

*A Handbook on*

# Instrumentation Engineering

*Contains well illustrated formulae  
& key theory concepts*

*~~~~~ for ~~~~~*

# GATE, PSUs

& OTHER COMPETITIVE EXAMS



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Publications



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### **A Handbook on Instrumentation Engineering**

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First Edition: 2016

Reprint: 2018

Reprint: 2020

Reprint: 2022

## Director's Message



**B. Singh (Ex. IES)**

During the current age of international competition in Science and Technology, the Indian participation through skilled technical professionals have been challenging to the world. Constant efforts and desire to achieve top positions are still required.

I feel every candidate has ability to succeed but competitive environment and quality guidance is required to achieve high level goals. At MADE EASY, we help you to discover your hidden talent and success quotient to achieve your ultimate goals. In my opinion GATE & PSU's exams are tool to enter in to main stream of Nation serving. The real application of knowledge and talent starts, after you enter in to the working system. Here in MADE EASY you are also trained to become winner in your life and achieve job satisfaction.

MADE EASY alumni have shared their winning stories of success and expressed their gratitude towards quality guidance of MADE EASY. Our students have not only secured All India First Ranks in ESE, GATE and PSU entrance examinations but also secured top positions in their career profiles. Now, I invite you to become alumni of MADE EASY to explore and achieve ultimate goal of your life. I promise to provide you quality guidance with competitive environment which is far advanced and ahead than the reach of other institutions. You will get the guidance, support and inspiration that you need to reach the peak of your career.

I have true desire to serve Society and Nation by way of making easy path of the education for the people of India.

After a long experience of teaching in Instrumentation Engineering over the period of time MADE EASY team realised that there is a need of good *Handbook* which can provide the crux of Instrumentation Engineering in a concise form to the student to brush up the formulae and important concepts required for GATE, PSUs and other competitive examinations. This *handbook* contains all the formulae and important theoretical aspects of Instrumentation Engineering. It provides much needed revision aid and study guidance before examinations.

**B. Singh (Ex. IES)**

CMD, MADE EASY Group

# Instrumentation Engineering

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## Transducers

- A transducer is a device which converts one kind of energy to the other.
- The conversion can be to or from electrical, electromechanical, photonic, photo-voltaic or any other form of energy.

## Sensors:

A sensor is a type of transducer which detects or measures a physical property and records, indicates or otherwise responds to it.

## Classification of Transducers

### Based on Principle of Transduction

- The transduction depends on how they transduce the input quantity into the resistance, capacitance or inductance which can be measured by bridge or any other electronic circuitry.

Electrical parameter and class of transducer	Principle of Operation and nature of device	Typical Application
Passive transducers (externally powered)		
1. Resistance		
Potentiometer device	Positioning of the slider by an external force varies the resistance in a potentiometer or a bridge circuit.	Pressure, displacement
Resistance strain gauge	Resistance of a wire or semiconductor is changed by elongation or compression due to externally applied stress.	Force, torque, displacement
Pirani gauge or hot wire meter	Resistance of a heating element is varied by convection cooling of a stream of gas.	Gas flow, gas pressure

<b>Resistance thermometer</b>	Resistance of pure metal wire with a large positive temperature co-efficient of resistance varies with temperature.	Temperature, radiant heat
<b>Thermistor</b>	Resistance of certain metal oxides with negative temperature coefficient of resistance varies with temperature.	Temperature, flow
<b>Resistance hygrometer</b>	Resistance of a conductive strip changes with moisture content.	Relative humidity
<b>Photoconductive cell</b>	Resistance of the cell as a circuit element varies with incident light.	Photosensitive relay

## 2. Capacitance

<b>Variable capacitance pressure gauge</b>	Distance between two parallel plate is varied by an externally applied force.	Displacement, pressure
<b>Capacitor microphone</b>	Sound pressure varies the capacitance between a fixed plate and a movable diaphragm.	Speech, music, noise

