Civil Engineering

5 Years Previous Solved Papers
Semester-IV : B.E./B.Tech
Compulsory and Major Elective Subjects

AKTU
Dr. APJ Abdul Kalam Technical University
Formerly known as UPTU

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Preface

University exams are of an immense value to every B.E./B.Tech students. Knowing that these exams are going to give them a degree status of an Engineer, it becomes very essential to understand the exam to score high. The proper route for understanding of exam is through previous years’ question papers. Previous years’ questions help every student to apprehend:

- Pattern and level of difficulty of questions
- Nature of questions asked from every subject
- Percentage numerical and percentage theory weightage of each subject
- Frequently repeated questions

Apart from all these it helps in revision of studied subject as solving previous years’ questions will make a student revise the entire syllabus according to exam point of view. University exam question papers consist of proof of theorem/rule, solving previous years’ papers helps in understanding of how to write answers to score full marks. It not only helps in better preparation but also helps in overcoming fear and boosts confidence.

MADE EASY sensed the need of a book containing Dr. APJ Abdul Kalam Technical University (AKTU) SOLVED Previous Years’ Questions of all semesters to help undergraduate students score well in their university exam.

MADE EASY team has put in sincere efforts to solve question papers of last 5 years of every subject of Civil branch and came up with this ultimate preparation tool which has semesterwise solved papers. This book will surely contribute to time management in exam and to the overall understanding of subject.

This book of IV semester- Civil Engineering contains solved papers of both compulsory and major elective subjects. I would like to acknowledge efforts of entire MADE EASY team who worked hard to solve previous years’ papers with detailed and accurate explanations to each and every question and I hope this book will stand up to the expectations of aspirants and my desire to serve student fraternity by providing best study material and quality guidance will get accomplished.

B. Singh (Ex. IES)
CMD, MADE EASY Group
# Contents

**Dr. APJ Abdul Kalam Technical University**  
**Semester-IV : Previous 5 Years Solved Papers**

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There is no image provided for this question. Please provide an image of the question so I can assist you better.
Solution:

(a)

\[ \Sigma M_A = 0 \]
\[ \Rightarrow R_B(l) = M \]
\[ \Rightarrow R_B = M/l \]
\[ \Sigma F_y = 0 \]
\[ \Rightarrow R_A + R_B = 0 \]
\[ \Rightarrow R_A = -R_B = -\frac{M}{l} \]

\[ \therefore R_A \text{ will act downwards.} \]

Measuring ‘x’ from A towards right.

In span AC

\[ M_x = -R_A \cdot x = -\frac{M}{l} \cdot x \]

At C,

\[ x = \frac{l}{2} \]

\[ \therefore M_C = -\frac{M}{2} \]

In span CB

\[ M_x = -R_A \cdot x + M \]

At B, \( x = l \) \[ M_B = -\frac{M}{l} \cdot l + M = 0 \]

At C,

\[ x = \frac{l}{2} \]

\[ \therefore M_x = -\frac{M}{l} \cdot \frac{l}{2} + M = +\frac{M}{2} \]

(b) **Perfect truss:** A pin jointed truss is said to be perfect if it is composed of members which are just sufficient to keep the truss in equilibrium when it is being loaded. A perfect plane truss follows the equation:

\[ n = 2j - 3 \]

where \( n = \text{Number of members in the truss} \)

\( j = \text{Number of joints in the truss} \)

**Imperfect truss:** A pin jointed truss is said to be imperfect if it contains members which are either less or more than the number of members required for perfect truss. For a plane truss,

if \( n > 2j - 3 \) then truss is a redundant structure

if \( n < 2j - 3 \) then truss is deficient

(c) **Degree of freedom:** It is the total number of independent movements (both translation and rotation) that is possible at a joint in a structure.

A single isolated node has six degrees of freedom viz. three translations (in x, y and z directions) and three rotations (about x, y and z-directions).
(e) \[
\sum F_y = 0 \\
\Rightarrow R_B + R_C = 10 \times 2 \\
\quad = 20 \text{ kN} \\
\sum M_C = 0 \\
\Rightarrow R_B(2) = 10 \times 2 \times 1 \\
\Rightarrow R_B = 10 \text{ kN} \\
\Rightarrow R_C = 10 \text{ kN} \\
\]
Measuring \(x\) from \(B\) towards right,
\[
SF_x = R_B - 10x
\]
which is linear variation
When \(SF_x = 0\)
\[
\Rightarrow R_B - 10x = 0 \\
\Rightarrow x = 1 \text{ m}
\]

(f) Static indeterminacy = \(8 - 3 = 5\)

(g) **Note:** Figure not given in the question.

