Indian Forest Service Main Examination
(2000-2018)

Civil Engineering
Paper-I

Also useful for Engineering Services Main Examination, Civil Services Main Examination and various State Engineering Services Examinations
Preface

Our country has a vast forest cover of near about 25% of geographical area and if man doesn’t learn to treat trees with respect, man will become extinct; Death of forest is end of our life. Scientific management and judicial exploitation of forest becomes first task for sustainable development.

Engineer is one such profession which has an inbuilt word “Engineer – skillful arrangement” and hence IFS is one of the most talked about jobs among engineers to contribute their knowledge and skills for the arrangement and management for sustainable development.

In order to reach to the estimable position of Divisional Forest Officer (DFO), one needs to take an arduous journey of Indian Forest Service Examination. Focused approach and strong determination are the pre-requisites for this journey. Besides this, a good book also comes in the list of essential commodity of this odyssey.

I feel extremely glad to launch the revised edition of such a book which will not only make Indian Forest Service Examination plain sailing, but also with 100% clarity in concepts.

MADE EASY team has prepared this book with utmost care and thorough study of all previous years’ papers of Indian Forest Service Examination. The book aims to provide complete solution to all previous years’ questions with accuracy.

On doing a detailed analysis of previous years’ Indian Forest Service Examination question papers, it came to light that a good percentage of questions have been asked in Engineering Services, Indian Forest Services and State Services exams. Hence, this book is a one stop shop for all Indian Forest Service Examination, CSE, ESE and other competitive exam aspirants.

I would like to acknowledge efforts of entire MADE EASY team who worked day and night to solve previous years’ papers in a limited time frame and I hope this book will prove to be an essential tool to succeed in competitive exams and my desire to serve student fraternity by providing best study material and quality guidance will get accomplished.

With Best Wishes

B. Singh
CMD, MADE EASY Group
# Previous Years Solved Papers

## Indian Forest Service Main Examination

### Civil Engineering

**Paper-I**

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**Strength of Materials**

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2. Shear Force and Bending Moment
3. Bending Stress and Shear Stress
4. Torsion
5. Principal Stress and Principal Strain
6. Deflection of Beams
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**Fluid Mechanics, Open Channel Flow and Hydraulic Machines**

1. Fluid Properties and Pressure Measurement
2. Fluid Statics
3. Fluid Kinematic
4. Fluid Dynamics
5. Pipe Flow
6. Dimensional Analysis
7. Laminar Flow, Turbulent Flow & Boundary Layer
8. Open Channel Flow
9. Hydraulic Machines

**Structural Analysis**

1. Determinacy and Indeterminacy
2. Influence Line Diagram and Rolling Loads
3. Arches and Suspension Bridges
4. Methods of Structural Analysis
5. Trusses
6. Matrix Method of Structural Analysis

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**Design of Steel Structures**

1. Structural Fasteners
2. Tension Member
3. Compression Member
4. Beams
5. Plate Girders and Industrial Roofs
6. Plastic Analysis
7. Miscellaneous

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**Design of Concrete and Masonry Structures**

1. Beams
2. Slabs
3. Columns

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**Geo Technical Engineering**

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2. Capillary, Permeability and Effective Stress
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4. Shear Strength of Soil
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Part-A
ENGINEERING MECHANICS, STRENGTH OF MATERIALS AND STRUCTURAL ANALYSIS

ENGINEERING MECHANICS :
Units and Dimensions, SI Units, Vectors, Concept of Force, Concept of particle and rigid body. Concurrent, Non-Concurrent and parallel forces in a plane, moment of force and Varignon’s theorem, free body diagram, conditions of equilibrium, Principle of virtual work, equivalent force system. First and Second Moment of area, Mass moment of Inertia. Static Friction, Inclined Plane and bearings.


STRENGTH OF MATERIALS :

STRUCTURAL ANALYSIS :
Castiglianio’s theorems I and II, unit load method, method of consistent deformation applied to beams and pin jointed trusses. Slope-deflection, moment distribution, Kani’s method of analysis and column Analogy method applied to indeterminate beams and rigid frames. Rolling loads and Influences lines : Influences lines for Shear Force and Bending moment at a section of a beam. Criteria for maximum shear force and bending Moment in beams traversed by a system of moving loads. Influences lines for simply supported plane pin jointed trusses.

Arches : Three hinged, two hinged and fixed arches, rib shortening and temperature effects, influence lines in arches.


Plastic Analysis of beams and frames : Theory of plastic bending, plastic analysis, statitical method, Mechanism method.

Unsymmetrical bending : Moment of inertia, product of inertia, position of Neutral Axis and Principle axes, calculation of bending stresses.

Part-B
DESIGN OF STRUCTURES : STEEL, CONCRETE AND MASONRY STRUCTURES.

STRUCTURAL STEEL DESIGN :

DESIGN OF CONCRETE AND MASONRY STRUCTURES :

Concept of mix design. Reinforced Concrete : Working Stress and Limit State method of design- Recommendations of I.S. codes design of one way and two way slabs, stair-case slabs, simple and continuous beams of rectangular, T and L sections. Compression members under direct load with or without eccentricity, Isolated and combined footings. Cantilever and Counterfort type retaining walls.

Water tanks : Design requirements for Rectangular and circular tanks resting on ground.

Prestressed concrete : Methods and systems of prestressing, anchorages, Analysis and design of sections for flexure based on working stress, loss of prestress.

Design of brick masonry as per I.S. Codes.

Design of masonry retaining walls.
Part-C

FLUID MECHANICS, OPEN CHANNEL FLOW AND HYDRAULIC MACHINES

FLUID MECHANICS

Fluid properties and their role in fluid motion, fluid statics including forces acting on plane and curve surfaces. Kinematics and Dynamics of Fluid flow: Velocity and accelerations, stream lines, equation of continuity, irrotational and rotational flow, velocity potential and stream functions, flownet, methods of drawing flownet, sources and sinks, flow separation, free and forced vortices, Control volume equation, continuity, momentum, energy and moment of momentum equations from control volume equation, Navier-Stokes equation, Euler’s equation of motion, application to fluid flow problems, pipe flow, plane, curved, stationary and moving vanes, sluice gates, weirs, orifice meters and Venturi meters.

Dimensional Analysis and Similarity: Buckingham’s Pi-theorem, dimensionless parameters, similarity theory, model laws, undistorted and distorted models.

Laminar Flow: Laminar flow between parallel, stationary and moving plates, flow through tube.

Boundary layer: Laminar and turbulent boundary layer on a flat plate, laminar sub-layer, smooth and rough boundaries, drag and lift.

Turbulent flow through pipes: Characteristics of turbulent flow, velocity distribution and variation of pipe friction factor, hydraulic grade line and total energy line, siphons, expansion and contractions in pipes, pipe networks, water hammer in pipes and surge tanks.

