ESE 2025
Main Examination

UPSC ENGINEERING SERVICES EXAMINATION

Topicwise
Conventional
Practice Questions

Mechanical Engineering

PAPER-I

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C O N T E N T S

1 Thermodynamics

1. Basic Concepts & Zeroth law of thermodynamics

Level-1

1.1 The temperature t on a thermometric scale is defined in terms of a property k by the relation

$$t = a \ln k + b$$

where a and b are constants. The values of k are found to be 1.83 and 6.78 at the ice point and the steam point, the temperature of which are assigned the numbers 0 to 100 respectively. Determine the temperature corresponding to a reading of k equal to 2.42 on the thermometer.

(10 Marks)

Solution:

Given : $k_0 = 1.83$ at $t = 0°$, $k_{100} = 6.78$ at $t = 100°$

To find, $\quad t = ?$ at $k = 2.42$

$$0 = a \ln 1.83 + b$$
$$100 = a \ln 6.78 + b \quad \text{...(ii)}$$

(ii) – (i) $\quad 100 = a(\ln 6.78 - \ln 1.83)$

$\therefore \quad a = \dfrac{100}{1.3097}$

$\therefore \quad a = 76.356$

So, from (i) $\quad b = -a \ln 1.83 = -46.143$

at $k = 2.42$,

$\therefore \quad t = a \ln k + b = 76.356 \ln (2.42) - 46.143$

$\quad t = 21.338°$

1.2 The resistance of the windings in a certain motor is found to be 80 ohms at room temperature (25°C), when operating at full load under steady state conditions. The motor is switched off and the resistance of the windings immediately measured again and is found to be 93 ohms. The windings are made of copper whose resistance at temperature t°C is given by

$$R_t = R_0(1 + 0.00393\, t)$$

Where R_0 is the resistance at 0°C. Find the temperature attained by the coil during full load.

(10 Marks)

Solution:

Given, $R_t = 80\ \Omega$ at $t = 25°C$, $R_t = 93\ \Omega$ at $t = t_1$, $R_t = R_0(1 + 0.00393t)$, $80 = R_0(1 + 0.00393 \times 25)$

$\therefore \quad R_0 = \dfrac{80}{1.09825}$

$\therefore \quad R_0 = 72.843\ \Omega$

$\therefore \quad R_t = R_0(1 + 0.00393\, t)$

$\quad 93 = 72.843(1 + 0.00393\, t_1)$

$1 + 0.00393\, t_1 = 1.2767$

$\therefore \quad t_1 = 70.411°C$

2. Energy & its Interaction

Level-1

2.1 A mass of 1.5 kg of air is compressed in a quasi-static process from 0.1 MPa to 0.7 MPa. for which pv = constant. The initial density of air is 1.16 kg/m^3. Find the work done by the piston to compress the air.

(10 Marks)

Solution:

Given :
$$m = 1.5 \text{ kg}, \quad \rho_1 = 1.16 \text{ kg/m}^3,$$
$$v_1 = \frac{V_1}{m} = \frac{1}{\rho_1}$$
$$P_1 = 0.1 \text{ MPa}, \quad P_2 = 0.7 \text{ MPa}$$

For air, as an ideal gas,

$$Pv = RT \qquad \qquad ...(i)$$

and
$$w = \int_1^2 P dv$$

from (i),
$$p_1 v_1 = P_2 v_2 = \text{constant} \qquad \qquad ...(ii)$$

$$0.1 \times 10^6 \times \frac{1}{1.16} = \text{const} = k (\text{Let}) \qquad \qquad \left[P_1 \times \frac{1}{\rho_1} = k \right]$$

$$\therefore \qquad k = 0.086 \times 10^6$$

$$\therefore \qquad w = \int_1^2 \frac{k}{v} dv = k \ln v \Big]_1^2 = k \ln \frac{v_2}{v_1} = k \ln \left(\frac{P_1}{P_2} \right) \qquad \text{From (ii)}$$

$$w = 0.086 \times 10^6 \ln \left(\frac{0.1}{0.7} \right) = -0.16775 \times 10^6 \text{ J/kg.}$$

$$\therefore \qquad W = m \times w = 1.5 \times -0.16775 \times 10^3 \text{ kJ} = = -251.626 \text{ kJ}$$

$$\therefore \qquad \text{work done by piston} = 251.626 \text{ kJ}$$

2.2 A mass of gas is compressed in a quasi-static process from 80 kPa, 0.1 m^3 to 0.4 MPa, 0.03 m^3. Assuming that the pressure and volume are related by PV^n = constant? Find the work done by the gas system.