(h) \[
10 \text{ kN}
\]
\[
2 \text{ m} \quad 4 \text{ m}
\]

\[
\frac{Z(L - Z)}{Z} = \frac{2(6 - 2)}{6} = 1.33
\]

\[
\text{ILD for BM at } C
\]

\[
\therefore \text{BM at } C = 10 \times 1.33 = 13.3 \text{ kNm}
\]

(i) **Muller Breslau's principle:** This principle states that application of a function (reaction, shear force, bending moment etc.) deflects the beam in a shape which is same as the ILD of the given function at its point of application. In other words it states that "the ordinate value of an influence line for any function on a structure is proportional to the ordinates of the deflected shape that is obtained by removing the restraint corresponding to the function from the structure and introducing a force that causes unit displacement in the positive direction".
(j) **Arch:** An arch always remains in compression and thus it is capable of carrying much higher load than that of a beam.

**Beam:** In a beam subjected to vertical loading, the bottom portion of the beam below the neutral axis comes in tension and top portion remains in compression. Almost all construction materials (like concrete, brick, stone etc.) are weak in tension and thus a beam fails much earlier than an arch.

### SECTION-B

Note: Attempt any five parts

[5 x 6 = 30]

### Q.2 How are structure classified? Explain with example.

**Solution:**

Structures are classified as *determinate* and *indeterminate* structures.

**Determinate structure:** In a determinate structure, all the reaction components can be determined by using the equations of equilibrium alone.

![Determinate Structure Diagram](image)

Here all the reaction components i.e. \( V_A, H_A, V_C, H_C \) can be determined from equilibrium equations alone viz.

\[
\sum F_x = 0 \\
\sum F_y = 0 \\
\sum M = 0
\]

**Indeterminate structure:** Here all the reaction components of the structure cannot be determined by equilibrium equations alone and additional compatibility equations are required.

![Indeterminate Structure Diagram](image)

Here, all the reaction components \( (V_A, H_A, M_A, V_B, H_B, M_B) \) cannot be determined by equilibrium equations alone and some additional equations are required.

### Q.3 What is principle of superposition? Explain with example.

**Solution:**

**Principle of superposition:** In Mechanics, principle of superposition states that in a linear system, the total deflection due a set of loads is equal to the sum of deflections caused due to loads acting individually.

**For example:** Consider a cantilever beam of span \( L \) and carrying a uniformly distributed load \( w \) per unit length throughout the span. It is also subjected to a point load \( P \) at the free end.
From principle of superposition, total deflection at B i.e.,
\[ \Delta_B = \Delta_{B1} + \Delta_{B2} = \frac{wL^4}{8EI} + \frac{PL^3}{3EI} \]
where \( EI \) = Flexural rigidity of beam

**Q.4** Find member forces in following truss:

**Solution:**

\[ BD = 2\sqrt{5} \text{ m} \]
\[ DC = \frac{DE}{\cos \theta} = \frac{2}{2\sqrt{5}} = \frac{\sqrt{5}}{2} \text{ m} \]
\[ H_B = 10 \text{ kN} \]

Now
\[ R_A + R_B = 20 \text{ kN} \]

Taking moments about \( B \), \( \Sigma M_B = R_A(2) + 10(4) - 20(2) = 0 \)

\[ R_A = \frac{20 \times 2 - 10 \times 4}{2} = 0 \]

\[ R_B = 20 \text{ kN}(\uparrow) \]

Joint D
\[ P_{DC} \sin \theta = 10 \]
\[ P_{DC} = 10\sqrt{5} \text{ kN (C)} \]
\[ P_{DC} \cos \theta + P_{DE} = 20 \]
\[ P_{DE} = 20 - \frac{10\sqrt{5}}{\sqrt{5}} = 0 \]

Joint E
\[ P_{EC} = P_{EA} = 0 \]

Joint B
\[ \Sigma V = 0 \]
\[ P_{BC} \cos \theta = R_B \]
\[ P_{BC} = \frac{20}{\cos \theta} = \frac{20}{2} \sqrt{5} = 10\sqrt{5} \text{ kN (C)} \]
\[ \Sigma H = 0 \]
\[ P_{BC} \cos (90^\circ - \theta) + P_{BA} = H_B \]
\[ 10\sqrt{5} \cdot \sin \theta + P_{BA} = 10 \]
\[ 10\sqrt{5} \cdot \frac{1}{\sqrt{5}} + P_{BA} = 10 \]
\[ P_{BA} = 0 \]

Joint C
\[ P_{CD} = P_{BC} \text{ and } P_{EC} = 0 \]
\[ P_{CA} = 0 \]

Q.5 State and explain Castigliano's II\textsuperscript{nd} theorem.

