td>2.</td>
<td>Shear Force and Bending Moment</td>
<td>13</td>
</tr>
<tr>
<td>3.</td>
<td>Principal Stress and Principal Strain</td>
<td>22</td>
</tr>
<tr>
<td>4.</td>
<td>Bending and Shear Stresses</td>
<td>29</td>
</tr>
<tr>
<td>5.</td>
<td>Deflection of Beams</td>
<td>37</td>
</tr>
<tr>
<td>6.</td>
<td>Torsion of Shafts and Pressure Vessels</td>
<td>54</td>
</tr>
<tr>
<td>7.</td>
<td>Theory of Columns &amp; Shear Centre</td>
<td>58</td>
</tr>
</tbody>
</table>
Syllabus: Bending moment and shear force in statically determinate beams; Simple stress and strain relationships; Theories of failures; Simple bending theory, flexural and shear stresses, shear centre; Uniform torsion, buckling of column, combined and direct bending stresses.

### Analysis of Previous GATE Papers

<table>
<thead>
<tr>
<th>Exam Year</th>
<th>1 Mark Ques.</th>
<th>2 Marks Ques.</th>
<th>Total Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991</td>
<td>5</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>1992</td>
<td>3</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>1993</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>1994</td>
<td>2</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>1995</td>
<td>2</td>
<td>–</td>
<td>2</td>
</tr>
<tr>
<td>1996</td>
<td>3</td>
<td>–</td>
<td>3</td>
</tr>
<tr>
<td>1997</td>
<td>2</td>
<td>–</td>
<td>2</td>
</tr>
<tr>
<td>1998</td>
<td>–</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1999</td>
<td>4</td>
<td>–</td>
<td>4</td>
</tr>
<tr>
<td>2000</td>
<td>7</td>
<td>–</td>
<td>7</td>
</tr>
<tr>
<td>2001</td>
<td>1</td>
<td>–</td>
<td>1</td>
</tr>
<tr>
<td>2002</td>
<td>2</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2003</td>
<td>2</td>
<td>5</td>
<td>12</td>
</tr>
<tr>
<td>2004</td>
<td>1</td>
<td>6</td>
<td>13</td>
</tr>
<tr>
<td>2005</td>
<td>2</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>2006</td>
<td>3</td>
<td>9</td>
<td>21</td>
</tr>
<tr>
<td>2007</td>
<td>3</td>
<td>5</td>
<td>13</td>
</tr>
<tr>
<td>2008</td>
<td>–</td>
<td>8</td>
<td>16</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Exam Year</th>
<th>1 Mark Ques.</th>
<th>2 Marks Ques.</th>
<th>Total Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>2</td>
<td>5</td>
<td>12</td>
</tr>
<tr>
<td>2010</td>
<td>5</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2011</td>
<td>1</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>2012</td>
<td>4</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>2013</td>
<td>4</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>2014 Set-1</td>
<td>1</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>2014 Set-2</td>
<td>1</td>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>2015 Set-1</td>
<td>1</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>2015 Set-2</td>
<td>2</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>2016 Set-1</td>
<td>–</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>2016 Set-2</td>
<td>–</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>2017 Set-1</td>
<td>2</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>2017 Set 2</td>
<td>1</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>2018 Set-1</td>
<td>2</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>2018 Set-2</td>
<td>–</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2019 Set-1</td>
<td>2</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2019 Set-2</td>
<td>2</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>
3.1 A failure theory postulated for metals is shown in a two dimensional stress plane. The theory is called

(a) Maximum distortion energy theory
(b) Maximum normal stress theory
(c) Maximum shear stress theory
(d) Maximum strain theory

[1991 : 1 Mark]

3.2 Which of the following Mohr’s circles qualitatively correctly represents the state of plane stress at a point in a beam above the neutral axis, where it is subjected to combined shear and bending compressive stresses?

(a)  
(b)  
(c)  
(d)  

[1993 : 1 Mark]

3.3 If an element of a stressed body is in a state of pure shear with a magnitude of 80 N/mm², the magnitude of maximum principal stress at that location is

(a) 80 N/mm²  
(b) 113.14 N/mm²  
(c) 120 N/mm²  
(d) 56.57 N/mm²

[1999 : 1 Mark]

3.4 Two perpendicular axes x and y of a section are called principal axes when

(a) Moments of inertia about the axes are equal ($I_x = I_y$)
(b) Product moment of inertia ($I_{xy}$) is zero
(c) Product of moment of inertia ($I_x$, $I_y$) is zero
(d) Moment of inertia about one of the axis is greater than the other

[1999 : 1 Mark]

3.5 Pick the incorrect statement from the following four statements

(a) On the plane which carries maximum normal stress, the shear stress is zero
(b) Principal planes are mutually orthogonal
(c) On the plane which carries maximum shear stress, the normal stress is zero
(d) The principal stress axes and principal strain axes coincide for an isotropic material

[2000 : 1 Mark]

3.6 The state of two dimensional stresses acting on a concrete lamina consists of a direct tensile stress, $\sigma_x = 1.5$ N/mm², and shear stress, $\tau = 1.20$ N/mm², which cause cracking of concrete. Then the tensile strength of the concrete (in N/mm²) is

(a) 1.50  
(b) 2.08  
(c) 2.17  
(d) 2.29

[2003 : 2 Marks]

3.7 In a two dimensional stress analysis, the state of stress at a point is shown below. If $\sigma = 120$ MPa and $\tau = 70$ MPa, $\sigma_x$ and $\sigma_y$ are respectively.
3.8 The components of strain tensor at a point in the plane strain case can be obtained by measuring longitudinal strain in following directions
(a) along any two arbitrary directions
(b) along any three arbitrary directions
(c) along two mutually orthogonal directions
(d) along any arbitrary direction

3.9 If principal stresses in a two-dimensional case are \(-10 \text{ MPa}\) and \(20 \text{ MPa}\) respectively, then maximum shear stress at the point is
(a) \(10 \text{ MPa}\)
(b) \(15 \text{ MPa}\)
(c) \(20 \text{ MPa}\)
(d) \(30 \text{ MPa}\)

3.10 Mohr’s circle for the state of stress defined by
\[
\begin{bmatrix}
30 & 0 \\
0 & 30
\end{bmatrix}
\] MPa is a circle with
(a) center at \((0, 0)\) and radius \(30 \text{ MPa}\)
(b) center at \((0, 0)\) and radius \(60 \text{ MPa}\)
(c) center at \((30, 0)\) and radius \(30 \text{ MPa}\)
(d) center at \((30, 0)\) and zero radius

3.11 An axially loaded bar is subjected to a normal stress of \(173 \text{ MPa}\). The shear stress in the bar is
(a) \(75 \text{ MPa}\)
(b) \(86.5 \text{ MPa}\)
(c) \(100 \text{ MPa}\)
(d) \(122.3 \text{ MPa}\)

3.12 Consider the following statements:
1. On a principal plane, only normal stress acts.
2. On a principal plane, both normal and shear stresses act.

3.13 The major and minor principal stresses at a point are \(3 \text{ MPa}\) and \(-3 \text{ MPa}\) respectively. The maximum shear stress at the point is
(a) zero
(b) \(3 \text{ MPa}\)
(c) \(6 \text{ MPa}\)
(d) \(9 \text{ MPa}\)

3.14 If a small concrete cube is submerged deep in still water in such a way that the pressure exerted on all faces of the cube is \(p\), then the maximum shear stress developed inside the cube is
(a) 0
(b) \(p/2\)
(c) \(p\)
(d) \(2p\)

3.15 The state of 2D stress at a point is given by a matrix
\[
\begin{bmatrix}
\sigma_{xx} & \tau_{xy} \\
\tau_{yx} & \sigma_{yy}
\end{bmatrix} = \begin{bmatrix}
100 & 30 \\
30 & 20
\end{bmatrix} \text{ MPa}
\]
The maximum shear stress in MPa is
(a) 50
(b) 75
(c) 100
(d) 110

3.16 Two triangular wedges are glued together as shown in the following figure. The stress acting normal to the interface, \(\sigma_n\) is \_________ \text{ MPa}.

3.17 For the plane stress situation shown in the figure, the maximum shear stress and the plane on which it acts are
3.18 For the stress state (in MPa) shown in the figure, the major principal stress is 10 MPa.

The shear stress $\tau$ is

(a) 10.0 MPa  (b) 5.0 MPa  (c) 2.5 MPa  (d) 0.0 MPa  

[2016 : 2 Marks, Set-II]

3.19 An element is subjected to biaxial normal tensile strains of 0.0030 and 0.0020. The normal strain in the plane of maximum shear strain is

(a) Zero  (b) 0.0050  (c) 0.0010  (d) 0.0025  

[2019 : 1 Mark, Set-I]

3.20 For a plane stress problem, the state of stress at a point P is represented by the stress element as shown in figure.

By how much angle ($\theta$) in degrees the stress element should be rotated in order to get the planes of maximum shear stress?