(10 Marks)

Solution:

Given :
$$P_1 = 80 \text{ kPa}$$
$$P_2 = 0.4 \text{ MPa} = 400 \text{ kPa}$$
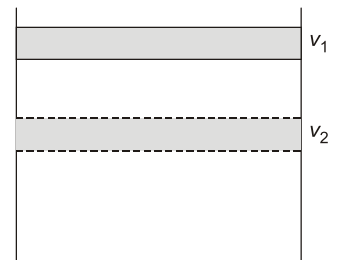$$V_1 = 0.1 \text{ m}^3$$
$$V_2 = 0.03 \text{ m}^3$$
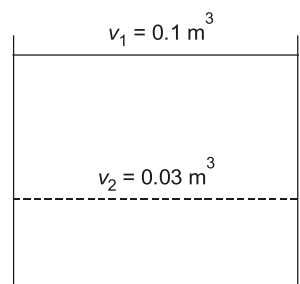$$Pv^n = \text{constant}$$
$$P_1 v_1^n = P_2 v_2^n$$
$$80 \times 0.1^n = 400 \times 0.03^n$$

$$\left(\frac{10}{3} \right)^n = 5$$

$$\therefore \qquad\qquad n = 1.3368$$

$$\therefore \qquad\qquad W = \int Pdv \ = \int \frac{C}{v^n}dv = \frac{cv^{1-n}}{1-n}\Bigg]_{V_1}^{V_2} \ = \frac{P_1v_1 - P_2v_2}{n-1}$$

$$= \frac{80 \times 0.1 - 400 \times 0.03}{1.3368 - 1}$$

$$W = -11.876 \text{ kJ}$$

2.3 A piston and cylinder machine containing a fluid system has a string device in the cylinder. The piston is frictionless, and it is held down against the fluid due to the atmospheric pressure of 101.325 kPa. The string device is turned 10,000 revolutions with an average torque against the fluid of 1.275 Nm. Meanwhile the piston of 0.6 m diameter moves out 0.8 m. Find the net work transfer for the system.
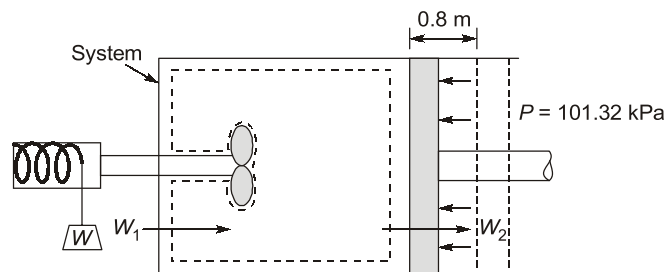
(10 Marks)

Solution:

Work done by the string device upon the system.

$$W_1 = 2\pi \ TN = 2 \ \pi \times 1.275 \times 10000 \text{ Nm}$$
$$= 80 \text{ kJ}$$

This is negative work for the system, work done by the system upon the surrounding

$$W_2 = (pA).L = \ 101.325 \times \frac{\pi}{4}(0.6)^2 \times 0.80 \ = 22.9 \text{ kJ}$$

This is positive work for the system. Hence, the work transfer for the system

$$W = W_1 + W_2$$
$$= -80 + 22.9$$
$$W = -57.1 \text{ kJ.}$$

2.4 To a closed system 150 kJ of work is supplied. If the initial volume is 0.6 m³ and pressure of the system changes as $P = 8 - 4V$, where P is in bar and V is in m³, determine the final volume and pressure of the system.

(10 Marks)

Solution:

Amount of work supplied to closed system = 150 kJ

initial volume = 0.6 m³

pressure volume relation ship, $P = 8 - 4V$

The work done during the process is given by,

$$W = \int_{V_1}^{V_2} PdV \ = 10^5 \int_{0.6}^{V_2} (8 - 4V) \ dV \ = 10^5\left[8 \ V - 4 \times \frac{V^2}{2}\right]_{0.6}^{V_2}$$

$$= 10^5\left[8(V_2 - 0.6) - 2(V_2^2 - 0.6^2)\right] \ = 10^5(8V_2 - 2V_2^2 - 4.08)\text{ Nm}$$

as this work is supplied to the system.

$$\therefore \qquad -150 \times 10^3 = 10^5(8V_2 - 2V_2^2 - 4.08)$$

$$2V_2^2 - 8V_2 + 2.58 = 0$$

$$V_2 = \frac{8 \pm \sqrt{64 - 4 \times 2 \times 2.58}}{4} = \frac{8 \pm 6.585}{4}$$

$$V_2 = 0.354 \, m^3$$

Positive sign is incompatible with the present problem, therefore it is not considered.