## 3. Inductance

<b>Magnetic circuit transducer</b>	Self-inductance or mutual inductance of a.c. excited coil is varied by changes in the magnetic circuit.	Pressure, displacement
<b>Reluctance pick up</b>	Reluctance of the magnetic circuits is varied by changing the position of the iron core of coil.	Pressure, displacement, vibration, position
<b>Differential transformer</b>	The differential voltage of two secondary windings of a transformer is varied by positioning the magnetic core through an externally applied force.	Pressure, force, displacement, position

<b>Eddy current gauge</b>	Inductance of a coil is varied by the proximity of an eddy current plate.	Displacement, thickness
<b>Magnetostriction gauge</b>	Magnetic properties are varied by pressure and stress.	Force, pressure, sound

#### 4. Voltage and Current

Hall effect pickup	A potential difference is generated across a semiconductor plate (germanium) when magnetic flux interacts with an applied current.	Magnetic flux, current power
Ionization chamber	Electron flow induced by ionization of gas due to radioactive radiation.	Particle counting radiation
Photoemissive cell	Electron emission due to incident radiation upon photoemissive surface.	Light and radiation
Photomultiplier tube	Secondary electron emission due to incident radiation on photosensitive cathode.	Light & radiation, photosensitive relays

#### Self-generating transducers (no external power) active transducer

Thermocouple and thermopile	An emf is generated across the junction of two dissimilar metals or semiconductors when that junction is heated.	Temperature, heat, flow, radiation
Moving coil generator	Motion of a coil in a magnetic field generates a voltage.	Velocity, vibrations
Piezoelectric pick-up	An emf is generated when an external force is applied to certain crystalline materials, such as quartz.	Sound, vibrations, acceleration, pressure changes.
Photovoltaic	A voltage is generated in a semiconductor junction device when radiant energy stimulates the cell.	Light meter, solar cell



## Primary and Secondary Transducers

- The transducers which convert the physical quantity into the mechanical displacement are called as the Primary transducers. eg. Bourdon Tube, Diaphragm, Bellows, etc.
- The transducers which convert the mechanical displacement into the electrical output are called as the Secondary transducers. eg. LVDT.

## Active and Passive Transducers

- Active transducers are those transducers which do not need any external power source to produce output.
- They produce the electrical power output themselves.  
eg. Piezoelectric transducers, thermocouple, photovoltaic cell
- Passive transducers are those transducers which draw power from an external source to produce their output.  
eg. Resistive, Capacitive, Inductive transducer.

## Analog and Digital Transducers

- The transducers which convert the input quantity into an analog output (i.e. the output which is continuous with time) are called analog transducers.  
eg. LVDT, Strain gauge etc.
- The transducers which convert the input quantity into an output which is in the form of pulses are called as the digital transducers.  
eg. Shaft encoders.

## Inverse Transducers

- Inverse Transducers are the device which convert the electric quantity into mechanical quantity.  
eg. Piezoelectric crystal.

## Characteristics of Transducers

### Transfer Function



- The transfer function is the ratio of Laplace transform of output to the Laplace transform of input.

### Sensitivity

- Sensitivity is defined as the ratio of the change in the output to the change in the input for a small time.

$$\text{Sensitivity} = \frac{\Delta Q_o}{\Delta Q_i}$$

### Accuracy

- Accuracy of a measuring system is defined as the closeness of the instrument output to the true value of the measured quantity.
- The accuracy of the instruments (which represents really its inaccuracy) can be specified in either of the following forms :

1. Percentage of true value

$$\frac{M.V - T.V}{T.V} \times 100$$

where

M.V. = measured value

T.V = true value

2. Percentage of full-scale deflection

$$\frac{M.V - T.V}{FSD} \times 100$$

FSD = full-scale deflection

### Precision

- Precision is defined as the ability of the instrument to reproduce a certain set of readings within a given accuracy.

### Resolution

- It is defined as the smallest increment in the measured value that can be detected with certainty by the instrument.
- The least count of any instrument is taken as the resolution of the instrument.

### Threshold

- It is defined as the minimum value of input below which no output can be detected.

### Range and Span

- The range of the instrument is specified by the lower and upper limits in which it is designed to operate for measuring, indicating or recording the measured variable.
- The algebraic difference between the upper and lower range values is termed as the **span** of the instrument.

## Resistive Transducers

- Resistive transducers are used for the measurement of a physical quantity in terms of the change in resistance produced by them.