(a) 31.7  (b) 13.3  (c) 26.6  (d) 48.3  

[2019 : 2 Marks, Set-II]
3.1 (c) 3.2 (c) 3.3 (a) 3.4 (b) 3.5 (c) 3.6 (c) 3.7 (c) 3.8 (b) 3.9 (b) 3.10 (d) 3.11 (b) 3.12 (a) 3.13 (b) 3.14 (a) 3.15 (a) 3.17 (d) 3.18 (b) 3.19 (d) 3.20 (a)

3.1 (c) 3.2 (c)

Theory
1. Maximum principal stress theory
2. Maximum principal strain theory
3. Maximum shear stress theory
4. Maximum strain energy theory
5. Maximum shear strain energy theory/Distortion energy theory/Mises-Hencky theory

Two dimensional stress plane

1. Rectangular

2. Rhombus

3. Hexagon

4. Ellipse

5. Ellipse

3.2 (c)

Let ‘A’ be a point in a beam above the neutral axis.

Plane stress diagram of point ‘A’,

Mohr circle will be best represent as,

3.3 (a)

For the case of pure shear, maximum principal stress occurs at the diagonals and is given by

\[ \sigma = \pm \tau \]

\[ = 80 \text{ N/mm}^2 \]

3.4 (b)

Principal axes are those axes about which product moment of inertia \(I_{xy}\) is zero and moment of inertia is either maximum or minimum.
3.5 (c)

Plane of maximum shear stress has normal stress of \( \frac{\sigma_1 + \sigma_2}{2} \).

3.6 (c)

Maximum principal stress

\[
\sigma = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2}
\]

\[
= \frac{1.5}{2} + \sqrt{\left(\frac{1.5}{2}\right)^2 + (1.20)^2}
\]

\[= 2.17 \text{ N/mm}^2\]

3.7 (c)

Let, \( \angle CAB = \theta \)

\[
\sin \theta = \frac{3}{5}, \cos \theta = \frac{4}{5} \quad \text{and} \quad \tan \theta = \frac{3}{4}
\]

Thus from force equilibrium,

\[
\sigma_x \times AB = AC \times (\sigma \cos \theta - \tau \sin \theta)
\]

\[
\Rightarrow \quad \sigma_x = \frac{5}{4} \times \left(120 \times \frac{4}{5} - 70 \times \frac{3}{5}\right)
\]

\[
\Rightarrow \quad \sigma_x = 67.5 \text{ MPa}
\]

And,

\[
\sigma_y \times BC = AC \times (\sigma \sin \theta + \tau \cos \theta)
\]

\[
\Rightarrow \quad \sigma_y = \frac{5}{3} \times \left(120 \times \frac{3}{5} + 70 \times \frac{4}{5}\right)
\]

\[
\Rightarrow \quad \sigma_y = 213.3 \text{ MPa}
\]

3.8 (b)

In case of plane strain condition, the components of strain tensor at a point are \( \varepsilon_x, \varepsilon_y \) and \( \phi_{xy} \).

Here we have three unknowns so we require 3 equations to find them and these unknowns can be find by the equations of longitudinal strain,

\[
\varepsilon_1 = \varepsilon_x \cos^2 \theta_1 + \varepsilon_y \sin^2 \theta_1 + \frac{\phi_{xy}}{2} \sin 2\theta_1
\]

\[
\varepsilon_2 = \varepsilon_x \cos^2 \theta_2 + \varepsilon_y \sin^2 \theta_2 + \frac{\phi_{xy}}{2} \sin 2\theta_2
\]

\[
\varepsilon_3 = \varepsilon_x \cos^2 \theta_3 + \varepsilon_y \sin^2 \theta_3 + \frac{\phi_{xy}}{2} \sin 2\theta_3
\]

Therefore to find out component of strain tensor in plain strain condition, we measure longitudinal strains \( \varepsilon_1, \varepsilon_2 \) and \( \varepsilon_3 \) along any three arbitrary direction.
Maximum shear stress,
\[
\tau = \frac{\sigma_1 - \sigma_2}{2} = \frac{P - P}{2} = 0
\]

**Note:** It is the case of hydrostatic force, Mohr circle of which is a point hence \(\tau_{\text{max}} = 0\).

**3.15 (a)**

\[
\sigma_{1/2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}
\]

\[
= \frac{100 + 20}{2} \pm \sqrt{\left(\frac{100 - 20}{2}\right)^2 + 30^2}
\]

\[
= 60 \pm 50
\]

\(\sigma_1 = 110; \quad \sigma_2 = 10\)

\[
\therefore \tau_{\text{max}} = \frac{110 - 10}{2} = 50 \text{ MPa}
\]

**Note:** \(\tau_{\text{max}}\) absolute \(= \frac{1.10 - 0}{2} = 0.55\)

But SS is not the option so taken \(\tau_{\text{max}}\) plane otherwise prefer \(\tau_{\text{max}}\) absolute

**3.16 Sol.**

**Method-I**

As plane \(AB\) and \(BC\) are principle planes, therefore Mohr's circle for the given condition is,

Here, normal stress is zero at 45° to the principle plane.

**Method-II**

\[
\sigma_R = 100 \text{ MPa}, \quad \sigma_{R_2} = -100 \text{ MPa}
\]

\[
\Rightarrow \quad \text{at 45°, it is a plane of maximum shear stress}
\]

\[
\sigma_n = \frac{\sigma_R + \sigma_{R_2}}{2} = 0
\]

**3.17 (d)**

Under hydrostatic loading condition, stresses at a point in all directions equal and hence no shear stress.
Alternatively,
\[ \tau = \frac{\sigma_1 - \sigma_2}{2} = \frac{50 - 50}{2} = 0 \]

Thus, Mohr’s circle reduces to a point.
Hence shear stress at all orientations is zero.

\[ \sigma_y = 5 \quad \sigma_z = 5 \]
\[ \Rightarrow \sigma_y + \sigma_z = \sigma_1 + \sigma_2 \]
\[ \Rightarrow 5 + 5 = 10 + \sigma_2 \]
\[ \Rightarrow \sigma_2 = 0 \]

Now,
\[ \sigma_{1/2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \]
\[ \therefore \sigma_1 = \frac{5 + 5}{2} \pm \sqrt{\left(\frac{5 - 5}{2}\right)^2 + \tau_{xy}^2} \]
\[ \Rightarrow 10 = 5 + \tau_{xy} \]
\[ \therefore \tau_{xy} = 5 \text{ MPa} \]

\[ \varepsilon_x = 0.0030 \]
\[ \varepsilon_y = 0.0020 \]
Normal strain in the plane of maximum shear strain
\[ = \frac{\varepsilon_x + \varepsilon_y}{2} = 0.0025 \]

\[ \sigma_x = 80 \]
\[ \sigma_y = -20 \]
\[ \tau_{xy} = -25 \]

Angle of plane of max shear
\[ \theta = \theta_p + 45^\circ \]
\[ \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{-50}{100} \]
\[ \theta_p = -13.28^\circ \]
\[ \therefore \theta = 31.71^\circ \]
4.1 For a given shear force across a symmetrical 'I' section, the intensity of shear stress is maximum at the
(a) extreme fibres
(b) centroid of the section
(c) at the junction of the flange and the web, but on the web
(d) at the junction of the flange and the web, but on the flange


4.2 The maximum bending stress induced in a steel wire of modulus of elasticity 200 kN/mm$^2$ and diameter 1 mm when wound on a drum of diameter 1 m is approximately equal to
(a) 50 N/mm$^2$
(b) 100 N/mm$^2$
(c) 200 N/mm$^2$
(d) 400 N/mm$^2$

[1992 : 1 Mark]

4.3 A homogeneous, simply supported prismatic beam of width $B$, depth $D$ and span $L$ is subjected to a concentrated load of magnitude $P$. The load can be placed anywhere along the span of the beam. The maximum flexural stress developed in beam is
(a) $\frac{2PL}{3BD^2}$
(b) $\frac{3PL}{4BD^2}$
(c) $\frac{4PL}{3BD^2}$
(d) $\frac{3PL}{2BD^2}$

[2004 : 2 Marks]

4.4 A beam with the cross-section given below is subjected to a positive bending moment (causing compression at the top) of 16 kN-m acting around the horizontal axis. The tensile force acting on the hatched area of the cross-section is

(a) zero
(b) 5.9 kN
(c) 8.9 kN
(d) 17.8 kN

[2006 : 2 Marks]

4.5 If a beam of rectangular cross-section is subjected to a vertical shear force $V$, the shear force carried by the upper one-third of the cross-section is
(a) zero
(b) $\frac{7V}{27}$
(c) $\frac{8V}{27}$
(d) $\frac{V}{3}$

[2006 : 2 Marks]

4.6 I-section of a beam is formed by gluing wooden planks as shown in the figure below. If this beam transmits a constant vertical shear force of 3000 N, the glue at any of the four joints will be subjected to a shear force (in kN per meter length) of

(a) 3.0
(b) 4.0
(c) 8.0
(d) 10.7

[2006 : 2 Marks]

4.7 The shear stress at the neutral axis in a beam of triangular section with a base of 40 mm and height 20 mm, subjected to a shear force of 3 kN is
(a) 3 MPa
(b) 6 MPa
(c) 10 MPa
(d) 20 MPa

[2007 : 2 Marks]

4.8 The maximum tensile stress at the section X-X shown in the figure below is

[Figure of I-section and T-section]
4.9 Consider a simply supported beam with a uniformly distributed load having a neutral axis (NA) as shown. For points P (on the neutral axis) and Q (at the bottom of the beam) the state of stress is best represented by which of the following pairs?

\[
\begin{align*}
\text{(a) } & \frac{8P}{bd} & \text{(b) } & \frac{6P}{bd} \\
\text{(c) } & \frac{4P}{bd} & \text{(d) } & \frac{2P}{bd}
\end{align*}
\]

[2008 : 2 Marks]

4.13 A simply supported reinforced concrete beam of length 10 m sags while undergoing shrinkage. Assuming a uniform curvature of 0.004 m^{-1} along the span, the maximum deflection (in m) of the beam at mid-span is _______.