Final volume, $V_2 = 0.354 \, m^3$

Final pressure, $P_2 = 8 - 4V = 8 - 4 \times 0.354$

$$P_2 = 6.584 \, bar$$

$$P_2 = 6.584 \times 10^5 \, N/m^2 \, or \, Pa$$

2.5 A cylinder contains 1 kg of certain fluid of an initial pressure of 20 bar. The fluid is allowed to expand reversibly behind a piston according to a law PV^2 = constant until the volume is doubled. The fluid is then cooled reversibly at constant pressure until the piston regains its original volume, heat is then supplied reversibly with the piston firmly locked in position until the pressure rises to the original value of 20 bar. Calculate the net work done by the fluid, for an initial volume of 0.05 m³.

(10 Marks)

Solution:

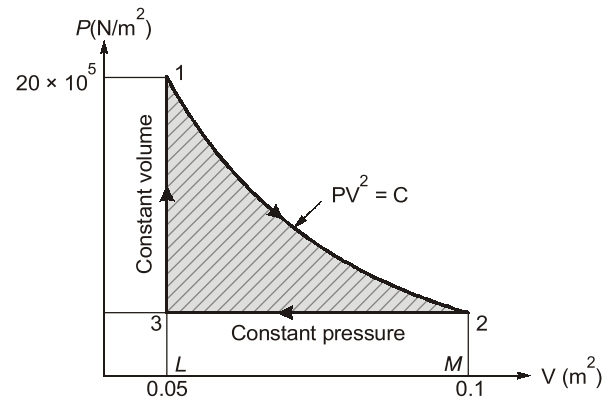Mass of fluid, m = 1 kg

$$P_1 = 20 \, bar = 20 \times 10^5 \, N/m^2$$

$$V_1 = 0.05 \, m^3$$

Considering the process 1 – 2

$$P_1 V_1^2 = P_2 V_2^2$$

$$P_2 = P_1 \left(\frac{V_1}{V_2}\right)^2 = 20 \left(\frac{V_1}{2V_1}\right)^2 \quad (\because V_2 = 2V_1 \text{ (given)})$$

$$= \frac{20}{4} = 5 \, bar$$

work done by the fluid from 1 to 2 = Area under = $\int_1^2 Pdv$

$$W_{1\text{-}2} = \int_{V_1}^{V_2} \frac{C}{v^2} dv$$

Where,

$$C = P_1 V_1^2 = (20 \times 10^5)(0.05)^2 \, Pa\text{-}m^6$$

$$\therefore \qquad W_{1\text{-}2} = 10^5 \times 20 \times 0.0025 \left[-\frac{1}{v}\right]_{0.05}^{0.1}$$

$$= 10^5 \times 20 \times 0.0025 \left(\frac{1}{0.05} - \frac{1}{0.1}\right) = 50000 \, Nm$$

work done on fluid from 2 to 3

$$= \text{Area under } 2\text{-}3 = P_2(V_2 - V_3)$$

$$= 10^5 \times 5 \times (0.1 - 0.05) = 25000 \, Nm$$

Work done during the process 3 – 1

$$= 0, \text{ because piston as locked in pistons}$$

(Volume remains constant)

$$\therefore \qquad \text{Net work done by the fluid} = \text{Enclosed area 1231} = 50000 - 25000$$

$$= 25000 \, Nm$$

Level-2

2.6 A fluid undergoes the following processes.

(i)   Heated reversibly at a constant pressure of 1.05 bar until it has a specific volume of 0.1 m^3/kg.

(ii)   It is then compressed reversibly according to a law Pv = constant to a pressure of 4.2 bar.

(iii)   It is then allowed to expand reversibly according to a law $Pv^{1.3}$ = constant.

(iv)   Finally it is heated at constant volume back to initial conditions.

The work done in the constant pressure process is 515 Nm and the mass of fluid present is 0.2 kg.

Calculate the net work done on or by the fluid in the cycle and sketch the cycle on P-v diagram.