Solution:

**Castigliano's II theorem:** It states that the partial derivative of total strain energy stored in a system (i.e. structure) with respect to a force acting at a point is equal to the deflection of the structure at the point of application of force in the direction of applied force.

Let there be a simply supported beam subjected to a system of forces as shown.

\[ \therefore \text{Strain energy stored in the system is equal to external work done,} \]
\[ \therefore W_{\text{ext}} = \int (P_1, P_2, ..., P_K, ..., P_{n-1}, P_n) = U \]

Now any of these loads say \( P_K \) is increased by a differential amount \( dP_K \). The strain energy of the system will change by \( \left( \frac{\partial U}{\partial P_K} \right) \cdot dP_K \). Thus total strain energy becomes,
\[ U_i = U + \sum_{j=1}^{N} F_j d \Delta_j + \frac{1}{2} dP_k d \Delta_k \]

Neglect \( \frac{1}{2} dP_k d \Delta_k \) being very small.

\[ U_i = U + \sum_{j=1}^{N} F_j d \Delta_j \]

Now the system of loading is reversed i.e. \( dP_k \) is applied first and then the system of loads \( f(p) \).

\[ W_{ext} = U_i = U + \left( \frac{1}{2} dP_k d \Delta_k \right) + dP_k \Delta_k \]

Neglect \( \frac{1}{2} dP_k d \Delta_k \) being very small.

\[ U_i = U + dP_k \Delta_k \]

But total strain energy stored in both the system must be equal,

\[ U + \frac{\partial U}{\partial P_k} dP_k = U + dP_k \Delta_k \]

\[ \frac{\partial U}{\partial P_k} = \Delta_k \]

\[ \Delta_k = \frac{\partial U}{\partial P_k} = \frac{\partial}{\partial P_k} (W_{ext}) \]

Extending the above result, we have

\[ \theta_k = \frac{\partial}{\partial M_k} (W_{ext}) \]

Thus for a linearly elastic structures, the partial derivative of total strain energy w.r.t. a load \( P_k \) gives deflection in its direction. (Castigliano's II theorem).

**Q.6** Explain method of substitute members for analysis of trusses with suitable example.

**Solution:**

**Method of substitute members for analysis of trusses:** The method of substitute members is used for analysis of complex trusses.

**Complex truss:** Complex trusses are those trusses which are although statically determinate but has set of simultaneous equations which cannot be easily solved.

**Summary of method of substitute members:** Let there be a truss as shown figure.

Number of member (m) = 9

Number of joints (j) = 6

\[ 2j - 3 = 2 \times 6 - 3 = 9 = m \]

Thus \( m = 2j - 3 \) and so truss is statically determinate.

But this truss cannot be solved easily either by method of joints or by method of sections.
• The given truss is reduced as shown below.

\[ \text{Figure I} \quad \text{Figure II} \quad \text{Figure III} \]

• In figure II, the member AD is removed and is substituted as member CE in figure III.

• The truss shown in figure II can now be solved starting with joint A. The forces in members of truss shown in figure II are denoted by \( F' \) (say).

• In figure III, the external force (P) has been removed and a unit load is applied along removed member AD.

Let the resulting forces in members are \( f \) (say).

• But actually there is no force in member EC as it is not there in the original truss.

\[ F_{EC} = F'_{EC} + x f_{EC} = 0 \]

where \( x \) is the factor which will make net force in member EC as zero.

**Procedure of method of substitute members:**

• Determine the reactions at the supports.

• Start from the joint where there are only two members similar to analysis of trusses by method of joints after determining the support reactions.

• At a joint where there are three unknown, remove one of the members from the joint and replace it by an imaginary member elsewhere in the truss.

• Apply load on the so formed simple truss and determine the forces in each member.

• Now remove the external loads and apply equal and opposite collinear unit loads on the truss at the two joints from which the member was removed.

• Determine the force in each member due to the unit loads.