[2015 : 2 Marks, Set-II]

4.14 A 450 mm long plain concrete prism is subjected to the concentrated vertical loads as shown in the figure. Cross section of the prism is given as 150 mm x 150 mm. Considering linear stress distribution across the cross-section, the modulus of rupture (expressed in MPa) is ___.

[2016 : 2 Marks, Set-II]

4.15 A cantilever beam of length 2 m with a square section of side length 0.1 m is loaded vertically at the free end. The vertical displacement at the free end is 5 mm. The beam is made of steel with Young’s modulus of 2.0 x 10^{11} N/m². The maximum bending stress at the fixed end of the cantilever is

(a) 20.0 MPa  
(b) 37.5 MPa  
(c) 60.0 MPa  
(d) 75.0 MPa

[2018 : 2 Marks, Set-I]

4.16 An 8 m long simply-supported elastic beam of rectangular cross-section (100 mm x 200 mm) is subjected to a uniformly distributed load of 10 kN/m over its entire span. The maximum principal stress (in MPa, up to two decimal places) at a point located at the extreme compression edge of a cross-section and at 2 m from the support is _______.

[2018 : 2 Marks, Set-II]

4.17 For a given loading on a rectangular plain concrete beam with an overall depth of 500 mm, the compressive strain and tensile strain developed at the extreme fibers are of the same magnitude of 2.5 x 10^{-4}. The curvature in the beam cross-
section \((\text{in } \text{m}^{-1})\), round off to 3 decimal places). 

\[ \text{[2019 : 1 Mark, Set-I]} \]

4.18 Cross section of a built-up wooden beam as shown in figure (not drawn to scale) is subjected to a vertical shear force of 8 kN. The beam is symmetrical about the neutral axis (NA), shown, and the moment of inertia about N.A. is \(1.5 \times 10^9 \text{ mm}^4\). Considering that the nails at the location \(P\) are spaced longitudinally (along the length of the beam) at 60 mm, each of the nails at \(P\) will be subjected to the shear force of

\[ \text{[2019 : 1 Mark, Set-II]} \]

\[ \text{All dimensions are in mm} \]

(a) 240 N \hspace{1cm} (b) 480 N
(c) 60 N \hspace{1cm} (d) 120 N

\[ \text{[2019 : 2 Marks, Set-I]} \]

4.19 For a channel section subjected to a downward vertical shear force at its centroid, which one of the following represents the correct distribution of shear stress in flange and web?

\[ \text{[2019 : 1 Mark, Set-II]} \]

...
### Answers  Bending and Shear Stresses

4.1 (b)  
4.2 (c)  
4.3 (d)  
4.4 (c)  
4.5 (b)  
4.6 (b)  
4.7 (c)  
4.8 (a)  
4.10 (a)  
4.12 (b)  
4.15 (b)  
4.18 (a)  
4.19 (c)

### Explanations  Bending and Shear Stresses

#### 4.1 (b)

![Shear stress diagram]

The intensity of shear stress is maximum at centroid of the section $\tau_{avg} = \frac{3}{2} \tau_{avg}$.

#### 4.2 (c)

As we know,

$$f = \frac{M}{I} y$$

$$E = 200 \text{ kN/mm}^2 = 2 \times 10^5 \text{ N/mm}^2$$

$$f = \frac{E y}{R} = \frac{2 \times 10^5}{500} \times 0.5$$

$$= 200 \text{ N/mm}^2$$

#### 4.3 (d)

When the concentrated load is placed at the midspan, maximum bending moment will develop at the midspan.

![Bending stress distribution]

Now,

$$f = \frac{M}{I} y$$

$$M = \frac{PL}{4}$$

$$\Rightarrow f = \frac{PL}{4} \times \frac{D}{2} = \frac{3PL}{8 BD^2}$$

#### 4.4 (c)

From similar triangles, we have,

$$f = \frac{M}{I} x$$

$$= \frac{16 \times 10^8 \times 12}{100 \times 150^3} \times 25 = 14.22 \text{ N/mm}^2$$

#### 4.5 (b)

Shear stress at distance $y$ from neutral axis,

$$\tau = \frac{SAy}{Ib} = \frac{V \times \left( \frac{d}{2} - y \right) \times b \times \left( \frac{d}{2} + y \right)}{Ib}$$

$$= \frac{V}{Ib} \times \left( \frac{d^2}{4} - y^2 \right)$$

$$\Rightarrow \tau = \frac{V}{2I} \times \left( \frac{d^2}{4} - y^2 \right)$$
\[ dF = \tau \times b \ dy = \frac{V \left( \frac{a^2}{4} - y^2 \right)}{2I} \times b \ dy \]

Integrating both sides, we get,

\[ \Rightarrow F = \frac{V b}{2I} \int \left( \frac{a^2}{4} - y^2 \right) dy \]

\[ = \frac{V b}{2I} \left[ \frac{a^2}{4} y - \frac{y^3}{3} \right]_{dy}^{d/2} \]

\[ = \frac{V b}{2I} \left[ \frac{a^3}{8} - \frac{a^3}{24} - \frac{a^3}{24} + \frac{a^3}{648} \right] \]

\[ = \frac{V b}{2I} \times \frac{a^3}{8} \times \frac{28}{81} \]

\[ = \frac{V b}{2bd^2} \times \frac{a^3}{8} \times \frac{28}{81} \times 12 = \frac{7V}{27} \]

4.6 (b)

Considering shaded area ‘abcd’ of plank,

**Shear flow.** \( q = \frac{VQ}{I} \)

Shear flow is measured as force per unit length along the longitudinal axis of a beam and over the cross-section is as shown above.

\[ Q = Ay \]

\[ Q = (75 \times 50) \times 125 = 468750 \text{ mm}^3 \]

Moment of inertia of I-section about NA,

\[ I = 350 \times 10^6 \text{ mm}^4 \]

\[ V = 3000 \text{ N} \]

\[ q = \frac{468750 \times 3000}{350 \times 10^6} = 4.018 \text{ N/mm} \]

So, resistance offered by glue to keep I-section intact,

\[ q = 4.018 \text{ N/mm} \approx 4 \text{ kN/m} \]

### 4.7 (c)

**Method-I**

Shear stress, \( \tau = \frac{SA\bar{y}}{Ib} \)

Width at a distance of \( \frac{40}{3} \) mm from the top

\[ = \frac{40 \times 40}{20 \times 3} = \frac{80}{3} \text{ mm} \]

\[ \therefore \tau = \frac{3 \times 10^3 \times \left( \frac{1}{2} \times \frac{80}{3} \times \frac{40}{3} \right)}{\left( \frac{1}{3} \times \frac{40}{3} \right) \times \frac{80}{3}} \]

\[ = \frac{3 \times 10^3 \times 3200 \times 40 \times 36 \times 3}{162 \times 3200 \times 20^3} = 10 \text{ MPa} \]

**Method-II**

\[ \tau_{N.A.} = \frac{4}{3} \tau_{avg} \]

\[ = \frac{4}{3} \times \frac{3000}{2 \times 40 \times 20} = 10 \text{ MPa} \]

### 4.8 (a)

The section at X–X may be shown as in the figure below:

The maximum tensile stress at the section X–X is,

\[ \sigma = \frac{P}{A} + \frac{M}{Z} \]

\[ = \frac{P}{b \times (d/2)} + \frac{P \times (d/4) \times 6}{b \times (d^2 / 4)} \]

\[ = \frac{2P}{bd} + \frac{6P}{bd} = \frac{8P}{bd} \]
Point P: Point P lies on NA, hence bending stress is zero at point P.
Point P also lies at mid span, so shear force, \( V = 0 \) \( \Rightarrow \) Shear stress, \( \tau = 0 \)
\( \therefore \) State of stress of point P will be,

Point Q: At point Q, flexural stress is maximum and nature of which is tensile due to downward loading.
Point Q lies at the extreme of beam, therefore, shear stress at point Q is zero.
\( \therefore \) State of stress of point Q will be,

Radius,
\[ R = \frac{1}{C} = 250 \text{ m} \]
\[ OA = \sqrt{250^2 - 5^2} \]
\[ = 249.95 \text{ m} \]
Deflection = \( AA' = 250 - 249.95 = 0.05 \text{ m} \)