(20 Marks)

Solution:

Given: m = 0.2 kg, $P_0 = P_1$ = 1.05 bar = 105 kPa, v_1 = 0.1 m^3/kg, w_1 = 515 Nm, P_2 = 4.2 bar, $v_3 = v_4$

State 0 = state 4

for process 0-1, constant pressure heat addition,

$$W_1 = mP_0(v_1 - v_0)$$
$$515 = 0.2 \times 105 \times 10^3 (0.1 - v_0)$$
$$v_0 = 0.0755 \ m^3/kg$$

For process 1-2, $P_1v_1 = P_2v_2$
$$1.05 \times 0.1 = 4.2 \times v_2$$
$$\therefore \qquad v_2 = 0.025 \ m^3/kg$$

$$\therefore \qquad W_2 = mP_1v_1 \ln\left(\frac{v_2}{v_1}\right) = 0.2 \times 105 \times 0.1 \ln\left(\frac{1}{4}\right)$$
$$= -2.9112 \ kNm = -2911.2 \ Nm. \qquad \ldots (i)$$

For process 3-4, constant volume heat rejection
$$\dot{W}_4 = 0 \qquad \ldots (ii)$$

process 2-3, $Pv^{1.3}$ = constant
$$P_2v_2^{1.3} = P_3v_3^{1.3}$$
$$420 \times (0.025)^{1.3} = P_3(0.0755)^{1.3}$$
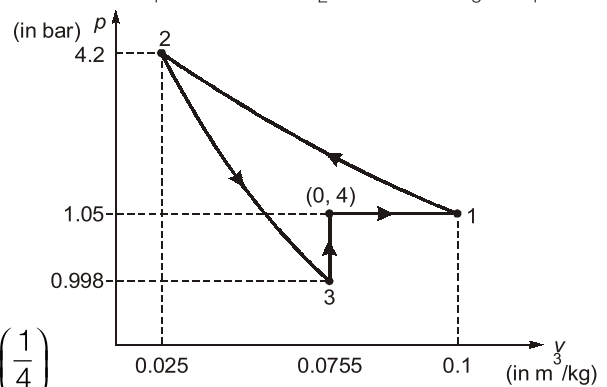$$\therefore \qquad P_3 = 99.825 \ kPa$$

$$\therefore \qquad \dot{W}_3 = \frac{\dot{m}(P_2v_2 - P_3v_3)}{n-1} = \frac{0.2(420 \times 0.025 - (99.82 \times 0.0755)}{(1.3-1)}$$
$$= 1.9755 \ kNm = 1975.5 \ Nm \qquad \ldots (iii)$$

$$\therefore \qquad \text{Net work} = W_{net} = \dot{W}_1 + \dot{W}_2 + \dot{W}_3 + \dot{W}_4 \qquad \text{[from (i), (ii) and (iii)]}$$
$$= 515 - 2911.2 + 1975.5 + 0$$

$$\dot{W}_{net} = -420.7 \ Nm$$

2.7 Figure shows a cylinder of 8 cm inside diameter having a piston loaded with a spring (stiffness 150 N/cm of compression). The initial pressure, volume and temperature of air in the cylinder are 3×10^5 N/m^2, 0.000045 m^3 and 20°C respectively. Determine the amount of heat added to the system. So that piston moves by 3.5 cm. Assume C_v = 0.717 kJ/kgK and R = 0.287 kJ/kgK.

(20 Marks)

Solution:

Given: D = 8 cm, k = 150 N/cm, P_1 = 300 kPa, V_1 = 0.000045 m^3, T_1 = 20°C, ΔL = 3.5 cm

Initial condition,

$$V_1 = 0.000045 \text{ m}^3$$

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4} \times 0.08^2$$
$$= 5.0265 \times 10^{-3} \text{ m}^2$$

$\therefore$ $L_1 = \dfrac{V_1}{A} = 0.00895 \text{ m}$

$\therefore$ $L_2 = 0.00895 + 0.035 = 0.04395$

$\therefore$ $V_2 = L_2 A = 0.0002209 \text{ m}^3$

$P_1 = 300 \text{ kPa}$

$P_2 A = P_1 A + k(L_2 - L_1)$

$\therefore$ $P_2 = P_1 + \dfrac{k}{A}(L_2 - L_1)$

$$= 300 \times 10^3 + \frac{15000}{5.0265 \times 10^{-3}} \times 0.035 = 404.446 \text{ kPa}$$