• Superimpose the two member forces with the factor ‘\( x \)’.

**Q.7** Draw SF diagram for followign beam:

\[ \text{Beam Diagram} \]

6 kN

3 kN/m

2 m

4 m

A

C

B
Solution:

\[ \Sigma F_y = 0 \]

\[ R_A + R_B = 6 + 3 \times 4 = 18 \text{kN} \]

\[ \Sigma M_B = 0 \]

\[ R_A(6) = 6(4) + 3 \times 4 \times 2 \]

\[ R_A = 8 \text{kN} \]

\[ R_B = 18 - 8 = 10 \text{kN} \]

Measuring 'x' from A towards right.

\[ S_{F_x}(\text{in span } CB) = R_A - 6 - 3(x - 2) \]

\[ = 8 - 6 - 3(x - 2) = 8 - 3x \]

when \[ S_{F_x} = 0 \]

\[ 8 - 3x = 0 \]

\[ x = \frac{8}{3} \text{m} \]

\[ \therefore SF = 0 \text{ at } x = \frac{8}{3} \text{m from A.} \]

At C, \( x = 2 \text{ m} \)

\[ S_{F_C} = 8 - 3(2) = 2 \text{kN} \]

Q.8 Explain Linear Arch and prove Eddy’s Theorem.

Solution:

Linear arch is an imaginary structure which is obtained by a funicular polygon for a given arch which has all joints like a truss and loaded at joints only. All the members of theoretical arch are subjected to axial compressive forces only and there is no shear force and bending moment in any member of theoretical arch.

Imagine a structure ACDEB consisting of members AC, CD, DE and EB, which are pin jointed like a truss, having shape of funicular polygon and loaded with \( W_1, W_2 \) and \( W_3 \) at joints. Such a structure is called as theoretical arch.

![Linear Arch Diagram](image)

**Eddy’s Theorem**: If a linear arch is superimposed on a given arch then bending moment at any section on given arch is proportional to the ordinate of intercept between given arch and theoretical arch.

\[ M_x \propto p \]

where \( p = \) intercept between given arch and theoretical arch.
Q.9 The equation of a three hinged parabolic arch with origin at its left hand support is \( y = x - \frac{x^2}{40} \). The span of arch is 40 m. Find normal thrust, radial shear and BM at a section 5 m from left hand if the arch is loaded with a udl of 30 kN/m upon its left half of the span only.

Solution:

\[ y = x - \frac{x^2}{40} \]

\[ x_p = 5 \text{ m} \quad \therefore \quad y_p = 5 - \frac{5^2}{40} = 4.375 \text{ m} \]

Central rise of arch is given by putting \( x = 20 \) m in equation of parabolic arch.

\[ \therefore \quad y_c = 20 - \frac{20^2}{40} = 10 \text{ m} \]

Now

\[ \Sigma F_y = 0 \]

\[ \Rightarrow \quad V_A + V_B = 30 \times 20 = 600 \text{ kN} \]

\[ \therefore \quad \Sigma M_B = 0 \]

\[ \Rightarrow \quad V_A(40) = 30 \times 20 \times 30 \]

\[ \Rightarrow \quad V_A = 450 \text{ kN} \]

\[ \therefore \quad V_B = 600 - V_A = 150 \text{ kN} \]

\[ \Sigma M_A = 0 \quad \text{(from right)} \]

\[ \Rightarrow \quad V(20) = H_B(V_c) \]

\[ \Rightarrow \quad 150(20) = H_B(10) \Rightarrow H_B = 300 \text{ kN} \]

\[ \therefore \quad H_A = H_B = 300 \text{ kN} \]

Now

\[ y = x - \frac{x^2}{40} \]

\[ \therefore \quad \frac{dy}{dx} = \left(1 - \frac{x}{20}\right) \]

\[ \tan \theta = \left( \frac{dy}{dx} \right)_{x=5} = 1 - \frac{5}{20} = \frac{3}{4} \]

Normal thrust \((N) = V \sin \theta + H \cos \theta \)
\[ = 300 \left( \frac{3}{5} \right) + 300 \left( \frac{4}{5} \right) = 420 \text{kN} \]

Radial shear \( (Q) = V \cos \theta - H \sin \theta \)

\[ = 300 \left( \frac{4}{5} \right) - 300 \left( \frac{3}{5} \right) = 60 \text{kN} \]

Bending moment at \( P (M_p) = V_a(5) - H(y_p) - 30 \times 5 \times \frac{5}{2} \)

\[ = 450(5) - 300(4.375) - 30 \times \frac{25}{2} \]

\[ = 2250 - 1312.5 - 375 = 562.5 \text{kNm} \]

**Q.10** Using unit load method, find horizontal deflection at the end \( E \) of following frame. Take \( EI \) as constant.