Method-II
\[ \delta = \frac{i^2}{8R} \]
\[ (2R - \delta) \delta = \frac{i^2}{4} \]
Deflection \( \delta = \frac{P l^3}{3EI} \)

\[ \Rightarrow 5 \times 10^{-3} \text{ m} = \frac{P (2)^3}{3 \times 2 \times 10^{11} \times (0.1)^4 / 12} \]

\[ \Rightarrow P = 3125 \text{ N} \]

Now, \( M = P l = 3125 \times 2 = 6250 \text{ Nm} \)

As, \( \frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R} \)

\[ \Rightarrow \sigma_{\text{max}} = \frac{M}{Z} \left( \frac{0.1}{3} \right)^3 = 6250 \times \frac{6}{1} = 37.5 \times 10^6 \text{ N/m}^2 = 37.5 \text{ MPa} \]

4.14 Sol.

4.16 Sol.

\[ BM_Q = 11.25 \times 150 \]

\[ = 16875 \times 10^6 \text{ N-mm} \]

\[ \Rightarrow \frac{\sigma}{y} = \frac{M_Q}{I} \]

where, \( y = \frac{150}{2} = 75 \text{ mm} \) and \( I = \frac{(150)^4}{12} \)

\[ \Rightarrow \sigma = \frac{16875 \times 10^6 \times 75}{(150)^4 / 12} = 3 \text{ MPa} \]

4.15 (b)

At extreme compression edge,
Bending stress,

\[ \sigma = \frac{My}{I} \]
\[
\sigma_{p1} = \frac{90 + 0}{2} = 45 \text{ MPa}
\]

Direct shear stress = 0

Principal stress,
\[
\sigma_{p1}/\sigma_{p2} = \frac{90 + 0}{2} \pm \frac{1}{2} \sqrt{(0 - 90)^2 + y(0)^2}
\]
\[
\sigma_{p1} = 90 \text{ MPa}
\]
So principal stress
\[
= 90 \text{ N/mm}^2 = 90 \text{ MPa}
\]

**4.17 Sol.**

Given: \( D = 500 \text{ mm} \)

Strain = \( \frac{\text{Stress}}{E} = \frac{f}{E} \)

\[
M = E = \frac{f}{R} = \frac{y}{y}
\]

\[
E = \frac{f}{R} = \frac{y}{y}
\]

\[
1 = \frac{f/E}{y} = \frac{\varepsilon}{y}
\]

\[
= 2.5 \times 10^{-4}
\]

\[
= 1 \times 10^{-6} = 0.001
\]

**4.18 (a)**

Shear flow,
\[
q = \frac{SAy}{l} = \frac{8000 \times 50 \times 100 \times 150}{1.5 \times 10^9} = 4 \text{ N/mm}
\]

Distance between two nails \( l = 60 \text{ mm} \)

:. S.F. resisted by each nail = \( q \times l = 240 \text{ N} \)

**4.19 (c)**

Shear flow distribution for channel.
5.1 A cantilever beam of span \( L \) is subjected to a downward load of 800 kN uniformly distributed over its length and a concentrated upward load \( P \), at its free end. For vertical displacement to be zero at the free end, the value of \( P \) is
(a) 300 kN  (b) 500 kN  (c) 800 kN  (d) 1000 kN

[1992 : 2 Marks]

5.2 In a real beam, at an end, the boundary condition of zero slope and zero vertical displacement exists. In the corresponding conjugate beam, the boundary conditions at this end will be
(a) shear force = 0 and bending moment = 0
(b) slope = 0 and vertical displacement = 0
(c) slope = 0 and bending moment = 0
(d) shear force = 0, and vertical displacement = 0

[1992 : 1 Mark]

5.3 A simply supported beam of span length \( L \) and flexural stiffness \( EI \) has another spring support at the centre of stiffness \( K \) as shown in figure. The central deflection of the beam due to a central concentrated load of \( P \) would be

\[
\frac{PL^3}{48EI + KL^3} \quad \text{(a)} \quad \frac{P}{48EI / L^3} - K \quad \text{(b)} \quad \left( \frac{PL^3}{48EI} \right) \left( \frac{P}{K} \right) \quad \text{(c)} \quad \frac{PL^3}{48EI / L^3} + K \quad \text{(d)}
\]

[1993 : 2 Marks]

5.4 A cantilever beam of span \( l \) subjected to uniformly distributed load \( w \) per unit length resting on a rigid prop at the tip of the cantilever. The magnitude of the reaction at the prop is
(a) \( \frac{1}{8}wl \)  (b) \( \frac{2}{8}wl \)
(c) \( \frac{3}{8}wl \)  (d) \( \frac{4}{8}wl \)

[1994 : 2 Marks]

5.5 The deflection of cantilever beam at free end \( B \) applied with a moment \( M \) at the same point is

\[
\frac{ML^2}{EI} \quad \text{(a)} \quad \frac{ML^2}{2EI} \quad \text{(b)} \quad \frac{ML^2}{3EI} \quad \text{(c)} \quad \frac{ML^2}{4EI} \quad \text{(d)}
\]

[1996 : 1 Mark]

5.6 \( M-\theta \) relationship for a simply supported beam shown below is given by

\[
\frac{MI}{EI} = 2\theta \quad \text{(a)} \quad \frac{MI}{EI} = 3\theta \quad \text{(b)} \quad \frac{MI}{EI} = 4\theta \quad \text{(c)} \quad \frac{MI}{EI} = 6\theta \quad \text{(d)}
\]

[1996 : 1 Mark]

5.7 A cantilever beam of span \( L \) is loaded with a concentrated load \( P \) at the free end. Deflection of the beam at the free end is

\[
\frac{PL^3}{48EI} \quad \text{(a)} \quad \frac{5PL^3}{384EI} \quad \text{(b)} \quad \frac{PL^3}{3EI} \quad \text{(c)} \quad \frac{PL^3}{6EI} \quad \text{(d)}
\]

[1997 : 1 Mark]
5.8 A cantilever beam is as shown in figure. The moment to be applied at free end for zero vertical deflection at that point is

![](image)

(a) 9 kNm clockwise  
(b) 9 kNm anti-clockwise  
(c) 12 kNm clockwise  
(d) 12 kNm anti-clockwise  

[1998 : 2 Marks]

5.9 The slope of the elastic curve at the free end of a cantilever beam of span \( L \), and with flexural rigidity \( EI \), subjected to uniformly distributed load of intensity \( w \) is

\[
\frac{wL^2}{6EI} \quad (a) \quad \frac{wL^3}{3EI} \quad (b) \quad \frac{wL^4}{8EI} \quad (c) \quad \frac{wL^3}{2EI} \quad (d)
\]

[1999 : 1 Mark]

5.10 A two span beam with an internal hinge is shown below

![](image)

The conjugate beam corresponding to this beam is

(a) \( a \quad \bullet \quad b \quad \bullet \quad c \quad \bullet \quad d \)  
(b) \( a \quad \bullet \quad b \quad \bullet \quad c \quad \bullet \quad d \)  
(c) \( a \quad \bullet \quad b \quad \bullet \quad c \quad \bullet \quad d \)  
(d) \( a \quad \bullet \quad b \quad \bullet \quad c \quad \bullet \quad d \)

[2000 : 1 Mark]

5.11 For the structure shown below, the vertical deflection at point A is given by

![](image)

(a) \( R L h^2 \)  
(b) \( R L h^2 \)  
(c) \( R L h^2 \)  
(d) \( R L h^2 \)

[2003 : 2 Marks]

5.12 In the propped cantilever beam carrying a uniformly distributed load of \( w \) N/m, shown in the following figure, the reaction at the support B is

\( \frac{PL^3}{81EI} \)  
\( \frac{2PL^3}{81EI} \)  
\( \frac{PL^3}{72EI} \)  
\( \text{Zero} \)

[2000 : 1 Mark]

5.13 A “H” shaped frame of uniform flexural rigidity \( EI \) is loaded as shown in the figure. The relative outward displacement between points K and O is

\( \frac{R L h^2}{E I} \)  
\( \frac{R L^2 h}{E I} \)  
\( \frac{R L h^2}{3E I} \)  
\( \frac{R L^2 h}{3E I} \)

[2003 : 2 Marks]
5.14 A simply supported beam of uniform rectangular cross-section of width b and depth h is subjected to linear temperature gradient, 0° at the top and T° at the bottom, as shown in the figure. The coefficient of linear expansion of the beam material is α. The resulting vertical deflection at the mid-span of the beam is

\[
\begin{align*}
\text{(a)} & \quad \frac{\alpha Th^2}{8L} \quad \text{upward} \\
\text{(b)} & \quad \frac{\alpha TL^2}{8h} \quad \text{upward} \\
\text{(c)} & \quad \frac{\alpha Th^2}{8L} \quad \text{downward} \\
\text{(d)} & \quad \frac{\alpha TL^2}{8h} \quad \text{downward}
\end{align*}
\]