Open channel flow: Uniform and non-uniform flows, momentum and energy correction factors, specific energy and specific force, critical depth, resistance equations and variation of roughness coefficient, rapidly varied flow, flow in contractions, flow at sudden drop, hydraulic jump and its applications surges and waves, gradually varied flow, classification of surface profiles, control section, step method of integration of varied flow equation, moving surges and hydraulic bore.

HYDRAULIC MACHINES AND HYDROPOWER:


Part-D

GEO TECHNICAL ENGINEERING

Types of soil, phase relationships, consistancy limits particles size distribution, classifications of soil, structure and clay mineralogy. Capillary water and structural water, effective stress and pore water pressure, Darcy’s Law, factors affecting permeability, determination of permeability, permeability of stratified soil deposits, Seepage pressure, quick sand condition, compressibility and consolidation, Terzaghi’s theory of one dimensional consolidation, consolidation test. Compaction of soil, field control of compaction. Total stress and effective stress parameters, pore pressure coefficients. Shear strength of soils, Mohr Coulomb failure theory, Shear tests. Earth pressure at rest, active and passive pressures, Rankine’s theory, Coulomb’s wedge theory, earth pressure on retaining wall, sheetpile walls, Braced excavation. Bearing capacity, Terzaghi and other important theories, net and gross bearing pressure. Immediate and consolidation settlement. Stability of slope, Total Stress and Effective Stress methods, Conventional methods of slices, stability number. Subsurface exploration, methods of boring, sampling, penetration tests, pressure meter tests. Essential features of foundation, types of foundation, design criteria, choice of type of foundation, stress distribution in soils, Boussinessq’s theory, Newmark’s chart, pressure bulb, contact pressure, applicability of different bearing capacity theories, evaluation of bearing capacity from field tests, allowable bearing capacity, Settlement analysis, allowable settlement. Proportioning of footing, isolated and combined footings, rafts, buoyancy rafts. Pile foundation, types of piles, pile capacity, static and dynamic analysis, design of pile groups, pile load test, settlement of piles, lateral capacity. Foundation for Bridges. Ground improvement techniques-preloading, sand drains, stone column, grouting, soil stabilisation.
1. Fluid Properties and Pressure Measurement

Q.1 Two coaxial cylinders 250 mm high have a liquid in between them, the outer cylinder has internal diameter 100 mm and the internal cylinder has external diameter 97.5 mm. Find the viscosity of liquid which produces a torque of 1 Nm upon the inner cylinder when outer one rotates at 90 rpm.

Solution:

For given assembly

Internal diameter, \( d_i = 97.5 \, \text{mm} \)
External diameter, \( d_e = 100 \, \text{mm} \)
Speed of external cylinder, \( N = 90 \, \text{rpm} \)
Height of cylinder, \( H = 250 \, \text{mm} \)

Tangential velocity of external cylinder,

\[
V = \frac{\pi d_e N}{60} = \frac{\pi \times 0.1 \times 90}{60} = 0.47 \, \text{m/s}
\]

Shear stress at internal cylinder,

\[
\tau = \frac{\mu \times 0.47}{0.1 - 0.0975} = 376 \, \mu
\]

Force on internal cylinder,

\[
F = \tau A = 376 \, \mu \times \pi \times 0.0975 \times 0.25 = 28.79 \, \mu
\]

Torque on internal cylinder,

\[
T = F \frac{d_i}{2} = \frac{28.79 \mu \times 0.0975}{2} = 0.712 \, \text{Pa} \cdot \text{s}
\]

Q.2 U-tube differential gauge is attached to two section A and B in a horizontal pipe in which oil of specific gravity 0.8 is flowing. The deflection in mercury in the gauge is 600 mm, lever near A being lower one. Calculate the differential pressure in \( \text{N/mm}^2 \). Draw a neat sketch of the arrangement.

Solution:

For arrangement
Oil density, \( \rho = 0.8 \times 1000 = 800 \text{ kg/m}^3 \)
Mercury density, \( \rho_m = 13600 \text{ kg/m}^3 \)

Let pressure at ‘A’ be \( P_A \), pressure at \( B \) be \( P_B \)
Equating pressure at bottom of gauge

\[
P_A + \rho g (H_B - H_A) = P_B + \rho_m g (H_B - H_A)
\]

\[
P_A - P_B = (\rho_m - \rho) g (H_B - H_A)
\]

\[
= (13600 - 800) \times 9.81 \times \frac{600}{1000} \times \frac{1}{1000} = 75.34 \text{ kPa}
\]

Alternate Solution:

From figure differential head,

\[
h = \left( \frac{13.6}{0.8} - 1 \right) \times 0.6 = 9.6 \text{ m} \text{ (of oil)}
\]

Differential pressure

\[
= \rho_o i \times g \times h = 800 \times 9.81 \times 9.6 = 75340.8 \text{ Pa or } 75.34 \text{ kPa}
\]

Q.3 Define capillarity. Derive an equation for capillarity rise between two thin vertical plates spaced ‘\( t \)’ distance apart. Calculate the distance between the plates when the capillarity rise is not to exceed 60 mm. Assume surface tension of water at 20°C as 0.075 N/meter.

[10 marks : 2001]

Solution:

It is a phenomenon of rise or fall of liquid surface related to adjacent general level of liquid, due to surface tension, when it is passing through tubes of small thickness.

For two vertical plates, ‘\( t \)’ distance apart

Let width of plate be ‘\( L \)’ and contact angle be ‘\( \theta \)’

Force due to surface tension = Force due to gravity

\[
\sigma (2L) \cos \theta = \rho g (L \times t) h
\]

Height of capillarity rise,

\[
h = \frac{2 \sigma \cos \theta}{\rho g t}
\]

For \( \sigma = 0.075 \text{ N/m} \) and \( h = 60 \text{ mm} \)

Assuming \( \theta = 0^\circ \) i.e., \( \cos \theta = 1 \)

\[
0.06 = \frac{2 \times 0.075 \times 1000}{9.81 \times 1000 \times t}
\]

\[
t = 0.255 \text{ mm}
\]

Q.4 Water seeps near the bottom of a buried gasoline (sp. gr. = 0.68) storage tank and rises to a depth of 1 m as shown in figure. If the free surface of gasoline is at 6 m from the tank bottom, find the gauge pressure at a point \( A \), inside the tank’s upper surface and at gasoline water interface. What will be the pressure at bottom of tank in metres of water?
Solution:

Density of water, \( \rho_w = 1000 \text{ kg/m}^3 \)
Density of gasoline, \( \rho_g = 0.6 \times 1000 = 600 \text{ kg/m}^3 \)
Pressure at \( A \), \( P_A = \rho_g g h_A = 600 \times 9.81 \times 3 = 17658 \text{ N/m}^2 = 17.658 \text{ kN/m}^2 \)
Pressure at water oil interface, \( P_B = \rho_g g h_B = 600 \times 9.81 \times 5 = 29430 \text{ N/m}^2 = 29.43 \text{ kN/m}^2 \)
Pressure at bottom,
\[
P_C = \rho_g g h_3 + \rho_w g (h_c - h_b) = 600 \times 9.81 \times 5 + 1000 \times 9.81 \times (6 - 5),
\]
\[
= 39240 \text{ N/m}^2 = 39.24 \text{ kN/m}^2
\]
\[
= \frac{39240}{9810} \approx 4 \text{ m of water}
\]

Q.5 Define bulk modulus of elasticity of a fluid, what is the SI unit of bulk modulus of elasticity? Discuss the factor affecting bulk modulus of elasticity of a fluid. Liquid are generally considered incompressible. Why?