$\therefore$ $W = \left(\dfrac{P_1 + P_2}{2}\right)(V_2 - V_1)$

$$= \left(\frac{300 + 404.446}{2}\right)(220.9 - 45) \times 10^{-6}$$

$W = 61.956 \text{ J}$

Now, $\dfrac{P_1 V_1}{T_1} = \dfrac{P_2 V_2}{T_2}$
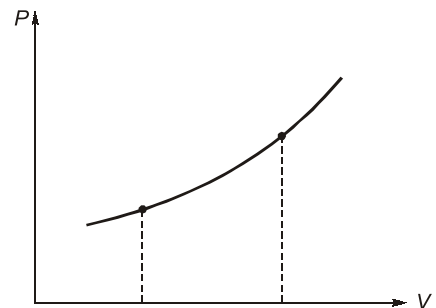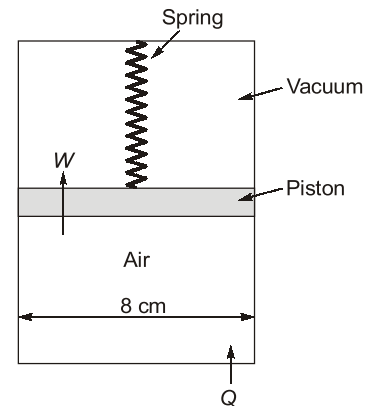
$\therefore$ $T_2 = \dfrac{404.446 \times 0.0002209}{300 \times 0.000045} \times 293$

$$= 1939.3 \text{ K}$$

$$m = \frac{P_1 V_1}{R T_1} = \frac{300 \times 45 \times 10^{-6}}{0.287 \times 293} = 0.0001605 \text{ kg}$$

$\therefore$ $\Delta U = m C_v(T_2 - T_1)$
$$= 0.0001605 \times 0.717(1939.3 - 293)$$
$$= 0.1845 \text{ kJ} = 189.5 \text{ J}$$

$\therefore$ $Q = \Delta U + W = 251.456 \text{ J}$

2.8 A system consisting of 1 kg of an ideal gas at 5 bar pressure and 0.02 m³ volume executes a cyclic processe comprising the following three distinct operations : (i) Reversible expansion to 0.08 m³ volume and 1.5 bar pressure, pressuring pressure to be a linear function of volume ($P = a + bV$), (ii) Reversible cooling at constant pressure and (iii) Reversible hyperbolic compression according to law PV = constant. This brings the gas back to initial conditions.
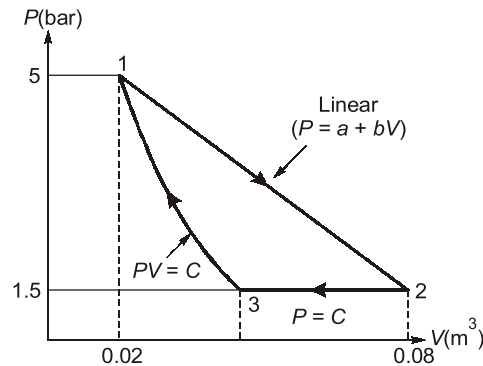(i)   Sketch the cycle on P-V diagram.
(ii)   Calculate the work done in each process stating whether it is done on or by the system and evaluate the net cyclic work and heat transfer.   **(20 Marks)**

Solution:

Given : m = 1 kg, P_1 = 5 bar, V_1 = 0.02 m³, V_2 = 0.08, P_2 = 1.5 bar

(i) P-V diagram :

(ii) Work done and heat transfer

Process 1 – 2 (linear law) :

$$P = a + bV$$

The value of constant a and b can be determined from the value of pressure and volume at the state points 1 and 2.