5.15 For the linear elastic beam shown in the figure, the flexural rigidity, EI is 781250 kNm². When \( w = 10 \text{ kN/m} \), the vertical reaction \( R_A \) at A is 50 kN. The value of \( R_A \) for \( w = 100 \text{ kN/m} \) is

\[
\begin{align*}
\text{(a)} & \quad 500 \text{ kN} \\
\text{(b)} & \quad 425 \text{ kN} \\
\text{(c)} & \quad 250 \text{ kN} \\
\text{(d)} & \quad 75 \text{ kN}
\end{align*}
\]

5.16 Consider the beam \( AB \) shown in the figure below. Part AC of the beam is rigid while Part CB has the flexural rigidity \( EI \). Identify the correct combination of deflection at end B and bending moment at end A, respectively

- \( \frac{Pl^3}{3EI}, 2PL \)
- \( \frac{Pl^3}{3EI}, PL \)
- \( \frac{8Pl^3}{3EI}, 2PL \)
- \( \frac{8Pl^3}{3EI}, PL \)

5.17 The reaction at C is

\[
\begin{align*}
\text{(a)} & \quad \frac{9Pa}{16L} \quad \text{(upwards)} \\
\text{(b)} & \quad \frac{9Pa}{16L} \quad \text{(downwards)} \\
\text{(c)} & \quad \frac{9Pa}{8L} \quad \text{(upwards)} \\
\text{(d)} & \quad \frac{9Pa}{8L} \quad \text{(downwards)}
\end{align*}
\]

5.18 The rotation at B is

\[
\begin{align*}
\text{(a)} & \quad \frac{5PLa}{16EI} \quad \text{(clockwise)} \\
\text{(b)} & \quad \frac{5PLa}{16EI} \quad \text{(anticlockwise)} \\
\text{(c)} & \quad \frac{59PLa}{16EI} \quad \text{(clockwise)} \\
\text{(d)} & \quad \frac{59PLa}{16EI} \quad \text{(anticlockwise)}
\end{align*}
\]

5.19 Vertical reaction developed at B in the frame below due to the applied load of 100 kN (with 150,000 mm² cross-sectional area and 3.125 x 10⁻⁹ mm² moment of inertia for both members) is
5.20 The stepped cantilever is subjected to moments $M$ as shown in the figure below. The vertical deflection at the free end (neglecting the self weight) is

\[
\begin{align*}
\text{(a)} & \quad \frac{ML^2}{8EI} \\
\text{(b)} & \quad \frac{ML^2}{4EI} \\
\text{(c)} & \quad \frac{ML^2}{2EI} \\
\text{(d)} & \quad \text{Zero}
\end{align*}
\]

\[\text{[2008 : 2 Marks]}\]

Linked Answer Questions 5.21 and 5.22.

Beam $GHI$ is supported by three pontoons as shown in the figure below. The horizontal cross-sectional area of each pontoon is $8 \text{ m}^2$, the flexural rigidity of the beam is $10000 \text{ kN-m}^2$ and the unit weight of water is $10 \text{ kN/m}^3$.

5.21 When the middle pontoon is removed, the deflection at $H$ will be

\[
\begin{align*}
\text{(a)} & \quad 0.2 \text{ m} \\
\text{(b)} & \quad 0.4 \text{ m} \\
\text{(c)} & \quad 0.6 \text{ m} \\
\text{(d)} & \quad 0.8 \text{ m}
\end{align*}
\]

\[\text{[2008 : 2 Marks]}\]

5.22 When the middle pontoon is brought back to its position as shown in the figure above, the reaction at $H$ will be

\[
\begin{align*}
\text{(a)} & \quad 8.6 \text{ kN} \\
\text{(b)} & \quad 15.7 \text{ kN} \\
\text{(c)} & \quad 19.2 \text{ kN} \\
\text{(d)} & \quad 24.2 \text{ kN}
\end{align*}
\]

\[\text{[2008 : 2 Marks]}\]

Linked Answer Questions 5.23 and 5.24.

In the cantilever beam $PQR$ shown in figure below, the segment $PQ$ has flexural rigidity $EI$ and the segment $QR$ has infinite flexural rigidity.

5.23 The deflection and slope of the beam at $Q$ are respectively

\[
\begin{align*}
\text{(a)} & \quad \frac{5W^3}{6EI} \quad \text{and} \quad \frac{3W^2L}{2EI} \\
\text{(b)} & \quad \frac{W^3}{3EI} \quad \text{and} \quad \frac{W^2L}{2EI} \\
\text{(c)} & \quad \frac{W^3}{2EI} \quad \text{and} \quad \frac{WL^2}{E} \\
\text{(d)} & \quad \frac{W^3}{3EI} \quad \text{and} \quad \frac{3WL^2}{2EI}
\end{align*}
\]

\[\text{[2009 : 2 Marks]}\]

5.24 The deflection of the beam at $R$ is

\[
\begin{align*}
\text{(a)} & \quad \frac{8W^3}{EI} \\
\text{(b)} & \quad \frac{5W^3}{6EI} \\
\text{(c)} & \quad \frac{7W^3}{3EI} \\
\text{(d)} & \quad \frac{8W^3}{6EI}
\end{align*}
\]

\[\text{[2009 : 2 Marks]}\]

5.25 A simply supported beam is subjected to a uniformly distributed load of intensity $w$ per unit length, on half of the span from one end. The length of the span and the flexural stiffness are denoted as $L$ and $EI$ respectively. The deflection at mid-span of the beam is

\[
\begin{align*}
\text{(a)} & \quad \frac{5wL^4}{6144EI} \\
\text{(b)} & \quad \frac{5wL^4}{768EI} \\
\text{(c)} & \quad \frac{5wL^4}{384EI} \\
\text{(d)} & \quad \frac{5wL^4}{192EI}
\end{align*}
\]

\[\text{[2012 : 2 Marks]}\]

5.26 A uniform beam $(EI = \text{constant}) PQ$ in the form of a quarter circle of radius $R$ is fixed at end $P$ and free at the end $Q$, where a load $W$ is applied as shown. The vertical downward displacement $\delta_Q$ at the loaded point $Q$ is given by

\[
\delta_Q = \beta \left( \frac{wR^3}{EI} \right)
\]

Find the value of $\beta$ correct to 4-decimal place.

\[\text{[2013 : 2 Marks]}\]
5.27 For the cantilever beam of span 3 m (shown below), a concentrated load of 20 kN applied at the free end causes a vertical displacement of 2 mm at a section located at a distance of 1 m from the fixed end. If a concentrated vertically downward load of 10 kN is applied at the section located at a distance of 1 m from the fixed end (with no other load on the beam), the maximum vertical displacement in the same beam (in mm) is ________.

5.28 The axial load (in kN) in the member PQ for the arrangement/assembly shown in the figure given below is ________.

5.29 The tension (in kN) in a 10 m long cable, shown in the figure, neglecting its self-weight is ________.

5.30 The beam of an overall depth 250 mm (shown below) is used in a building subjected to two different thermal environments. The temperatures at the top and bottom surfaces of the beam are 36°C and 72°C respectively. Considering coefficient of thermal expansion (α) as 1.50 x 10⁻⁶ per °C, the vertical deflection of the beam (in mm) at its mid-span due to temperature gradient is ________.

5.31 A horizontal beam ABC is loaded as shown in the figure below. The distance of the point of contraflexure from end A (in m) is ________.

5.32 A steel strip of length, L = 200 mm is fixed at end A and rests at B on a vertical spring of stiffness, k = 2 N/mm. The steel strip is 5 mm wide and 10 mm thick. A vertical load, P = 50 N is applied at B, as shown in the figure. Considering E = 200 GPa, the force (in N) developed in the spring is ________.

5.33 Two beams are connected by a linear spring as shown in the following figure. For a load P as shown in the figure, the percentage of the applied load P carried by the spring is ________.

(a) 120 (b) 75 (c) 60 (d) 45
5.34 A 3 m long simply supported beam of uniform cross-section is subjected to a uniformly distributed load of \( w = 20 \text{ kN/m} \) in the central 1 m as shown in the figure.

\[
\begin{align*}
&\text{w} = 20 \text{ kN/m} \\
&P \quad 1 \text{ m} \quad 1 \text{ m} \quad 1 \text{ m} \\
&\text{EI} = 30 \times 10^8 \text{ Nm}^2
\end{align*}
\]

If the flexural rigidity (\( EI \)) of the beam is \( 30 \times 10^8 \text{ Nm}^2 \), the maximum slope (expressed in radians) of the deformed beam is

(a) \( 0.681 \times 10^{-7} \) \hspace{1cm} (b) \( 0.943 \times 10^{-7} \) \hspace{1cm} (c) \( 4.310 \times 10^{-7} \) \hspace{1cm} (d) \( 5.910 \times 10^{-7} \)

[2016 : 2 Marks, Set-I]

5.35 Two beams \( PQ \) (fixed at \( P \) with a roller support at \( Q \), as shown in Figure I, which allows vertical movement) and \( XZ \) (with a hinge at \( Y \)) are shown in the Figures I and II respectively. The spans of \( PQ \) and \( XZ \) are \( L \) and \( 2L \) respectively. Both the beams are under the action of uniformly distributed load (\( w \)) and have the same flexural stiffness, \( EI \) (where, \( E \) and \( I \) respectively denote modulus of elasticity and moment of inertia about axis of bending). Let the maximum deflection and maximum rotation be \( \delta_{\text{max}1} \) and \( \theta_{\text{max}1} \), respectively, in the case of beam \( PQ \) and the corresponding quantities for the beam \( XZ \) be \( \delta_{\text{max}2} \) and \( \theta_{\text{max}2} \), respectively.