Solution:

Bulk Modulus: It is a measure of compressibility of a fluid. It is a ratio of change in pressure to change in volume per unit volume. It is the ratio of pressure to volumetric strain

Bulk modulus,
\[
k = \frac{\Delta P}{\Delta V/V}
\]

SI unit of bulk modulus of elasticity is expressed in terms of N/m² i.e., Pascal

Factors affecting bulk modulus:
1. As pressure of fluid increases, bulk modulus of elasticity of fluid increases.
2. With increase in temperature of a liquid the bulk modulus of liquid decreases.
3. With increase in temperature of a gas the bulk modulus of gas increases.

As in case of liquid the bulk modulus of elasticity is very high. So even with very large increase in pressure change in volume is very small, so liquid are considered incompressible.

Q.6 A three-cylinder car has pistons of 75 mm and cylinders of 75.1 mm. Find the percentage change in force required to drive the piston, when the lubricant warms from 25°C to 100°C. The dynamic viscosity of the lubricant at 25°C is 2 Ns/m² and at 100°C, it is 0.4 Ns/m².

Solution:

Given, \( d_1 \), Piston diameter = 75 mm
\( d_2 \), Cylinder diameter = 75.1 mm
\[
\therefore \quad \text{Clearance} = \frac{d_1 - d_2}{2} = \frac{0.1}{2} = 0.05 \text{ mm}
\]
Also, \( \mu_{25} = 2 \text{ Ns/m}^2, \mu_{100} = 0.4 \text{ NS/m}^2 \)

As per Newtonian's law of viscosity, \( \tau = \mu \frac{du}{dy} \)

Force required to drive the piston = \( F = \tau A_s \)
\[
\therefore \quad F = \left( \mu \frac{du}{dy} \right) A_s
\]
\[
\frac{F_1}{F_2} = \frac{\mu_1}{\mu_2} \quad \text{(The other factors remains constant)}
\]

\[
\Rightarrow \quad \frac{F_{25^\circ}}{F_{100^\circ}} = \frac{\mu_{25^\circ}}{\mu_{100^\circ}} = \frac{2.0}{0.4} = 5.0
\]

\[
\% \text{ change in force, } \Delta F = \left( \frac{F_{25^\circ} - F_{100^\circ}}{F_{25^\circ}} \right) \times 100 = 1 - \frac{1}{5} = 80\%
\]

\[\Rightarrow \quad 80\% \text{ force reduces to drive the same piston.}\]

### 2. Fluid Statics

**Q.7** A rectangular gate 2 m wide and 6 m high is hinged at base and makes an angle 60° with the base of channel which is horizontal. To keep the gate in stable position a force of 29.43 kN is applied at right angle to the plate. Find the depth of water at which gate begins to fall neglecting the weight of gate and friction at the hinges.

**Solution:**

For rectangular plate of \((2 \times 6)\) m²

Area = 12 m²

**Assumption:** Force is applied at the to end of gate.

**Height of centroid of plate** = \(3 \sin 60^\circ = 2.6\) m

**Average pressure on plate,** \(P = \rho g(H - 2.6)\)

**Centre of pressure,** \(h_{cp} = (H - 2.6) + \frac{2 \times 6^3 \times \sin 60^\circ}{2 \times 6 \times (H - 2.6)}\)

**Before it,** \(h_{cp} = \bar{y} + \frac{I_{cg} \sin^2 \theta}{\bar{y} \cdot A} = (H - 2.6) + \frac{2.25}{H - 2.6}\)

**Height of centre of pressure,** \(y = H - h_{cp}\)

Taking moment about hinge

\[1000 \times 29.43 \times 6 = \frac{1000 \times 9.81 \times (H - 2.6)}{\sin 60^\circ} \times \left[ 2.6 - \frac{2.25}{H - 2.6} \right] \times 12\]

\[\Rightarrow \quad 2.6 \times (H - 2.6) - 2.25 = 1.3\]

**Height of water surface,** \(H = 3.965\) m

**For depth of water greater than,** \(H = 3.965\) m, gate will begin to fall.

**Alternate Solution:**

Assume gate crest is below WT

\[OA = 6\) m \]

\[F = 29.43\) kN \]

Let height of water surface from \(A = x\)

**Pressure at \(A\)** \(p_A = s_w g x\)

**Pressure at \(O\)** \(p_0 = s_w g (x + 6 \sin 60^\circ)\)

\[= s_w g (x + 3\sqrt{3})\]
\[ P = \frac{1}{2} \times (p_A + p_0) \times OA \times 2 \]
\[ = \frac{1}{2} s_w g (2x + 3\sqrt{3}) \times 6 \times 2 \]
\[ = 6s_w g (2x + 3\sqrt{3}) \]

Resultant of \( P \) will out at distance \( l \) from \( O \).

\[ l = \left( \frac{p_2 + 2p_A}{p_0 + p_A} \right) \times \frac{A_0}{3} = \left( \frac{s_w g (x + 3\sqrt{3}) + 2s_w gx}{s_w g (x + 3\sqrt{3})} \right) \times \frac{6}{3} \]
\[ = 2 \left( \frac{3x + 3\sqrt{3}}{2x + 3\sqrt{3}} \right) \]

In equilibrium
\[ \Sigma M_0 = 0 \]
\[ \Rightarrow F \times A_0 = P \times l \]
\[ \Rightarrow 29.43 \times 6 = 6s_w g (2x + 3\sqrt{3}) \times 2 \left( \frac{3x + 3\sqrt{3}}{2x + 3\sqrt{3}} \right) \]
\[ \Rightarrow 29.43 \times 6 = 6s_w g (3x + 3\sqrt{3}) = 6 \times 9.81 (3x + 3\sqrt{3}) \]
\[ x = -1.23 \text{ m} \]

\[ \Rightarrow \text{Gate crest is 1.23 on above WT.} \]
\[ \therefore \text{Depth of gate below WT} \]
\[ H = 6 \sin 60^\circ + (-1.23) \]
\[ = 3.96 \text{ m} \]

**Q.8** A square plate of side 1 m is immersed in water with its diagonal vertical. The upper corner is 0.5 m below the free surface, as shown in figure below. Find the hydrostatic force on the plate and depth of centre of pressure from free surface of water.