Expression for the resistance of a conductor is given as:

$$R = \frac{\rho l}{A} \Omega$$

where,  $\rho$  = Resistivity of the conductor ( $\Omega\text{-m}$ )

$l$  = Length of the conductor (m)

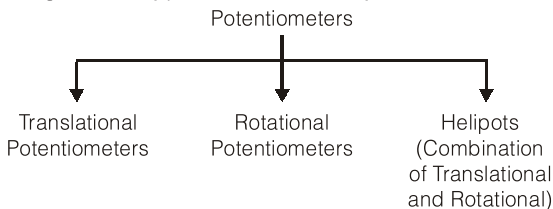
$A$  = Area of the cross-section of the conductor ( $\text{m}^2$ )

- Resistance of the conductor can be changed by change in either of the given three parameters (i.e. resistivity, length or the area of cross-section).

## Potentiometer Transducers

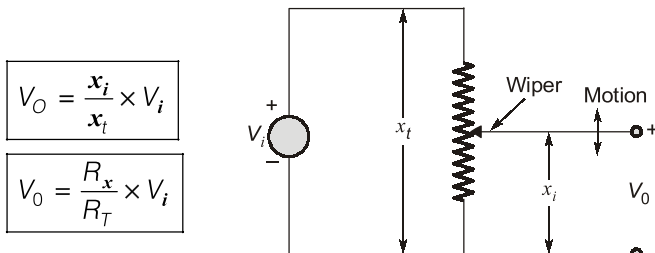
- These are the passive transducers used for the measurement of translatory and the rotational motions.

Depending on the type of motion, they can be classified as:



## Translational potentiometer Transducers

- The output signal of these transducers is a DC or AC voltage which is proportional to the potentiometer resistance that is proportional to the slider linear displacement



$$\text{Sensitivity (S)} = \frac{V_0}{x_i} = \frac{V_i}{x_t} \text{ V/m}$$

where,  $V_i$  and  $V_0$  = input and output voltages respectively (V).

$x_t$  = total length of the translational

potentiometer (m).

$x_i$  = displacement of the wiper from its zero position (m).

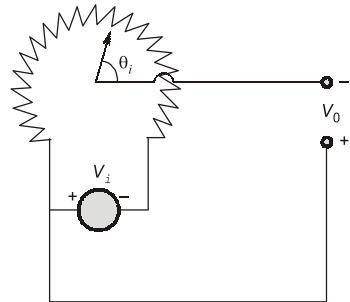
$R_x$  = resistance at output terminal of the potentiometer ( $\Omega$ ).

$R_t$  = resistance of the potentiometer ( $\Omega$ ).

### Rotational Potentiometer Transducers

- The output signal of these transducers is a DC or AC voltage which is proportional to the potentiometer resistance that is proportional to the slider angular displacement

$$V_0 = \frac{\theta_i}{\theta_t} \times V_i$$



Where,  $\theta_i$  = Input Angular displacement of wiper in degrees

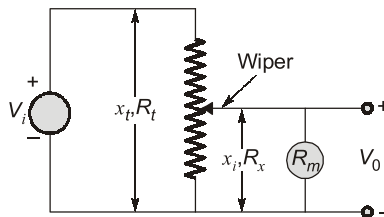
$\theta_t$  = Total wiper angular displacement in degrees

$$\text{Sensitivity (S)} = \frac{V_0}{\theta_i} = \frac{V_i}{\theta_t} \text{ V/degree}$$

**Note:** .....

- Pot is a zero order instrument.
  - For measurement of angular and translational displacement helipot is used.
  - Material used for pot are platinum, Nickel chromium, Nickel Copper etc.
- .....

### Loading Effect in Potentiometer



**Case I**  $R_m = \infty \Rightarrow V_0 = \frac{x_i}{x_t} V_i = k V_i$

**Case II**  $R_m = \infty \Rightarrow V_0 = \frac{k}{k(1-k) \left( \frac{R_t}{R_m} \right) + 1} V_i$

**Note:** .....

- For higher linearity of the potentiometer, the potentiometer resistance ( $R_p$ ) must be as low as possible, and meter resistance ( $R_m$ ) must be high (ideally infinite).
  - For higher sensitivity of the potentiometer, the potentiometer resistance ( $R_p$ ) must be high.
- .....