Which one of the following relationship is true?

(a) \( \delta_{\text{max}1} \neq \delta_{\text{max}2} \) and \( \theta_{\text{max}1} \neq \theta_{\text{max}2} \)

(b) \( \delta_{\text{max}1} = \delta_{\text{max}2} \) and \( \theta_{\text{max}1} \neq \theta_{\text{max}2} \)

(c) \( \delta_{\text{max}1} \neq \delta_{\text{max}2} \) and \( \theta_{\text{max}1} = \theta_{\text{max}2} \)

(d) \( \delta_{\text{max}1} = \delta_{\text{max}2} \) and \( \theta_{\text{max}1} = \theta_{\text{max}2} \)

[2016 : 2 Marks, Set-I]

5.36 Two prismatic beams having the same flexural rigidity of 1000 kN-m² are shown in the figures.

\[
\begin{align*}
&\text{6 kN/m} \\
&\delta_1 \\
&4 \text{ m} \quad 120 \text{ kN} \\
&\delta_2 \\
&1 \text{ m} \quad 1 \text{ m}
\end{align*}
\]

If the mid-span deflections of these beams are denoted by \( \delta_1 \) and \( \delta_2 \) (as indicated in the figures), the correct option is

(a) \( \delta_1 = \delta_2 \) \hspace{1cm} (b) \( \delta_1 < \delta_2 \)

(c) \( \delta_1 > \delta_2 \) \hspace{1cm} (d) \( \delta_1 >>> \delta_2 \)

[2017 : 2 Marks, Set-II]

5.37 The figure shows a simply supported beam \( PQ \) of uniform flexural rigidity \( EI \) carrying two moments \( M \) and \( 2M \)

\[
\begin{align*}
&L/3 \\
&P \quad M \\
&L/3 \\
&2M \\
&Q
\end{align*}
\]

The slope at \( P \) will be

(a) \( 0 \) \hspace{1cm} (b) \( ML/(9EI) \)

(c) \( ML/(6EI) \) \hspace{1cm} (d) \( ML/(3EI) \)

[2018 : 2 Marks, Set-I]
**Answers**

<table>
<thead>
<tr>
<th>5.1 (a)</th>
<th>5.2 (a)</th>
<th>5.3 (a)</th>
<th>5.4 (c)</th>
<th>5.5 (b)</th>
<th>5.6 (a)</th>
<th>5.7 (c)</th>
<th>5.8 (c)</th>
<th>5.9 (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.10 (d)</td>
<td>5.11 (c)</td>
<td>5.12 (b)</td>
<td>5.13 (a)</td>
<td>5.14 (d)</td>
<td>5.15 (b)</td>
<td>5.16 (a)</td>
<td>5.17 (c)</td>
<td>5.18 (a)</td>
</tr>
<tr>
<td>5.19 (a)</td>
<td>5.20 (c)</td>
<td>5.21 (b)</td>
<td>5.22 (c)</td>
<td>5.23 (a)</td>
<td>5.24 (c)</td>
<td>5.25 (b)</td>
<td>5.29 (b)</td>
<td>5.34 (*)</td>
</tr>
<tr>
<td>5.35 (d)</td>
<td>5.36 (a)</td>
<td>5.37 (c)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Explanations**

### 5.1 (a)

For the vertical displacement to be zero at the free end, the vertical downward displacement due to UDL should be equal to upward displacement due to load $P$.

\[
\frac{wL^4}{8EI} = \frac{PL^3}{3EI}
\]

\[
\Rightarrow P = \frac{3}{8}wL = \frac{3}{8} \times 800 = 300 \text{ kN}
\]

### 5.2 (a)

The end conditions in a conjugate beam will be such that if a real beam at a support has non-zero slope or deflection, the conjugate beam will have non-zero SF or BM respectively.

- Slope $= 0$, $\text{SF} = 0$
- Deflection $= 0$, $\text{BM} = 0$

### 5.3 (a)

Let reaction due to the spring be $R$.

Now, by compatibility condition

\[
\frac{PL^3}{48EI} - \frac{RL^3}{48EI} = \Delta_{\text{spring}}
\]

### 5.4 (c)

Let $R$ be the prop-reaction at the tip of a cantilever beam.

\[
\Rightarrow \frac{wl^4}{8EI} - \frac{Rl^3}{3EI} = 0
\]

\[
\Rightarrow R = \frac{3}{8}wl
\]

### 5.5 (b)

From moment area theorem,

- Downward deflection of beam $AB$ at free and $B'$,

\[
\delta_y = A\bar{X} = \frac{MLL}{EI} \cdot \frac{L}{2} = \frac{ML^2}{2EI}
\]
\[ \theta_C - \theta_A = \text{Area of } M/EI \text{ diagram} \]

\[ 0 - \theta_A = -\frac{M}{EI} \times \frac{l}{2} = -\frac{Ml}{2EI} \]

\[ \therefore \quad \theta_A = \frac{Ml}{2EI} \text{ (anticlockwise)} \]

\[ \therefore \quad \frac{Ml}{EI} = 2\theta \]

5.7 (c)

\[ \Delta_{BA} = A\Delta = \frac{PL}{3EI} \quad \text{for moment area method} \]

\[ \Delta_{BA} = \frac{1}{2} \times L \times \frac{PL}{EI} \times \frac{2L}{3} = \frac{PL^3}{3EI} \]

5.8 (c)

\[ \delta_{AB} = \text{Moment of area of } \frac{M}{EI} \text{ diagram.} \]

5.9 (a)

\[ \theta_B - \theta_A = \frac{1}{3} \times L \times \frac{Wl^2}{2EI} = \frac{Wl^3}{6EI} \]

5.10 (d)

By moment area method

By superposition

(\downarrow) defl. due to load = (\uparrow) defl. due to moment

\[ \frac{P^3}{3EI} = \frac{Ml^2}{2EI} \]

\[ \frac{9 \times (2)^3}{3EI} = \frac{M \times (2)^2}{2EI} \]

\[ \frac{9 \times 8}{3 \times 2} = M \]

\[ \Rightarrow \quad 12 \text{kNm} = M \text{ (CW)} \]
For a fixed beam, deflection and slope are **zero at fixed end**, so in the conjugate beam bending moment and shear force should be zero. Hence fixed end becomes free end.

<table>
<thead>
<tr>
<th>Real beam</th>
<th>Conjugate beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed End</td>
<td>Free End</td>
</tr>
<tr>
<td>Internal Hinge</td>
<td>Internal Hinge Support</td>
</tr>
<tr>
<td>Internal Hinge Support</td>
<td>Internal Hinge</td>
</tr>
<tr>
<td>Hinge Support at Ends</td>
<td>Hinge Support at Ends</td>
</tr>
</tbody>
</table>

**5.11 (c)**

Consider free body diagram,

By superposition

Deflection at \( A \) = \[
\frac{PL^3}{3EI} - \frac{ML^2}{2EI}
\] = \[
\frac{P(3L)^3}{3EI} - \frac{2PL(3L)^2}{2EI}
\] = \[
\frac{P \times 27L^3}{3EI} - \frac{2PL9L^2}{2EI} = 0
\]

**5.12 (b)**

Deflection at propped end = 0

\[
\frac{w \times L^4}{8EI} - \frac{R_b \times L^3}{3EI} = 0
\]

\[
\Rightarrow R_b = \frac{3}{8} wL
\]

**5.13 (a)**

The bending moment in the member \( JN = R \times h \) (sagging)

\[
\therefore \text{Slope at } J \text{ or } N = \frac{R \times h \times L}{2EI}
\]

Thus, outward displacement between points

\[
K \text{ and } O = \frac{R \times h L}{2EI} + \frac{R \times h L}{2EI} \times h = \frac{R h^2 L}{EI}
\]
5.15 (b)

The deflection at the free end for,

\[ w(10 \text{kN/m}) = \frac{wl^4}{8EI} = \frac{10 \times (5)^4 \times 1000}{8 \times 781250} = 1 \text{ mm} \]

The gap between the beam and rigid platform is 6 mm. Hence, no reaction will be developed when \( w = 10 \text{kN/m} \).

Now, deflection at the free end for \( w(100 \text{kN/m}) \) will be \( 10 \times 1 \text{ mm} = 10 \text{ mm} \)

But, this cannot be possible because margin of deflection is only 6 m.