Solution:

\[
\begin{array}{c}
\text{Segment} \quad | \quad \text{ED} \quad | \quad \text{DC} \quad | \quad \text{CB} \quad | \quad \text{BA} \\
\text{Origin} \quad | \quad E \quad | \quad D \quad | \quad C \quad | \quad B \\
\text{Limits} \quad | \quad 0 \text{ to } 1.5 \text{ m} \quad | \quad 0 \text{ to } 1.5 \text{ m} \quad | \quad 0 \text{ to } 2 \text{ m} \quad | \quad 0 \text{ to } 2 \text{ m} \\
\text{M (kNm)} \quad | \quad 0 \quad | \quad -20x \quad | \quad -20(1.5) = -30 \quad | \quad -20(1.5) - 10x = -30 - 10x \\
\text{m} \quad | \quad 0 \quad | \quad 0 \quad | \quad -x \quad | \quad -(x + 2) \\
\end{array}
\]

Let \( \Delta_{EH} = \text{Horizontal deflection of point } E. \)

\[ EI \Delta_{EH} = \int Mmdx \]

\[ = \int_{ED} Mmdx + \int_{DC} Mmdx + \int_{CB} Mmdx + \int_{BA} Mmdx \]
\[
\begin{align*}
&= 0 + 0 + \int_0^2 (-30)(-x) \, dx + \int_0^2 (-30 - 10x) - (x - 2) \, dx \\
&= 15 \left[ x^2 \right]_0^2 + \int_0^2 (50x + 60 + 10x^2) \, dx \\
&= 15(4) + 25(4) + 60(2) + \frac{10}{3} (8) \\
&= 60 + 100 + 120 + \frac{80}{3} \\
&= \frac{920}{3} = 306.67
\end{align*}
\]

\[\therefore \quad \Delta_{ef} = \frac{306.67}{EI} \text{ where } EI \text{ is in kNm}^2.\]

**Q.11** Find deflection at free end of following beam using conjugate beam method. Take EI as constant.

![Diagram of beam with loads and reactions](Image)

**Solution:**

![Diagram of conjugate beam with reactions and bending moments](Image)

\[
\begin{align*}
R_A + R_B &= 3(5) = 15 \text{ kN} \\
\sum M_A &= 0 \\
\Rightarrow \quad &R_B (4) = 3 \times 5 \times \frac{5}{2} \quad \Rightarrow \quad R_B = 9.375 \text{ kN} \\
\therefore \quad &R_A = 15 - R_B = 15 - 9.375 = 5.625 \text{ kN}
\end{align*}
\]

**Bending Moment**

**Span AB**

\[
M = R_A \cdot x - 3 \cdot \frac{x^3}{2} = 5.625x - 1.5x^2
\]

For maximum bending moment, \(\frac{dM}{dx} = 0\)

\[
\Rightarrow \quad 5.625 - 3x = 0 \quad \Rightarrow \quad x = 1.875 \text{ m from A.}
\]
\[ M_{\text{max}} = 5.625(1.875) - 1.5(1.875)^2 = 5.27 \text{ kN}\cdot\text{m} \]

Also when \( M = 0 \)

\[ 5.625 m - 1.5 x^2 = 0 \]

\[ x = 3.75 \text{ m from } A. \]

At \( B, M_B = 5.625 (4) - 1.5(4)^2 = -1.5 \text{ kN}\cdot\text{m} \)