**Solution:**

For square plate,

Side length, \( L = 1 \text{ m} \)

Area, \( A = L^2 = (1)^2 = 1 \text{ m}^2 \)

\[ \bar{x} = \frac{\sqrt{2}L}{2} + 0.5 = \frac{1}{\sqrt{2}} + 0.5 = 1.207 \text{ m} \]

Hydrostatic force on plate, \( F_x = \rho g \bar{x} A = 1000 \times 9.81 \times 1.207 \times 1 = 11840.67 \text{ N} \)
Depth of centre of pressure,
\[ \bar{h} = \bar{x} + \frac{I_{cg}}{x^2} \]
where, \( \theta = 90^\circ \)

and
\[ I_{cg} = \frac{L^4}{12} = \frac{1}{12} = 0.083 \text{ m}^4 \]
i.e.,
\[ \bar{h} = 1.207 + \frac{0.083}{1.207 \times 1} = 1.276 \text{ m} \]

Q.9 A 2.5 diameter tank of height 2.5 m is closed at the top and contains a liquid of specific gravity 0.75 upto height of 2.0 m. If space above the liquid is under a pressure of -3 kPa (suction), calculate
(a) Force acting on the bottom of tank when it is accelerated vertically upwards at 0.5 times g and
(b) The acceleration required for maintaining zero absolute pressure at tank bottom. Take atmospheric pressure as 100 kN/m² and water density as 1000 kg/m³.

Solution:
\[ \rho = 0.75 \times 1000 = 750 \text{ kg/m}^3 \]

(a) Pressure at bottom of the tank,
\[ \rho_{en} = p + \rho gh + \rho ga \]
\[ = p + \rho (g + a)h \]
\[ = -3 + 750 \times (9.81 + 0.5 \times 9.81) \times 2 \times 10^{-3} \]
\[ = 19.07 \text{ kPa} \]
\[ \therefore \text{ Force acting on the bottom of the tank} \]
\[ F = \rho \cdot \frac{\pi D^2}{4} = 19.07 \times \frac{\pi}{4} \times 2.5^2 = 93.62 \text{ kN} \]

(b) absolute pressure at the bottom of tank
\[ p = (100 - 3) + \rho (g + a)h \]
\[ O = 97 \times 10^3 + 750 (9.81 + a) \times 2 \]
\[ \Rightarrow a = -74.47 \text{ m/s}^2 \]
\[ = 74.47 \text{ m/s}^2 \]

Q.10 It is frequently desirable to install automatic gates to prevent a flood. One such gate ABC is shown in the figure.
The gate $ABC$ is 1 m square and hinged at $B$. It will open automatically when water depth $h$ becomes high enough. Determine the minimum value of $h$ at which the gate will open. 

Solution:

Pressure distribution on Gate

\[ P_A = \gamma_w Z = \gamma_w h \text{ kN/m}^2 \]
\[ P_B = \gamma_w (h + 1) \text{ kN/m}^2 \]

The gate will open for minimum depth $h$ when the resultant force will just pass over point $B$.

\[ y_c = \frac{(P_B + 2P_A)}{(P_B + P_A)} \times \frac{AC}{3} = \left(\frac{1+3h}{1+2h}\right) \times \frac{1}{3} \]
\[ y_c = 0.40 \text{ m} = \left(\frac{1+3h}{1+2h}\right) \times \frac{1}{3} \]
\[ \therefore \quad 1.20 + 2.40 \ h = 1 + 3 \ h \]
\[ \Rightarrow \quad h = \frac{1.20 - 1}{3 - 2.40} = 0.333 \text{ m} \]

3. Fluid Kinematics

Q.11 Define stream function, the stream function of a flow is given as $\psi = 2x^2 - 2y^2$. Find whether the flow is rotational and calculate the velocity at point (3, 5).

Solution:

Stream Function: It is a scalar function of space and time such that it’s partial derivative with respect to any direction gives the velocity component at right angle. (In counter clockwise direction) to the direction. i.e., for stream function, $\psi$

\[ \frac{\partial \psi}{\partial x} = V \]
\[ \frac{\partial \psi}{\partial y} = -U \]
where velocity \( \vec{v} = U \hat{i} + V \hat{j} \)

For given stream function, \( \psi = 2x^2 - 2y^2 \)

\[
U = \frac{\partial \psi}{\partial y} = -\partial \left( 2x^2 - 2y^2 \right) = 4y
\]

\[
V = \frac{\partial \psi}{\partial x} = \frac{\partial}{\partial x} \left( 2x^2 - 2y^2 \right) = 4x
\]

So

\[
\vec{v} = 4y \hat{i} + 4x \hat{j}
\]

Rotational velocity, \( \omega_z = \frac{1}{2} \left( \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y} \right) = \frac{1}{2} \left( \frac{\partial (4x)}{\partial x} - \frac{\partial (4y)}{\partial y} \right) = \frac{4 \cdot 4 - 4}{2} = 0 \)

as rotational velocity is zero, flow is irrotational.

Velocity at (3, 5) \( \vec{v} = (4 \times 5) \hat{i} + (4 \times 3) \hat{j} = 20 \hat{i} + 12 \hat{j} \)

Magnitude of velocity \( = \sqrt{20^2 + 12^2} = 23.32 \text{ m/s} \)

Q.12 Determine the U-component of velocity distribution to satisfy continuity equation given that

\( V = ax^3 - by^3 + cz \)

and \( W = bx^3 + cy - 2axz \)

[10 marks : 2005]

Solution:

For a velocity \( \vec{v} = U \hat{i} + V \hat{j} + W \hat{k} \)

Condition to satisfy continuity equation, for incompressible flow,

\[
\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0
\]

\[
\frac{\partial U}{\partial x} = -\frac{\partial \left( ax^3 - by^3 + cz \right)}{\partial y} - \frac{\partial \left( bx^3 + cy - 2axz \right)}{\partial z}
\]

\[
\frac{\partial U}{\partial x} = 3bx^2 + 2ax
\]

\[
U = \int \left( 3bx^2 + 2ax \right) dx
\]

\[
U = 3bx^2 + ax^2 + D \quad \text{(Where D is a constant)}
\]

Q.13 If velocity field is given by \( U = x^2 - y^2 + x \) and \( V = -(2xy + y) \), determine the velocity potential function and stream function.