Thus \( w = 100 \text{kN/m} \) will induce a reaction \( R_B \) at \( B \).

\[ \frac{wl^4}{8EI} - \frac{R_B l^3}{3EI} = \text{Permissible deflection} \]

\[ \Rightarrow \frac{100 \times (5)^4}{8 \times 781250} - \frac{R_B \times (5)^3}{3 \times 781250} = \frac{6}{1000} \]

\[ \Rightarrow 10 - \frac{6}{1000} \times \frac{R_B \times 125}{3 \times 781250} \]

\[ \Rightarrow R_B = 75 \text{kN} \]

\[ \therefore R_A = (100 \times 5 - 75) \]

\[ = 425 \text{kN} \]

5.16 (a)

Part \( AC \) of the beam is rigid. Hence \( C \) will act as a fixed end. Thus the deflection at \( B \) will be given as \( \delta_B = \frac{PL^3}{3EI} \)

But the bending moment does not depend on the rigidity or flexibility of the beam.

\[ \therefore \text{BM at } A = P \times 2L = 2PL \]

5.17 (c)

The moment at point \( B = 2Pa \)

In the cantilever beam \( ABC \), the deflection at \( C \) due to moment \( 2Pa \) will be given as,

\[ \delta_c = \delta_B + \theta_B \times L \]

\[ \delta_c = \frac{ML^2}{2EI} + \frac{ML}{EI} \times L \]

\[ = \frac{2PaL^2}{2EI} + \frac{2Pal}{EI} \times L \]

\[ = \frac{3Pal^2}{EI} \] (downwards)

\[ \therefore \text{The reaction at } C \text{ will be upwards,} \]

\[ \delta_c = \frac{R(2L)^3}{3EI} \]

\[ = \frac{8Rl^3}{3EI} \] (upwards)

Thus,

\[ \delta_c = \frac{3Pal^2}{EI} \]

\[ = \frac{8Rl^3}{3EI} \]

\[ \Rightarrow \frac{3}{8} \times \frac{PL}{EI} \text{ (upwards)} \]

5.18 (a)

The rotation at \( B \):

(i) Due to moment,

\[ \theta_B1 = \frac{2Pa \times L}{EI} \] (clockwise)

(ii) Due to reaction \( R \),

\[ \theta_B2 = \frac{RI^2}{2EI} + \frac{RL^2}{EI} = \frac{3RI^2}{2EI} \]

\[ = \frac{27}{16} \times \frac{PaL}{EI} \] (anticlockwise)

\[ \therefore \theta_B = \theta_B1 - \theta_B2 \]

\[ = \left( 2 - \frac{27}{16} \right) \frac{PaL}{EI} \]

\[ = \frac{5}{16} \frac{PaL}{EI} \] (clockwise)
Deflection at $A$ in beam $AB = \text{Compression in column } AC$,

$$\frac{(100 - R)L^3}{3EI} = \frac{RL}{AE}$$

$$\Rightarrow \frac{(100 - R) \times (1 \times 1000)^3}{3 \times 3.125 \times 10^9} = \frac{R}{150000}$$

$$\Rightarrow 100 - R = 0.0625$$
$$\Rightarrow 100 = 94.1 \times R$$
$$\Rightarrow R = 94.1 \text{ kN}$$
$$\therefore V_B = 100 - R$$
$$= 100 - 94.1$$
$$= 5.9 \text{ kN}$$

Using **Moment Area Method**, we have

$$\text{Deflection at } B \text{ w.r.t. } A$$

$$= \text{Moment of area of } \frac{M}{EI} \text{ diagram between } A \text{ and } B \text{ about } B$$

$$= \frac{M}{EI} \times L \times \frac{L}{2} = \frac{ML^2}{2EI}$$

**5.21 (b)**

The reactions at the ends are zero as there are hinges to left of $G$ and right of $I$. Hence when the middle pontoon is removed, the beam $GHI$ acts as a simply supported beam.

The deflection at $H$ will be due to the load at $H$ as well as due to the downward settlement of pontoons at $G$ and $I$ in water. Since the loading is symmetrical, both the pontoons will be immersed to same height and produce equal reaction (24 kN). Let it be $\delta_1$.

$$\therefore \delta_1 \times \text{ area of cross-section of pontoon} \times \text{ unit weight of water} = 24$$

$$\Rightarrow \delta_1 \times 8 \times 10 = 24$$

$$\Rightarrow \delta_1 = 0.3 \text{ m}$$

Also, deflection at $H$ due to bending in beam,

$$\delta_2 = \frac{PL^3}{48EI}$$

$$= \frac{48 \times 10^3}{48 \times 10^4} = 0.1 \text{ m}$$

$$\therefore \text{ Final deflection at } H,$$

$$\Delta = \delta_1 + \delta_2 = 0.3 + 0.1 = 0.4 \text{ m}$$

**5.22 (c)**

Let the elastic deflection at $H$ be $\delta_2$.

$$\therefore \delta_2 = \frac{(P - R)L^3}{48EI} \quad \text{ ... (i)}$$
The reactions at G and I will be same, as the beam is symetrically loaded.

Let the reaction at each G and I be $Q$.

Using principle of buoyancy, we get,

\[ \delta_1 \times \text{area of cross-section of pontoon} \times \gamma_w = Q \]

\[ \Rightarrow \delta_1 \times 8 \times 10 = Q \]

\[ \Rightarrow \delta_1 = \frac{Q}{80} \quad \text{...(ii)} \]

Also, we have,

\[ Q + Q + R = P \]

\[ \Rightarrow 2Q + R = 48 \quad \text{...(iii)} \]

Also, $(\delta_1 + \delta_2) \times \text{area of cross-section of pontoon} \times \gamma_w = R$

\[ \Rightarrow (\delta_1 + \delta_2) \times 8 \times 10 = R \]

\[ \Rightarrow \delta_1 + \delta_2 = \frac{R}{80} \quad \text{[from (ii)]} \]

\[ \Rightarrow \frac{Q}{80} + \delta_2 = \frac{R}{80} \quad \text{[from (iii)]} \]

\[ \Rightarrow \delta_2 = \frac{2R - 48 + R}{160} \]

\[ \Rightarrow \delta_2 = \frac{3R - 48}{160} \quad \text{[from (i)]} \]

\[ \Rightarrow (48 - R) \times \frac{10^3}{48 \times 10^4} = \frac{3R - 48}{160} \]

\[ \Rightarrow R = 19.2 \text{ kN} \]

The given cantilever beam can be modified into a beam as shown below,

\[ \text{Deflection at } Q = \frac{Wl^3}{3EI} + \frac{WL \times l^2}{2EI} = \frac{2WL^3}{3EI} + \frac{3WL^3}{6EI} = \frac{5WL^3}{6EI} \]

\[ \text{Slope at } Q = \frac{WL^2}{2EI} + \frac{WL \times l}{EI} = \frac{WL^2}{2EI} + \frac{2WL^2}{2EI} = \frac{3WL^2}{2EI} \]

Since the portion QR of the beam is rigid, QR will remain straight.

\[ \text{Deflection of } R = \Delta_1 + \Delta_2 = \text{Deflection at } Q + \text{Slope at } Q \times L \]

\[ = \frac{5WL^3}{6EI} + \frac{3WL^2}{2EI} \times L \]

\[ = \frac{5WL^3 + 9WL^3}{6EI} = \frac{14WL^3}{6EI} = \frac{7WL^3}{3EI} \]

\[ \text{Method-I} \]

\[ \delta = \frac{5 \times (w/2) l^4}{384 \times EI} \]

\[ = \frac{5 \times (w/2) l^4}{384 \times EI} \]

\[ \delta = \frac{5}{768} \frac{w^4}{EI} \]

\[ \text{Method-II} \]
Total deflection,
\[ \frac{\delta}{2} + \frac{\delta}{2} = \frac{5}{384} \frac{WL^4}{EI} \]
\[ \frac{\delta}{2} = \frac{5}{768} \frac{WL^4}{EI} \]
\[ \frac{\delta}{2} = \frac{5}{768} \frac{WL^4}{EI} \]

\[ U = \int \frac{M^2 dx}{2EI} = \frac{\pi}{\pi/2} \int_0^{\pi/2} \frac{(WR \sin \theta)^2 R d\theta}{2EI} \]
\[ = \frac{p^2 R^3 \pi}{2EI} \left[ \frac{1 - \cos 2\theta}{2} \right]_0^{\pi/2} \]
\[ = \frac{p^2 R^3 \pi}{4EI} \left[ 2 \theta - \sin \theta \right]_0^{\pi/2} \]
\[ = \frac{p^2 R^3 \pi}{8EI} \]
\[ \therefore \delta_Q = \frac{\partial U}{\partial W} = \frac{2PR^3 \pi}{8EI} = \frac{PR^3 \pi}{4 EI} \]
\[ \therefore \delta_Q = \frac{\pi}{4} \left( \frac{WR^3}{EI} \right) \]
\[ \therefore \beta = \frac{\pi}{4} = 0.7854 \]

From Betti's law \( P_1 \times \Delta_{x_2} = P_2 \times \Delta_{z_1} \)
\[ \Rightarrow 10 \times 2 = 20 \times \Delta_{z_1} \]
\[ \Rightarrow \Delta_{z_1} = 1 \text{ mm} \]

Free body diagram,
For principle of superposition,

\[ \frac{160 \times 2^3}{3EI} + \frac{160 \times 2^3}{2EI} \times 2 \times \frac{V_QL^3}{3EI} = \frac{V_QL}{AE} \quad \text{...(l)} \]