Deflection at \( C \) = Moment at \( C \) of conjugate beam

\[
\begin{align*}
&= \int_{\frac{D}{A}} y \cdot dx(5 - x) - \int_{\frac{B}{D}} ydx(5 - x) - \int_{\frac{C}{B}} 3 \frac{(5 - x)^2}{EI} dx(5 - x) \\
&= \int_{0}^{3.75} \frac{(5.625x - 1.5x^2)(5 - x)}{EI} dx - \int_{3.75}^{4} \frac{(5.625x - 1.5x^2)(5 - x)}{EI} dx - \frac{3}{2} \int_{4}^{5} (5 - x)^2 dx \\
&= \int_{0}^{3.75} \left( \frac{28.125x - 13.125x^2 + 1.5x^3}{EI} \right) dx - \int_{3.75}^{4} \left( \frac{28.125x - 13.125x^2 + 1.5x^3}{EI} \right) dx + \frac{3}{4} \int_{4}^{5} \left( (5 - x)^4 \right) dx \\
&= \left[ \frac{28.125}{2}(3.75^2) - \frac{13.125}{3}(3.75^3) + \frac{1.5}{4}(3.75^4) \right] \frac{1}{EI} - \left[ \frac{28.125}{2}(4^2 - 3.75^2) \\
&- \frac{13.125}{3}(4^3 - 3.75^3) + \frac{1.5}{4}(4^4 - 3.75^4) \right] \frac{1}{8EI} - \frac{3}{8EI} \right] \\
&= \left[ 197.754 - 230.713 + 74.158 \right] \frac{1}{EI} \frac{1}{EI} \left[ 27.246 - 49.287 + 21.842 \right] - \frac{3}{8EI} \\
&= \frac{241.2}{EI} - \frac{3}{8EI} = - \frac{238.2}{8EI} = - \frac{40.25}{EI}
\end{align*}
\]

**Q.12** Find max. BM developed anywhere on the girder of span 20 m due to rolling loads of 250 kN and 150 kN spaced 6 m apart with 150 kN load as leading load while rolling from left to right. Also find equivalent UDL to give same BM.

**Solution:**

![Diagram](image)

Maximum BM occurs at mid-span.

Let 150 kN load is at mid-span.

\[ \begin{align*}
\Sigma M_B &= 0 \\
\Rightarrow \quad R_A(20) &= 150(10) + 250(16) \\
\Rightarrow \quad R_A &= 275 \text{ kN} \\
\therefore \quad R_B &= (250 + 150) - R_A = 400 - 275 = 125 \text{ kN} \\
\therefore \quad M_C &= R_B(10) = 125(10) = 1250 \text{ kNm}
\end{align*} \]
Let 250 kN load is at mid-span.

\[ \sum M_B = 0 \]
\[ \Rightarrow \quad R_A \times 20 = 150 \times 4 + 250 \times 10 \]
\[ \Rightarrow \quad R_A = 155 \text{ kN} \]
\[ \therefore \quad M_C = R_A \times 10 = 155 \times 10 = 1550 \text{ kNm} \]
\[ : \quad \text{Absolute maximum BM} = 1550 \text{ kNm} \]

Let \( w = \) Equivalent UDL to give the same BM

\[ : \quad M = \frac{wL^2}{8} \]
\[ \Rightarrow \quad 1550 = \frac{w(20)^2}{8} \]
\[ \Rightarrow \quad w = 31 \text{ kN/m} \]

**Q.13** Prove that when a series of point loads rolls upon a simply supported girder, then for max. BM to occur under a chosen wheel load the span must equally divide the distance between the chosen wheel load and the resultant of all loads on the span.

**Solution:**

Let there be a system of \( n \) point loads \( W_1, W_2, \ldots, W_n \) moving along the span AB from left to right.

It is required to find the maximum bending moment under the load \( W_i \).

Let \( W_R \) and \( W'_R \) = Resultant of all the point loads on the span and those on the left of \( W_i \) located at \( x_R \) and \( x'_R \) from \( W_i \) respectively.

BM at \( C \),

\[ M_C = R_Ax - W'R'x'R = \frac{W_R}{L}x(L-x+x_R) - W'R'x'_R \]
For maximum BM at C,

\[
\frac{dM_c}{dx} = 0 = \frac{W_R}{L} \frac{d}{dx} \left[ x(L + x_R) - x^2 \right] - \frac{d}{dx} (W'R'x'R')
\]

\[
(L + x_R) - 2x = 0
\]

\[
L - x = x - x_R
\]

Here \((L - x)\) = Distance of point C from support B.

\((x - x_R)\) = Distance of CG of loads from support A.