[10 marks : 2006]

Solution:

For given flow

\( U = x^2 - y^2 + x \)

\( V = -(2xy + y) \)

Checking

(i) \( \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = \frac{\partial \left( x^2 - y^2 + x \right)}{\partial x} + \frac{\partial (2xy - y)}{\partial y} = 2x + 1 - 2x - 1 = 0 \Rightarrow \text{Flow is possible} \)

(ii) \( \omega_z = \frac{1}{2} \left( \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y} \right) = \frac{1}{2} \left( \frac{\partial (-2xy - y)}{\partial x} - \frac{\partial \left( x^2 - y^2 + x \right)}{\partial y} \right) \)
\[
\frac{1}{2}[-2y + 2y] = 0 \Rightarrow \text{Flow is irrotational}
\]

As flow is irrotational, velocity potential function exists

\[
U = -\frac{\partial \phi}{\partial x}
\]

\[
\Rightarrow \quad \partial \phi = -Udx = -(x^2 - y^2 + x)dx
\]

\[
\Rightarrow \quad \int \partial \phi = -\int (x^2 - y^2 + x)dx
\]

\[
\Rightarrow \quad \phi = -\frac{x^3}{3} + xy^2 - \frac{x^2}{2} + f(y) + C \quad \text{...(i)}
\]

Also

\[
V = -\frac{\partial \phi}{\partial y}
\]

\[
\Rightarrow \quad \partial \phi = -Vdy = -(2xy + y) \ dy
\]

\[
\Rightarrow \quad \int \partial \phi = \int [2xy + y] \ dy
\]

\[
\Rightarrow \quad \phi = xy^2 + \frac{y^2}{2} + f(x) + C \quad \text{...(ii)}
\]

From eq. (i) and (ii)

\[
\phi = -\frac{x^3}{3} + xy^2 + \frac{y^2}{2} - \frac{x^2}{2} + C \quad \text{where C is constant}
\]

\[
U = -\frac{\partial \psi}{\partial y}
\]

\[
\Rightarrow \quad \partial \psi = -Udy = -(x^2 - y^2 + x)dy
\]

\[
\Rightarrow \quad \int \partial \psi = -\int (x^2 - y^2 + x)dy
\]

\[
\Rightarrow \quad \psi = -x^2y + \frac{y^3}{3} - xy + f(x) + C_1 \quad \text{...(iii)}
\]

Also

\[
V = \frac{\partial \psi}{\partial x}
\]

\[
\Rightarrow \quad \partial \psi = Vdx = -(2xy + y) \ dx
\]

\[
\Rightarrow \quad \psi = -x^2y - xy + f(y) + C_1 \quad \text{...(iv)}
\]

From eq. (iii) and (iv)

\[
\psi = -x^2y + \frac{y^3}{3} - xy + C_1 \quad \text{(Where } C_1 \text{ is constant)}
\]

Q.14 The velocity component in two dimensional flow field for an incompressible fluid are expressed as

\[
u = \frac{y^3}{3} + 2x - x^2y \quad \text{and} \quad v = xy^2 - 2y - \frac{x^3}{3}
\]

(i) Verify that the functions represent a possible case of fluid flow.

(ii) Show that these functions represents a possible case of an irrotational flow.

[10 marks : 2010]

**Solution:**

(i) For an incompressible fluid flow to be possible,

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \quad \text{should be equal to zero}
\]

For given flow

\[
\frac{\partial u}{\partial x} = 2 - 2xy
\]
\[ \frac{\partial v}{\partial y} = 2x - 2 \]
\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 2 - 2xy + 2xy - 2 = 0 \Rightarrow \text{Flow is possible} \]

(ii) For a flow rotational velocity is represented as
\[ \omega_z = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \]
For given flow
\[ \omega_z = \frac{1}{2} \left( y^2 - x^2 - y^2 + x^2 \right) = 0 \]
As \( \omega_z = 0 \Rightarrow \text{Flow is irrotational.} \)

Q.15 What is meant by local and convective acceleration? For a one dimensional flow, describe by \( U(x, t) \), derive the expression for a convective acceleration in terms of velocity and its gradient. [8 marks : 2015]

Solution:

**Local acceleration:** It is the rate of change of fluid velocity at a point with respect to time. If it is zero, flow is called steady.

**Convective acceleration:** It is the rate of change of fluid velocity, due to change in the position. If it is zero, flow is called uniform.

For a 3D, unsteady flow

\[ \ddot{a} = \frac{\partial \dot{V}}{\partial x} + \frac{\partial \dot{V}}{\partial y} + \frac{\partial \dot{V}}{\partial z} + \frac{\partial \dot{V}}{\partial t} \]

For a one dimensional flow \( U(x, t) \)
\[ \ddot{U} = \frac{\partial U}{\partial x} + \frac{\partial U}{\partial t} \]

\[ \ddot{a} = \frac{\partial U}{\partial x} \times \frac{dx}{dt} + \frac{\partial U}{\partial t} \times \frac{dt}{dt} = \frac{U \frac{\partial U}{\partial x}}{\partial t} + \frac{\partial U}{\partial t} \]

Q.16 A 3D flow field is given by
\[ \dot{V} = \left( 2x^2 + 3y \right)i + \left( -2xy + 3y^2 + 3zy \right)j + \left( -\frac{3}{2}x^2 + 2xz - 9y^2z \right)k \]

Determine the acceleration at \((1, 1, 1)\). [8 marks : 2016]

Solution:

Given,
\[ \dot{V} = \left( 2x^2 + 3y \right)i + \left( -2xy + 3y^2 + 3zy \right)j + \left( -\frac{3}{2}x^2 + 2xz - 9y^2z \right)k \]

We know, acceleration,
\[ a = a_x i + a_y j + a_z k \]
\[ a_x = \frac{\partial u}{\partial x} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \]
\[ = \left( 2x^2 + 3y \right) \times 4x + \left( -2xy + 3y^2 + 3zy \right) \times 3 + \left( -\frac{3}{2}x^2 + 2xz - 9y^2z \right) \times 0 \]

At point \((1, 1, 1)\)

\[ a_x = (2 \times 1^2 + 3 \times 3) \times 4 + (-2 \times 1 \times 1 + 3 \times 1^2 + 3 \times 1 \times 1) \times 3 \]
\[ = 5 \times 4 + (-2 + 3 + 3) \times 3 \]
\[ = 20 + 12 = 32 \text{ units} \]

\[ a_y = \frac{\partial v}{\partial y} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \]
\[
\begin{align*}
&= 0 + \left(2x^2 + 3y\right)(-2y) + \left(-2xy + 3y^2 + 3yz\right)(-2x + 6y + 3z) \\
&\quad + \left(-\frac{3}{2}z^2 + 2xz - 9y^2 z\right) \times 3y \\
&= 5 \times (-2) + (-2 + 3 + 3) (-2 + 6 + 3) + (-1.5 + 2 - 9) \times 3 \\
&= -10 + 4 \times 7 - 8.5 \times 3 \\
&= -10 + 28 - 25.5 = -7.5 \text{ units}
\end{align*}
\]

Similarly,
\[
\begin{align*}
a_1 &= \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \\
&= 0 + \left(2x^2 + 3y\right) \times 2z + \left(-2xy + 3y^2 + 3yz\right)(-18yz) \\
&\quad + \left(-\frac{3}{2}z^2 + 2xz - 9y^2 z^2\right) \times \left(-3z + 2x - 9y^2\right) \\
&= 5 \times 2 + (-2 + 3 + 3) \times (-18) + (1.5 + 2 - 9) (-3 + 2 - 9) \\
&= 10 - 18 \times 4 - 8.5 \times (-10) \\
&= 10 - 72 + 85 = 23 \text{ units}
\end{align*}
\]

\[
\therefore \quad a = 32i - \frac{15}{2}j + 23k
\]