Deflections due to axial forces will be very less as compared to bending forces.
So we can neglect the axial deformation.
\[ \therefore \text{From equation (i),} \]
\[ \frac{160 \times 2^3}{3EI} + \frac{160 \times 2^3}{2EI} \times 2 - \frac{V_Q4^3}{3EI} = 0 \]
\[ \Rightarrow \frac{160 \times 2^3}{3} + \frac{160 \times 2^2 \times 2}{2} = \frac{V_Q \times 4^3}{3} \]
\[ \Rightarrow \]
\[ V_Q = 50 \text{ kN} \]
Given, $\alpha = 1.50 \times 10^{-5}/\text{C}$

\[ \delta = \frac{\alpha T l^2}{8h} \]

\[ = \frac{1.50 \times 10^{-5} \times (72 - 36) \times 3^2}{8 \times (250 \times 10^{-3})} \]

\[ = 2.43 \text{ mm} \]

**Method-II**

\[ (2R - \delta) \delta = \frac{L^2}{4} \]

\[ \delta = \frac{L^2}{8R} \]

\[ \frac{R}{L + L\alpha \Delta T_1} = \frac{R + h}{L + L\alpha \Delta T_2} \]

\[ \frac{\alpha (\Delta T_2 - \Delta T_1)}{1 + \alpha \Delta T_1} = \frac{h}{R} \]

Since $\alpha \Delta T_1 \ll 1$

\[ R = \frac{h}{\alpha (\Delta T_2 - \Delta T_1)} \]

\[ \delta = \frac{2\alpha (\Delta T_2 - \Delta T_1)}{8h} \]

\[ = \frac{3^2 \times 1.5 \times 10^{-5} \times 36}{8 \times 250 \times 10^{-3}} \]

\[ = 2.43 \text{ mm} \]

**5.31 Sol.**

Reaction at $B$,

\[ \Delta \text{ due to } R_B = \frac{R_B l^3}{3EI} \]
\[ \Delta \text{ due to } M = \frac{M^2}{2EI} \]

\[ \Delta_B = 0 \text{ (Compatibility condition)} \]

\[ \begin{align*}
10 \times (0.75)^3 + 2.5 \times 0.75^2 \times R_B \times 0.75^3 &= 0 \\
9R_B &= 135 \\
64EI &= 64EI
\end{align*} \]

\[ \therefore \quad R_B = 15 \text{ kN} \]

BM at a distance \( x \) from free end,

\[ \text{BM}_x = 10 \times x - 15 \times (x - 0.25) = 0 \]

\[ 10x = 15x - 3.75 \]

\[ 5x = 3.75 \]

\[ \therefore \quad x = 0.75 \text{ m} \]

\[ \therefore \quad \text{From end A, distance is 0.25 m.} \]

\[ \Delta_{\text{spring}} = \Delta_B - \Delta_D \]

Compression of spring

\[ \frac{(P - R) L^3}{3EI} - \frac{RL^3}{3EI} = \frac{R}{K_s} \]

\[ \frac{(P - R) L^3}{3EI} = \frac{R \times (2L^3)}{3EI} \]

\[ (P - R) - R = 2R \]

\[ P = 4R \]

\[ R = \frac{P}{4} \]

\[ \% \text{ force carried by spring} = 25\% \]

Deflection of point B = Deflection of spring

\[ \Delta_B = \frac{(P - R) L^3}{3EI} = \frac{R}{K} \]

Where, \( R = \) Force in the spring,

\[ \Rightarrow \quad \frac{(50 - R) 200^3}{3 \times 200 \times 10^3 \times 5 \times 10^3 / 12} = \frac{R}{2} \]

Due to symmetrical loading,

\[ R_B = R_Q = \frac{w \times 1}{2} = \frac{w}{2} \]
\[ M_x = R_p x \left/ \frac{-w}{2}(x-1)^2 \right. \left/ + \frac{w}{2}(x-2)^2 \right. \]

According to the Macaulay method,

\[ EI \frac{d^2y}{dx^2} = R_p x \left/ \frac{-w}{2}(x-1)^2 \right. \left/ + \frac{w}{2}(x-2)^2 \right. \]

After integrating once

\[ EI \frac{dy}{dx} = \frac{R_p x^2}{2} + c_1 \left/ \frac{-w}{6}(x-1)^3 \right. \left/ + \frac{w}{6}(x-2)^3 \right. \]

Due to symmetrical loading slope will be zero at mid section \((x = 1.5 \text{ m})\),
\[
0 = \frac{R_p (1.5)^2}{2} + c_1 - \frac{w}{6} (1.5 - 1)^3
\]
\[
= \frac{9R_p}{8} + c_1 - \frac{9w}{48} + \frac{w}{16} - \frac{c_1 - w}{48}
\]
\[
\therefore c_1 = \frac{w}{48} = \frac{9w}{16} = -13\frac{w}{24}
\]
\[
\therefore \text{Equation of slope,}
\]

\[ EI \frac{dy}{dx} = \frac{R_p x^2}{2} - 13\frac{w}{24} \left/ \frac{-w}{6}(x-1)^3 \right. \left/ + \frac{w}{6}(x-2)^3 \right. \frac{1}{6} \]

The slope will be maximum at the support,
\[
\theta_{\max} = \frac{-13w}{24E} = \frac{-13 \times 20 \times 10^3}{24 \times 30 \times 10^6} = -0.361 \times 10^{-3} \text{ radians}
\]

**Method-II**

**Moment Area Method,**

\[ M_A = P \times \frac{1}{2} (x-1)^2 \quad (x = 1 \text{ m}) \]
\[ M_B = P \times \frac{1}{2} (x-1)^2 \quad (x = 1.5 \text{ m}) \]
\[
M_B = 10 \times 1.5 - \frac{20 \times 0.5^2}{2} = 12.5 \text{ kNm}
\]

**Deflection in beam xy at y = Deflection in beam yz at y,**
\[
\Rightarrow \frac{wL^4}{8EI} - \frac{RL^3}{3EI} = \frac{RL^3}{3EI} + \frac{wL^4}{8EI}
\]
\[
\therefore R = 0
\]

In beam PQ also at support Q, vertical reaction in zero because of roller support.

So, beam PQ, xy and yz are same.
\[
\therefore Q_{\max} = Q_{\max} \text{ and } \delta_{\max} = \delta_{\max}
\]

**5.35 (d)**

\[
\theta_A - \theta_C = \text{Area of } M/EI \text{ diagram between points } P \text{ and } B.
\]
\[
\theta_A = \frac{1}{2} \times 1 \times \frac{10}{EI} + \left( 0.5 \times \frac{10}{EI} \right) + \left( \frac{2}{3} \times 0.5 \times \frac{2.5}{EI} \right)
\]
\[
= \frac{5}{EI} + \frac{5}{EI} + \frac{0.833}{EI}
\]
\[
\theta_A = -\frac{10.833}{EI} = -\frac{10.833 \times 10^3}{30 \times 10^6}
\]
\[
= -0.3611 \times 10^{-3} \text{ radians}
\]

**5.36 (a)**

\[
\delta_1 = \frac{5}{384} \times \frac{wl^4}{EI}
\]
\[
= \frac{5}{384} \times \frac{6(4)^4}{1000} = 20 \times 10^{-3} \text{ m}
\]
\[
\delta_2 = \frac{Pl^3}{48EI} = \frac{120 \times 2^3}{48 \times 1000} = 20 \times 10^{-3} \text{ m}
\]
\[
\therefore \delta_1 = \delta_2
\]
5.37 (c)

**Method-I**

**Conjugate Beam**

Now \( R_1 + R_2 = \frac{1}{2} \times L \times \frac{M}{E} \frac{L}{6E} \)

\[ \Sigma M_Q = 0 \]

\[ R_1 = \frac{1}{2} \times \frac{L}{3} \times \frac{M}{E} \left( \frac{2L}{3} + \frac{L}{9} \right) + \frac{1}{2} \times \frac{L}{3} \times \frac{M}{E} \frac{L}{9} = 0 \]

\[ R_1 = \frac{7ML}{54EI} + \frac{4ML}{54EI} - \frac{ML}{27EI} = \frac{ML}{6EI} \]

\[ \therefore \text{ Slope at } P, \theta_P = SF_p = \frac{ML}{6EI} \]

**Method-II**

**Moment area method:**

\[ \delta_{QP} = \text{Deflection of point Q wrt to tangent at point P} \]

\[ \theta_Q = \frac{\delta_{QP}}{L} \]

\[ \delta_{QP} = \left( -\frac{1}{2} \times \frac{M}{E} \times \frac{L}{3} \left( \frac{2L}{3} + \frac{L}{9} \right) \right) + \left( -\frac{1}{2} \times \frac{M}{E} \times \frac{L}{3} \left( \frac{L}{3} + \frac{L}{9} \right) \right) + \left( \frac{1}{2} \times \frac{M}{E} \times \frac{L}{3} \left( \frac{2L}{9} \right) \right) \]

\[ = \frac{ML^2}{6EI} \]

\[ \theta_Q = \frac{\delta_{QP}}{L} = \frac{ML^2}{6EI} = \frac{ML}{6EI} \]