## Strain Gauges Transducers

These types of transducers are based on the principle that if a conductor is stretched or compressed, its resistance will change, because of change in its length, area and resistivity.

$$\text{Gauge factor}(G_F) = \frac{\text{Per unit change in Resistance}}{\text{Per unit change in Length}} = \frac{\Delta R / R}{\Delta L / L} = \frac{\Delta R / R}{\epsilon}$$

Where,  $\Delta R$  being change in resistance  $R$  due to axial strain( $\epsilon$ )

$$G_F = 1 + 2\nu + \frac{\Delta\rho/\rho}{\epsilon}$$

Where  $\epsilon = \text{Axial Strain} = \frac{\Delta L}{L}$

$\nu = \text{Poisson ratio}$

$$= \frac{\text{Lateral strain}}{\text{longitudinal strain}} = \frac{\text{Transverse strain}}{\text{Axial strain}}$$

$$\nu = \frac{-\partial D / D}{\partial L / L} = -\frac{\epsilon_t}{\epsilon}$$

## Types of Strain Gauges

### Unbonded metal wire strain gauges

- In such strain gauges a resistance wire is stretched between the two frames, one being the moving frame and the other one is fixed.
- They are made up of various alloys like copper-nickel, chrome-nickel or nickel-iron.

### Bonded wire strain gauge

- In such strain gauge, the wire is spread uniformly and hence they can be used to measure the stress which is spread uniformly over it.
- The material used is same as that used by unbonded metal wire strain gauge.

### Bonded metal foil strain gauge

- These type of strain gauges are the extension to the metal wire strain gauges.
- They have large surface area hence they have large heat dissipation capacity. So they are used at the higher operating range of temperature.

### Semiconductor strain gauge

- These type of strain gauge have higher sensitivity and higher gauge factor, also they have higher accuracy.
- For P type semiconductor strain gauge  $G_F$  is positive and for N type semiconductor strain gauge  $G_F$  is negative

### Note:

- Gauge factor for metals is nearby 2.
- Poisson ratio of metals lies between 0 to 0.5.
- For metallic and P type strain gauges  $G_F$  is positive that is on tensile stress resistance increases ( $R + \Delta R$ ) where as on the compressive stress resistance decreases ( $R - \Delta R$ ) .
- For N type strain gauges  $G_F$  is negative that is on tensile stress resistance decreases ( $R - \Delta R$ ) where as on the compressive stress resistance increases ( $R + \Delta R$ ) .

### Signal Conditioning for Strain Gauges

- The signal conditioning of strain gauge involves the use of the Wheatstone Bridge Circuit.
- The Wheatstone Bridge can operate in two modes.

Null Mode (No deflection)	Deflection Mode
<ul style="list-style-type: none"> <li>• More accurate</li> <li>• Used for static input only</li> <li>• Less used</li> <li>• Slow in operation</li> </ul>	<ul style="list-style-type: none"> <li>• Accuracy is low</li> <li>• Used for both static and dynamic inputs</li> <li>• More used</li> <li>• Fast in operation</li> </ul>

### NULL MODE

- In the null-mode the strain gauge behaves as one of the arm of the Wheatstone bridge circuit.
- The resistance  $R_2$  (kept variable) is varied until the galvanometer ( $G$ ) shows a zero or null deflection in the voltmeter.
- This change in the value of  $R_2$  (i.e.  $\Delta R_2$ ) is calibrated in term of the applied force.