**Q.17** State and explain forced vortex as occurring in a centrifugal pump. [5 marks : 2010]

**Solution:**

In a forced vortex, mass of fluid is rotated with external torque, which results in increase in pressure. Centrifugal pump works on the principle of forced vortex flow. In forced vortex flow, water rotates with a constant angular velocity. Flow on a impeller of pump also rotates with constant angular velocity, which imparts energy to water in form of centrifugal head. At the starting of pump

\[
U_2^2 - U_1^2 = 2gH_m
\]

\[
\left(\frac{\pi D_i N}{60}\right)^2 - \left(\frac{\pi D_o N}{60}\right)^2 = 2gH_m
\]

which is similar to forced vortex flow equation

\[
\omega^2 (r^2 - r_0^2) = 2gZ
\]

Thus on the principle of forced vortex, mechanical energy is converted into water energy in centrifugal pumps.

**Q.18** Write the formula for rotation of a fluid element about z-axis in terms of velocity gradient in the Cartesian coordinate system. For a free vortex, although streamlines are circular, the flow is irrotational. Explain why? [10 marks : 2015]

**Solution:**

In a rotational flow about z-axis i.e., x-y plane, due to viscous forces rotation takes place.

Let velocity in x-y plane  \( \tilde{v} = u\hat{i} + v\hat{j} \)

Rotation due to \( u \hat{i} \)

\[
\tan (d\alpha) = \frac{dx}{dy} = \frac{-\left(u + \frac{\partial u}{\partial y} dy\right) dt + u dt}{dy}
\]
\[
\frac{d\alpha}{dt} = -\frac{\partial u}{\partial y}
\]

(Taking anticlockwise rotation positive)

Rotation due to \( V \hat{j} \)

\[
\tan(\beta) = \tan(\beta) = \frac{v + \frac{\partial v}{\partial x} \ dx}{dx} \ dt - vdt
\]

\[
\frac{\partial \beta}{\partial t} = \frac{\partial v}{\partial x}
\]

Net rotational velocity,

\[
\omega_z = \left( \frac{\partial u}{\partial t} + \frac{\partial \beta}{\partial t} \right)
\]

\[
= \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)
\]

(As 2 particles are used in calculation)

In free vortex flow, even though stream lines are circular but as no external torque is provided, angular momentum remains constant i.e.,

\[
mVr = \text{Constant}
\]

\[
V = \frac{1}{2}
\]

Let

\[
V = \frac{C}{r}
\]

Using

\[
\partial P = \frac{\rho V^2}{r} \ dr - \left( \frac{\partial P}{\partial z} \right) dz
\]

\[
\partial P = \frac{\rho \rho V^2}{r^2} \ dr - \rho g \ dz
\]

Integrating between any two point (1) and (2) in vortex

\[
\int_1^2 \partial P = \int_1^2 \frac{\rho V^2}{r^2} \ dr - \int_1^2 \rho g \ dz
\]

\[
P_2 - P_1 = \frac{\rho \rho V^2}{2} \left[ \frac{1}{r_2^2} - \frac{1}{r_1^2} \right] - \rho g \ z_2 + \rho g \ z_1
\]

\[
P_2 + \rho g \ z_2 + \frac{1}{2} \rho V_2^2 = P_1 + \rho g \ z_1 + \frac{1}{2} \rho V_1^2
\]

(: Total energy constant)

As total energy remain conserved between two stream lines flow is essentially irrotational.

**Q.19** The velocity potential for a two-dimensional flow is \( \phi = x \ (2y - 1) \). Determine the value of stream function at a point \( P(4, 5) \). Comment whether the flow is rotational or irrotational. Also determine the velocity at the point \( P(4, 5) \).

[8 marks : 2017]

**Solution:**

Given, Velocity potential, \( \phi = x(2y - 1) \)

\[
\therefore \quad u = -\frac{\partial \phi}{\partial x} = -(2y - 1) = -\frac{\partial \psi}{\partial y}
\]

\[
\Rightarrow \quad \psi = \int (2y - 1) dy = y^2 - y + f(x)
\]

\[
V = -\frac{\partial \phi}{\partial y} = -2x = -\frac{\partial \psi}{\partial x} = f'(x)
\]
\[ \int -2x \, dx = -x^2 + c \]

\[ \psi = y^2 - x^2 - y + c \]

\[ \psi(4, 5) = 5^2 - 4^2 - 5 + c = 4 + c = 4 \] (assume \( c = 0 \))

\[ \frac{\partial \phi}{\partial x} = 2x - 1, \quad \frac{\partial^2 \phi}{\partial x^2} = 0 \]

\[ \frac{\partial \phi}{\partial y} = 2x, \quad \frac{\partial^2 \phi}{\partial y^2} = 0 \]

\[ \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 2 + 0 = 2 \neq 0 \]

Flow is not irrotational flow as laplace eqn is not satisfied.

\[ u = -(2y - 1) = -2(2 \times 5 - 1) = -9 \]

\[ v = -2x = -2 \times 4 = 8 \]

\[ |\vec{v}| = \sqrt{9^2 + 8^2} = 12.01 \text{ unit} \]

4. Fluid Dynamics

Q.20 The head of water over on orifice of 100 mm diameter is 5 metre. The water coming out of orifice is collected in circular tank of 2 meter diameter. The rise of water in the tank is 0.45 meter in 30 secs. Also the coordinate of a certain point measured from vena contracta are 100 cms horizontal and 5.2 cms vertical. Calculate the hydraulic coefficients of orifice.

Solution:

For orifice,

Head, \( h = 5 \) m

Orifice diameter, \( d = 100 \text{ mm} = 0.1 \) m

Orifice area, \( a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.1)^2 = 7.85 \times 10^{-3} \) m\(^2\)

Volume collected \( \frac{\pi}{4} \times 2^2 \times 0.45 = 1.414 \) m\(^3\)

Actual discharge, \( Q_{act} = \frac{\text{Volume collected}}{\text{Time taken}} = \frac{1.414}{30} = 0.047 \text{ m}^3/\text{s} \)

Theoretical discharge, \( = a \sqrt{2gh} = 7.85 \times 10^{-3} \times \sqrt{2 \times 9.81 \times 5} = 0.0778 \text{ m}^3/\text{s} \)

Coefficient of discharge, \( C_D = \frac{\text{Actual discharge}}{\text{Theoretical discharge}} = \frac{0.047}{0.0778} = 0.60 \)

Theoretical velocity, \( V_t = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 5} = 9.90 \) m/s

Let actual velocity be \( \dot{V} \)

Horizontal distance, \( x = \dot{V} t \)

Vertical distance, \( y = \frac{1}{2} gt^2 \)

i.e., \( x^2 = \dot{V}^2 \left( \frac{2y}{g} \right) \)
\[ V = \sqrt{\frac{g x^2}{2y}} = \sqrt{\frac{9.81 \times (1)^2}{2 \times (0.052)}} = 9.71 \text{ m/s} \]

Coefficient of velocity,
\[ C_v = \frac{V}{V_m} = \frac{9.71}{9.90} = 0.98 \]

Q.21 A pipeline carrying water has a 60° reducing bend in a horizontal plane. The cross-sectional areas at inlet and outlet of the bend are 1.0 m² and 0.5 m² respectively. The pressure at inlet and outlet of the bend are 40 kN/m² and 30 kN/m² respectively. The discharge in pipe is measured as 10 m³/s. Calculate the magnitude and direction of force required to hold the bend in position. Take the density of water as 1000 kg/m³.