**Q.14** Develop SFD and BMD of following beam:

![Beam Diagram]

**Solution:**

![SFD and BMD Diagram]

\[
\Sigma F_y = 0
\]

\[
R_A + R_B = \frac{1}{2} (4)(6) + 10 + 3(2 + 2) = 34 \text{ kN} \quad \text{...(i)}
\]

Let \( \bar{x} \) = Distance of centroid of load on position AC from C.

\[
\bar{x} = \frac{AC}{3} = \frac{4 \text{ m}}{3}
\]

\[
\Sigma M_B = 0
\]

\[
R_A(8) = \frac{1}{2} (4)(6)(4 + \bar{x}) + 10(2) + 3(4)(2)
\]
\[ 8R_A = 12\left(4 + \frac{4}{3}\right) + 20 + 24 \]

\[ R_A = 13.5 \text{ kN} \]

\[ R_B = 34 - R_A = 34 - 13.5 = 20.5 \text{ kN} \]

In span AC

\[ SF_x = R_A - \frac{1}{2}x\left(\frac{3}{2}x\right) = 13.5 - \frac{3}{4}x^2 \]

(Parabolic variation)

At \( C, x = 4 \text{ m}, SF_C = 13.5 - \frac{3}{4}(4)^2 = 13.5 - 3(4) = 1.5 \text{ kN} \)

**Q.15** Find forces in the members, \( FE, BE \) and \( BC \) of the following truss.

**Solution:**

\[ \sin \theta = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} \]

\[ \cos \theta = \frac{1}{\sqrt{2}} \]

\[ \cos \phi = \frac{2}{\sqrt{13}} \]

\[ \sin \phi = \frac{3}{\sqrt{13}} \]

\[ R_A + RD = 35 + 50 = 85 \text{ kN} \]

Taking moments about \( D, \) \[ \Sigma M_D = R_A(6) + 35(4) - 50(2) = 0 \]
\[ R_A = \frac{35 \times 4 + 50 \times 2}{6} = 40 \text{ kN} \]

\[ \therefore R_D = 85 - R_A = 85 - 40 = 45 \text{ kN} \]

**Joint A:**

\[ P_{AB} \sin \theta = R_A \]

\[ P_{AB} = 40\sqrt{2} \text{ kN (T)} \]

\[ P_{AB} \cos \theta = P_{AF} \]

\[ P_{AF} = 40\sqrt{2} \cdot \frac{1}{\sqrt{2}} = 40 \text{ kN (T)} \]

**Joint F:**

\[ P_{FE} = P_{FA} = 40 \text{ kN (T)} \]

\[ P_{FB} = 35 \text{ kN (T)} \]

**Joint B:**

\[ \Sigma H = 0 \]

\[ P_{BA} \cos \theta - P_{BC} \cos \theta - P_{BE} \cos \theta = 0 \]

\[ \Rightarrow 40\sqrt{2} = P_{BC} + P_{BE} \]...

\[ \Sigma V = 0 \]

\[ P_{AB} \sin \theta + P_{BE} \sin \theta = P_{BC} \sin \theta + P_{BF} \]

\[ 40\sqrt{2} \cdot \frac{1}{\sqrt{2}} + \frac{P_{BE}}{\sqrt{2}} = P_{BC} + 35 \]

\[ P_{BC} - P_{BE} = 5\sqrt{2} \]...

From (i) and (ii)

\[ P_{BC} = \frac{45\sqrt{2}}{2} \text{ kN (C)} \]

\[ P_{BE} = \frac{35\sqrt{2}}{2} \text{ kN (C)} \]

**Joint C:**

\[ \Sigma H = 0 \]

\[ P_{CB} \cdot \cos \theta = P_{CD} \cdot \cos \phi \]

\[ \Rightarrow P_{CD} = \frac{45\sqrt{2}}{2} \cdot \frac{1}{\sqrt{2}} = \frac{45\sqrt{13}}{2 \times 2 \times \sqrt{2}} = \frac{45\sqrt{13}}{4} \text{ kN (C)} \]

\[ \Sigma V = 0 \]

\[ P_{CB} \sin \theta + P_{CD} \sin \phi = P_{DE} \]

\[ \Rightarrow P_{DE} = \frac{45\sqrt{2}}{2} \cdot \frac{1}{\sqrt{2}} + \frac{45\sqrt{13}}{4} \cdot \frac{3}{\sqrt{13}} \]

\[ \Rightarrow P_{DE} = \frac{45}{2} + \frac{135}{4} \cdot \frac{3}{\sqrt{13}} = \frac{225}{4} \text{ kN (T)} \]

**Joint D:**

\[ \Sigma H = 0 \]

\[ P_{DE} = P_{DC} \cdot \cos \phi = \frac{45\sqrt{13}}{4} \cdot \frac{2}{\sqrt{13}} = \frac{45}{2} \text{ kN (T)} \]

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