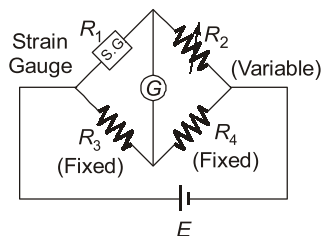
$$F \propto \epsilon_a \propto \Delta R_1 \propto \Delta R_2$$

$$\therefore \Delta R_2 \propto \epsilon_a \propto F$$

where,

$F$  = applied force;  $\epsilon_a$  = applied axial strain

$\Delta R_2$  = change in resistance  $R_2$ , to produce null deflection

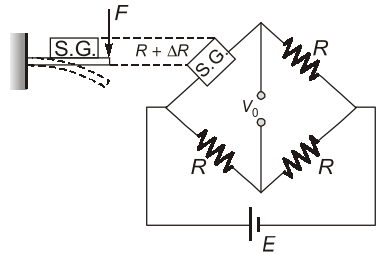


## DEFLECTION MODE

### Quarter Bridge

- One strain gauge is used
- This configuration has temperature related error which is cancel out by the connecting dummy strain gauge in the adjacent arm of the active strain gauge in the quarter bridge arrangement.

$$V_0 = \frac{E}{4} G_F \cdot \epsilon_a = \frac{E}{4} \frac{\Delta R}{R}$$



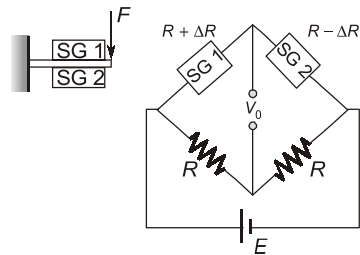
### Half Bridge

- Two strain gauges are used
- Sensitivity is improved than the quarter

$$S_{\text{Half Bridge}} = 2 S_{\text{Quarter Bridge}}$$

- Temperature error is removed

$$V_0 = \frac{E}{2} G_F \cdot \epsilon_a = \frac{E}{2} \frac{\Delta R}{R}$$



SG 1  $\Rightarrow$  Tensile Force  $\Rightarrow$  Positively strained  $\rightarrow R + \Delta R$

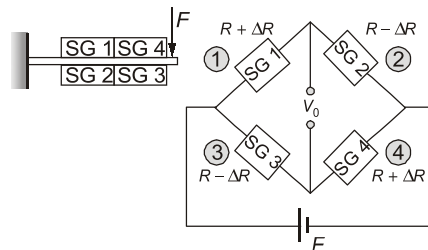
SG 2  $\Rightarrow$  Compressive Force  $\Rightarrow$  Negatively strained  $\rightarrow R - \Delta R$

### Full Bridge

- Four strain gauges are used
- Sensitivity of full bridge is more than the half bridge.

$$S_F : S_H : S_Q = 4 : 2 : 1$$

- Temperature error is removed.
- This method is always preferred for the stress and force measurement.



$$V_0 = E G_F \cdot \epsilon_a = E \frac{\Delta R}{R}$$

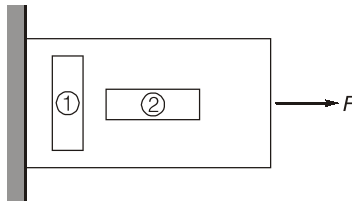
- 1 and 2 can be interchanged.
- 2 and 3 can be interchanged.

**Note:** .....

- For getting maximum sensitivity connect the strain gauges with opposite strain in the adjacent arm (i.e. in 1, 2 or 3, 4) and with same strain in opposite arm (i.e. in 1, 4 or 2, 3).
- .....

### Strain Measurement By Poissons Method

- In this method two strain gauges are bounded at right angles to each other (i.e. at  $90^\circ$ ).



(i) if force is away (  $\longrightarrow F$  )

①  $\Rightarrow$  Compressive

②  $\Rightarrow$  Tensile

(ii) if force is towards (  $\longleftarrow F$  )

①  $\Rightarrow$  Tensile

②  $\Rightarrow$  Compressive

The output voltage is given as

$$V_0 = \frac{E}{4} G_f \cdot \varepsilon (1 + \mu)$$

$\mu \Rightarrow$  Poissons constant

The output voltage is increased  $(1 + \mu)$  times.





## Capacitive Transducers

- Capacitive transducers are the transducers whose capacitance is changed by the change in the physical variables like displacement, force, and pressure.
- Capacitance of a parallel plate capacitor.