[10 marks : 2011]

Solution:

At inlet of the pipe

Pressure, \( P_1 = 40 \text{ kN/m}^2 \)
Area, \( A_1 = 1.0 \text{ m}^2 \)
Velocity, \( V_1 = \frac{Q}{A_1} = \frac{10}{1} = 10 \text{ m/s} \)

At outlet section of the pipe

Pressure, \( P_2 = 30 \text{ kN/m}^2 \)
Area, \( A_2 = 0.5 \text{ m}^2 \)
Velocity, \( V_2 = \frac{Q}{A_2} = \frac{10}{0.5} = 20 \text{ m/s} \)

Applying momentum equation (Newton’s 2nd law)

In \( x \)-direction

\[ F_x + P_1 A_1 - P_2 A_2 \cos 60° = -p Q V_1 + p Q V_2 \cos 60° \]
\[ \Rightarrow F_x + 40 \times 1 - 30 \times 0.5 \times \frac{1}{2} = -10 \times 10 + 10 \times 20 \times \frac{1}{2} \]
\[ \Rightarrow F_x = -32.5 \text{ kN} \]

In \( y \)-direction

\[ F_y - P_2 A_2 \sin 60° = p Q V_2 \sin 60° \]
\[ \Rightarrow F_y = 10 \times 20 \times \frac{\sqrt{3}}{2} - 30 \times 0.5 \times \frac{\sqrt{3}}{2} = 160.2 \text{ kN} \]

Resultant force:

\[ F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{32.5^2 + 160.2^2} = 163.48 \text{ kN} \]

\[ \theta = \tan^{-1} \left( \frac{F_y}{F_x} \right) = \tan^{-1} \left( \frac{160.20}{32.50} \right) = 78.53° \]

\( F_x = 32.5 \text{ kN} \) \( 160.2 \text{ kN} = F_y \)
Q.22 A venturimeter is fitted in 40 cms diameter horizontal pipeline, which has a throat diameter of 15 cms. The pressure intensity at the inlet is 1.4 kg/cm² and at throat it is 40 cms of mercury of vacuum pressure. Determine the flow of water. Assume 5% of differential head loss between inlet and throat. Find also the coefficient of discharge of venturimeter.

[15 marks : 2013]

Solution:

Given:
At inlet (section 1)
Pressure, \( P_1 = 1.4 \text{ kg/cm}^2 \)
Diameter, \( D_1 = 40 \text{ cm} \)
At throat (section 2)
Pressure, \( P_2 = 40 \text{ cm of mercury vacuum} \)
\( \frac{-400}{760} \times 1.033 = -0.54 \text{ kg/cm}^2 \)
Diameter, \( D_2 = 15 \text{ cms} \)
Head loss \( h_i = 5\% \text{ of differential head} = 0.05 \text{ h} \)

Applying equation of continuity
\[ A_1V_1 = A_2V_2 \]
\[ \Rightarrow \frac{\pi}{4} \times 40^2 \times V_1 = \frac{\pi}{4} \times 15^2 \times V_2 \]
\[ \Rightarrow V_1 = 0.14 V_2 \]

Coefficient of discharge,
\[ C_d = \sqrt{\frac{h-h_i}{h}} = \sqrt{\frac{h-0.05h}{h}} = 0.9747 \]

Applying Bernoulli’s equation
\[ \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z \]
\[ \frac{1.4 \times 10^4}{1000} + \frac{(0.14V_2)^2}{2 \times 9.81} = \frac{-0.54 \times 10^4}{1000} + \frac{V_2^2}{2 \times 9.81} \]
\[ V_2 = 19.70 \text{ m/s} \]

Theoretical discharge,
\[ Q_{th} = A_2V_2 \]
\[ = \frac{\pi}{4} \times 15^2 \times 10^{-4} \times 19.70 = 0.348 \text{ m}^3/\text{s} \]

Actual discharge,
\[ Q_{act} = C_d Q_{th} \]
\[ = 0.9747 \times 0.348 = 0.339 \text{ m}^3/\text{s} \]

Q.23 An orifice meter having orifice diameter of 10 cms is fitted in a 200 mm dia. pipe, which is laid horizontally. The manometer reads 30 cms of height of mercury. Determine the discharge of oil flow of specific gravity 0.8. Consider coefficient of discharge = 0.60.

[8 marks : 2013]

Solution:

Assuming inlet as section 1 and throat at section 2
At inlet
Diameter, \( D = 200 \text{ mm} \)
At throat
Diameter, \( D_2 = 10 \text{ cms} \)
Using equation of continuity

\[ A_1 V_1 = A_2 V_2 \]

\[ \frac{\pi}{4} \times \left( \frac{200}{10} \right)^2 \times V_1 = \frac{\pi}{4} \times 10^2 \times V_2 \]

\[ \Rightarrow \]

\[ V_2 = 4V_1 \]

Differential head,

\[ h = 0.30 \left( \frac{13.6}{0.8} - 1 \right) = 4.8 \text{ m} \]

Using equation of energy conservation

\[ \frac{V_2^2 - V_1^2}{2g} = 4.8 \]

\[ (4V_1)^2 - V_1^2 = 4.8 \times 2 \times 9.81 \]

\[ V_1 = 2.35 \text{ m/s} \]

Now,

\[ A_1 = \frac{\pi}{4} \times 0.2^2 = 0.0314 \text{ m}^2 \]

\[ A_2 = \frac{\pi}{4} \times 0.1^2 = 0.00785 \text{ m}^2 \]

Theoretical discharge,

\[ Q_{th} = A_1 V_1 = 0.0738 \text{ m/s}^2 = 0.0738 \text{ m}^3/\text{s} \]

Actual discharge,

\[ Q_{act} = C_d Q_{th} = 0.6 \times 0.0736 = 0.044 \text{ m}^3/\text{s} \]

Q.24 A 40 x 20 cms venturimeter fitted in a vertical pipe of 40 cms diameter, which carries oil of sp. gr. 0.9. The difference in elevation of inlet and throat is 40 cms. The U-tube manometer reads 30 cms of mercury deflection. Determine:

(i) The discharge in a pipe

(ii) Difference of pressure between inlet and throat

\( C_d \) of venturimeter = 0.98, and flow is vertically upward.