$$5 = a + 0.02\ b \qquad\qquad\qquad ...(i)$$
$$1.5 = a + 0.08\ b \qquad\qquad\qquad ...(ii)$$

from equation (i) and (ii), we get

$$b = -58.33 \text{ and } a = 6.167$$

$$W_{1-2} = \int_1^2 PdV = \int_1^2 (a + bV)dV = \int_1^2 (6.167 - 58.33\,V)\,dV$$

$$= 10^5 \left| 6.167\,V - 58.33 \times \frac{V^2}{2} \right|_{0.02}^{0.08}$$

$$= 10^5 \left| 6.167(0.08 - 0.02) - 58.33 \times \frac{(0.08^2 - 0.02^2)}{2} \right| \times 10^{-3} \text{kJ}$$

$$W_{1-2} = 19.5 \text{ kJ}$$

Process 2–3 (constant pressure) :

$$P_3 = P_2 = 1.5 \text{ bar}$$

The volume V_3 can be worked out from the hyperbolic compression 3–1, as follows:

$$P_1 V_1 = P_3 V_3$$

or

$$V_3 = \frac{P_1 V_1}{P_3} = \frac{5 \times 0.02}{1.5} = 0.0667 \text{ m}^3$$

Now work for 2-3 process,

$$W_{2\text{-}3} = P_2\,[V_3 - V_2] = (1.5)\,(10^5)\,[0.0667 - 0.08]$$
$$= -1995 \text{ J} = -1.995 \text{ kJ}$$

Process 3 – 1(Hyperbolic process)

$$W_{3-1} = P_3 V_3 \ln\left(\frac{V_1}{V_3}\right) = (10^5 \times 1.5) \times 0.0667 \ln\left(\frac{0.02}{0.0667}\right) \times 10^{-3} \text{kJ}$$

$$W_{3-1} = -12.05 \text{ kJ}$$

New work done, $W_{net} = W_{1-2} + W_{2-3} + W_{3-1}$

$$= 19.5 + (-1.995) + (-12.05) = 5.455 \text{ kJ}$$

Heat transferred during the complete cycle,

$$\oint \delta Q = \oint \delta W = 5.455 \text{ kJ}$$

3. First law of thermodynamics

Level-1

3.1 A certain gas of mass 4 kg is contained within a piston cylinder assembly. The gas undergoes a process for which $PV^{1.5}$ = constant. The initial state is given by 3 bar, 0.1 m³. The change in internal energy of the gas in the process is $u_2 - u_1 = -4.6$ kJ/kg. Find the net heat transfer for the process when the final volume is 0.2 m³. Neglect the changes in K.E and P.E.

(10 Marks)

Solution:

Given: m = 4 kg, P_1 = 3 bar, V_1 = 0.1 m³, V_2 = 0.2 m³, ΔU = –4.6 kJ/kg

For a process $\qquad\qquad PV^{1.5}$ = constant

$$P_1 V_1^{1.5} = P_2 V_2^{1.5}$$

$$300 \times (0.1)^{1.5} = P_2 \times (0.2)^{1.5}$$

$\therefore \qquad\qquad P_2$ = 106.066 kPa

$\therefore \qquad \Delta W = \int P dV = \dfrac{P_1 V_1 - P_2 V_2}{n-1} = \dfrac{300 \times 0.1 - 106.066 \times 0.2}{1.5 - 1}$ = 17.574 kJ

$\therefore \qquad \Delta U = \dot{m} \times (-4.6) = 4 \times (-4.6) = -18.4$ kJ

$\therefore \qquad Q = \Delta U + \Delta W = -18.4 + 17.574 = -0.826$

$\qquad\qquad Q = -0.826$ kJ

3.2 A system undergoes a process 1-2 in which it absorbs 200 kJ energy as heat while it does 100 kJ work. Then it follows the path 2-3 in which it rejects 50 kJ energy as heat when 80 kJ work is done on it. If it is required to restored the system to state 1 through an adiabatic path, calculate the work and heat interactions along the adiabatic path. Also calculate the net work and heat interaction.

(10 Marks)

Solution:

Application of the first law of thermodynamics to the process 1-2 gives.

$$U_2 - U_1 = Q_{12} - W_{12} = 200 - 100 = 100 \text{ kJ}$$
$$U_3 - U_2 = Q_{23} - W_{23} = -50 - (-80) = 30 \text{ kJ}$$

For the complete cycle $\Delta U = 0$

$$(U_2 - U_1) + (U_3 - U_2) + (U_1 - U_3) = 0$$
$$100 + 30 + (U_1 - U_3) = 0$$

or $\qquad\qquad U_1 - U_3 = -130 \text{ KJ}$

The process 3 – 1 is desired to be adiabatic therefore $Q_{31} = 0$

The first law of thermodynamics for the process 3 – 1, gives,

$$U_1 - U_3 = Q_{31} - W_{31}$$

or $\qquad\qquad -130 = 0 - W_{31}$

or $\qquad\qquad W_{31} = 130 \text{ kJ}$