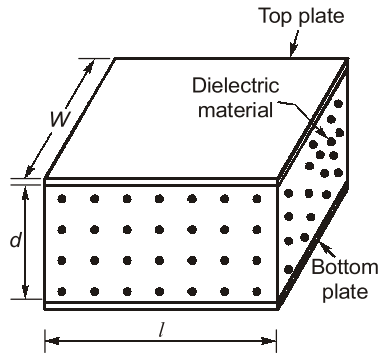
$$C = \frac{A \epsilon_r \epsilon_0}{d}$$

$A$  = overlapping area of the plates ( $\text{m}^2$ ).

$d$  = distance between two plates (m).

$\epsilon_0$  = permittivity of free space, ( $8.85 \times 10^{-12}$ ) F/m.

$\epsilon_r$  = relative permittivity



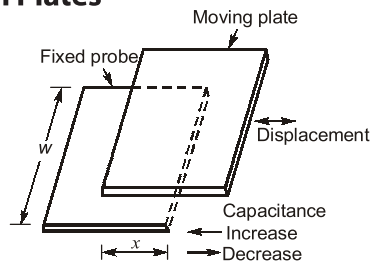
## Transducers Using Change in Area of Plates

### Sheet type capacitor

$$C = \frac{A\epsilon}{d} = \frac{(w \cdot x) \times \epsilon}{d} F$$

$x$  = length of overlapping part of plates (m).

$w$  = width of overlapping part of plates (m).



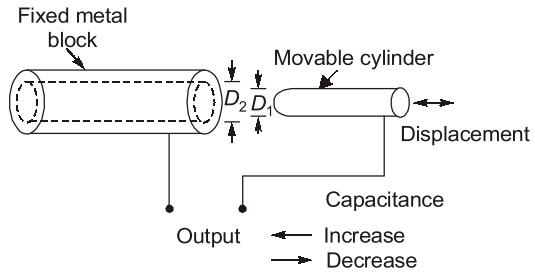
$$\text{Sensitivity}(S) = \frac{\partial C}{\partial x} = \frac{\epsilon w}{d} F/m$$

**Note:** .....

- This type of capacitive transducer is suitable for measurement of linear displacement ranging 1 mm to 10 mm.
- .....

## Cylindrical Capacitor

$$C = \frac{2\pi \epsilon_0 x}{\text{Log}_e \left[ \frac{D_2}{D_1} \right]}$$



$x$  = Length of overlapping part of cylinder (m)

$D_2$  = inner diameter of outer cylindrical electrode (m)

$D_1$  = Outer diameter of inner cylindrical electrode (m)

$$\text{Sensitivity (S)} = \frac{\partial C}{\partial x} = \frac{2\pi \epsilon}{\text{Log}_e \left[ \frac{D_2}{D_1} \right]} \text{ F/m}$$

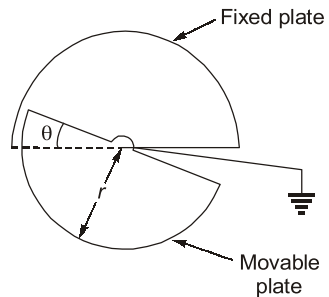
## Measurement of Angular Displacement

$$C = \frac{\epsilon \theta r^2}{2d} \text{ F}$$

$\theta$  = Angular displacement in rad.

$r$  = radius of the movable plate

$$\text{Sensitivity (S)} = \frac{\partial C}{\partial \theta} = \frac{\epsilon r^2}{2d} \text{ F/rad}$$

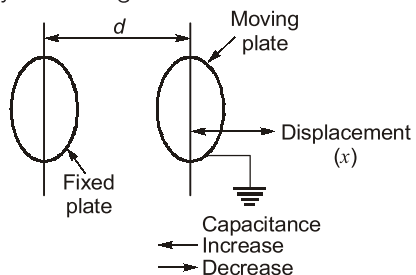


## Transducer Using Change in Distance Between Plates

- The capacitance can be varied by the change of distance ' $d$ ' between the plates.

$$C = \frac{\epsilon A}{d}$$

$$\text{Sensitivity (S)} = \frac{\partial C}{\partial d} = \frac{-\epsilon A}{d^2}$$