Solution:

(i)

\[ \rho_{oil} = 900 \text{ kg/m}^3 \]

\[ \rho_{mercury} = 13600 \text{ kg/m}^3 \]

\[ D_1 = 40 \text{ cm}, D_2 = 20 \text{ cm} \]

\[ Q = A_1 V_1 = A_2 V_2 \text{ (continuity equation)} \]

\[ \Rightarrow \]

\[ \frac{\pi}{4} \times 40^2 V_1 = \frac{\pi}{4} \times 20^2 V_2 \]

\[ V_2 = 4V_1 \]

From Bernoulli’s equation

\[ \frac{P_1}{\rho_0 g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho_0 g} + \frac{V_2^2}{2g} + Z_2 \]

\[ \Rightarrow \]

\[ \frac{(P_1 - P_2)}{\rho_0 g} = \left( \frac{V_2^2 - V_1^2}{2g} \right) + (Z_2 + Z_1) \]

Pressure equation,

\[ P_1 + \rho_0 g(x + d) - \rho_m gd - \rho_0 g(x + z) = P_2 \]

\[ \Rightarrow \]

\[ (P_1 - P_2) = (\rho_m - \rho_0)gd + \rho_0 gZ \]

\[ = (13600 - 900) \times 9.8 \times 0.30 + 900 \times 9.81 \times 0.40 \]

\[ = 40.908 \times 10^3 \text{ N/m}^2 = 40.91 \text{ kN/m}^2 \]
from equation (1)

\[
\frac{40.91 \times 10^3}{900 \times 9.81} = \frac{(4V_1^2 - V_2^2)}{2 \times 9.81} + 0.40
\]

\[V_1 = 2.35 \text{ m/s}\]

\[Q_p = A_1V_1 = \frac{\pi}{4} \times 0.4^2 \times 2.35 = 0.295 \text{ m}^3/\text{s}\]

Actual discharge

\[Q_{act} = CdQ_p = 0.98 \times 0.295 = \textbf{0.289 m}^3/\text{s}\]

**Q.25** The loss of head from the entrance to the throat of a 250 mm x 125 mm venturimeter is 1/8 times the velocity head at throat. Take the deflection of water mercury manometer as 101.6 mm. Compute the discharge.

[8 marks : 2016]

**Solution:**

Diameter at the inlet, 
\[d_1 = 250 \text{ mm}\]

\[\therefore \text{ Area, } \quad a_1 = \frac{\pi}{4}(d_1^2) = \frac{\pi}{4}(0.25)^2 = 0.0491 \text{ m}^2\]

Diameter at the throat, 
\[d_2 = 125 \text{ mm}\]

\[\therefore \text{ Area, } \quad a_2 = \frac{\pi}{4}(d_2^2) = \frac{\pi}{4}(0.125)^2 = 0.0123 \text{ m}^2\]

Reading of differential manometer, 
\[x = 101.6 \text{ mm}\]

\[\therefore \text{ Difference of pressure head, } \quad h = x \left( \frac{S_m - S_w}{S_w} \right)\]

\[\Rightarrow \quad \left( \frac{P_1}{\rho g} + z_1 \right) - \left( \frac{P_2}{\rho g} + z_2 \right) = h = 0.1016\left[13.8 - 1\right]\]

\[\Rightarrow \quad \left( \frac{P_1}{\rho g} + z_1 \right) - \left( \frac{P_2}{\rho g} + z_2 \right) = 1.3 \text{ meter of water} \quad \ldots(\text{i})\]

Loss of head, 
\[h_L = \frac{1}{8} \times \frac{V_2^2}{2g} = \frac{0.125V_2^3}{2g}\]

Now, applying Bernoulli’s equation at inlet and float of venturimeter.

\[\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g} + h_L\]

\[\Rightarrow \quad \left( \frac{P_1}{\rho g} + z_1 \right) - \left( \frac{P_2}{\rho g} + z_2 \right) + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = h_L = \frac{0.125V_2^3}{2g}\]

Substituting value of eq. (i) is above equation

\[
\left[1.3 + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right] = \frac{0.125V_2^3}{2g}
\]

\[1.3 + \frac{V_1^2}{2g} - \frac{1.125V_2^3}{2g} = 0\]

Applying continuity equation at inlet and throat,

\[a_1V_1 = a_2V_2\]

\[V_1 = \frac{a_2}{a_1}V_2 = \frac{\pi(0.125)^2}{\pi(0.250)^2} \times V_2\]
⇒ \( V_2 = 4V_1 \)

On substitution, we get

\[ 1.3 + \frac{\frac{V_1^2}{2g}}{2g} - 1.125 \times \frac{16V_1^2}{2g} = 0 \]

⇒ \[ \frac{17V_1^2}{2g} = 1.3 \]

⇒ \( V_1 = 1.225 \text{ m/s} \)

\[ \therefore \text{ Discharge} = a_1V_1 = 0.0491 \times 1.225 = 0.06 \text{ m}^3/\text{s} \]

\[ \text{5. Pipe Flow} \]

Q.26 In a pipeline of diameter ‘\( d \)’ there is a large number of laterals tapping uniformly at the average rate of \( Q \), where \( Q \) equals the discharge at entrance of main pipe divided by numbers of lateral in length ‘\( f \)’ of main pipe as shown in the figure. Prove that loss of head due to friction will be equal to 1/3rd of loss of head due to friction in the same pipe without any laterals.

Solution:

For pipe, let friction factor be ‘\( f \)’, discharge entering be \( Q \)

Lateral tapping discharge, \( Q_i = \frac{Q}{n} \)

where, \( n \) is number of laterals

Pipe length, \( L = (n + 1)S \)

Head loss due to friction, \( h_f = \frac{8fLQ^2}{\pi^2gd^5} = \sum_{i=1}^{n} \frac{8fS(iQ_i)^2}{\pi^2gd^5} = \frac{8f(n+1)(2n+1)fLQ^2}{6\pi^2gd^5} \)

\[ = \frac{8f(n+1)(2n+1)fLQ^2}{6\pi^2gd^5} \times \left( \frac{L}{n+1} \right) \times \left( \frac{Q}{n} \right) \]

i.e., \( h_f = \frac{8f(n+1)(2n+1)fLQ^2}{6\pi^2gd^5} \times \left( \frac{L}{n+1} \right) \times \left( \frac{Q}{n} \right) \) \( \ldots \text{(i)} \)

For pipe without any lateral, \( h_e = \frac{8fLQ^2}{\pi^2gd^5} \) \( \ldots \text{(ii)} \)

From eq. (i) and (ii)

\[ \frac{h_f}{h_e} = \frac{(2n+1)}{6n} = \left( \frac{2 + \frac{1}{n}}{6} \right) = \frac{1}{3} \]

(As \( n \) is very large)