## Variation of Dielectric Constant Between Plates

$$C = \frac{\epsilon_0 W l_1}{d} + \frac{\epsilon_0 \epsilon_r W l_2}{d}$$

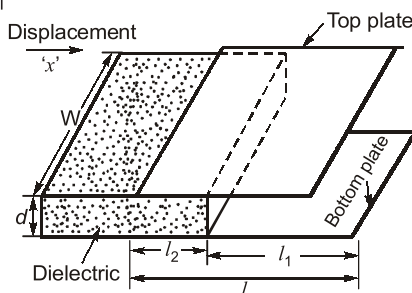
$\epsilon_r$  = permittivity of dielectric medium

$\epsilon_0$  = permittivity of free space

Change in capacitance due to displacement  $x$  of dielectric medium in the direction as shown in the figure

$$\Delta C = \epsilon_0 \frac{W x}{d} (\epsilon_r - 1)$$

$$\text{Sensitivity}(S) = \frac{\partial C}{\partial x} = \epsilon_0 \frac{W}{d} (\epsilon_r - 1)$$



## Measurement of liquid level by varying dielectric constant

$$C = 2\pi\epsilon_0 \frac{\epsilon_1 h_1 + \epsilon_2 h_2}{\log_e(r_2/r_1)}$$

$h_1$  = height of the liquid (m).

$h_2$  = height of cylinder above the liquid (m).

$\epsilon_1$  = relative permittivity of liquid.

$\epsilon_2$  = relative permittivity of vapour above liquid.

$r_2$  = inner radius of outer cylinder (m).

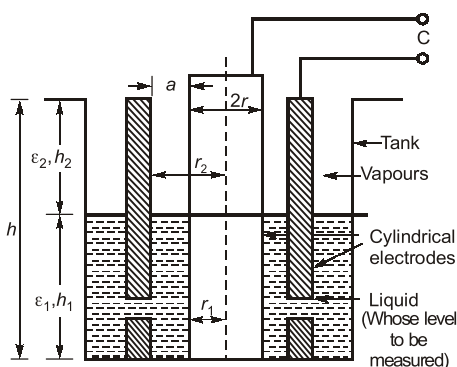
$r_1$  = outer radius of inner cylinder (m).

$\epsilon_0$  = permittivity of free space (F/m).

$h \gg r_2$  and  $r_2 \gg r_2 - r_1 \gg a$

Now,  $r_2 = r + a$  and  $r_1 = r$

$$\therefore C = 2\pi\epsilon_0 \frac{\epsilon_1 h_1 + \epsilon_2 h_2}{\log_e\left(1 + \frac{a}{r}\right)}$$



## Differential Arrangement

- This arrangement is used to make linear relationship between change in capacitors ( $\Delta C$ ) and displacement  $x$  applied to the moving plate.

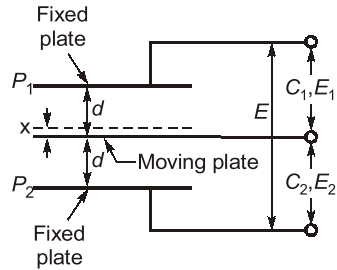
$$\Delta C = C_1 - C_2 = C_0 \frac{x}{d}$$

$C_0$  = nominal capacitance value

(no displacement is applied)

$x$  = displacement of moving plate

$$S = \frac{\partial C}{\partial x} = \frac{C_0}{d} \text{ pF/m}$$



Differential output voltage

$$\Delta E = E_1 - E_2 = E \frac{x}{d}$$

## Frequency response on Capacitance Transducers

$$\frac{E_o(s)}{X_i(s)} = \frac{\tau s}{1 + \tau s}$$

$$\text{Magnitude}(M) = \left| \frac{E_o(j\omega)}{X_i(j\omega)} \right| = \frac{\omega \tau}{\sqrt{1 + (\omega \tau)^2}}$$

$$\phi = \frac{\pi}{2} - \tan^{-1}(\omega \tau) \text{ rad}$$

### Note:

- Under static inputs  $\omega = 0$

$$\therefore M = \frac{0}{\sqrt{1 + 0}} = 0$$

Thus, capacitive transducers are not used under static conditions.

- Capacitive transducers have high input impedance, Thus less loading.
- Capacitive transducers are used to measure dynamic inputs.
- Capacitive inputs transducers have frequency response same, as of high pass filter.
- Capacitive transducers have high sensitivity