Indian Forest Service Main Examination

Civil Engineering Paper-I

Also useful for Engineering Services Main Examination,
Civil Services Main Examination and
various State Engineering Services Examinations
Preface

Our country has a vast forest cover of near about 25% of geographical area and if man doesn't learn to treat trees with respect, man will become extinct; Death of forest is end of our life. Scientific management and judicial exploitation of forest becomes first task for sustainable development.

Engineer is one such profession which has an inbuilt word “Engineer – skillful arrangement” and hence IFS is one of the most talked about jobs among engineers to contribute their knowledge and skills for the arrangement and management for sustainable development.

In order to reach to the estimable position of Divisional Forest Officer (DFO), one needs to take an arduous journey of Indian Forest Service Examination. Focused approach and strong determination are the pre-requisites for this journey. Besides this, a good book also comes in the list of essential commodity of this odyssey. I feel extremely glad to launch the revised edition of such a book which will not only make Indian Forest Service Examination plain sailing, but also with 100% clarity in concepts.

MADE EASY team has prepared this book with utmost care and thorough study of all previous years’ papers of Indian Forest Service Examination. The book aims to provide complete solution to all previous years' questions with accuracy.

On doing a detailed analysis of previous years’ Indian Forest Service Examination question papers, it came to light that a good percentage of questions have been asked in Engineering Services, Indian Forest Services and State Services exams. Hence, this book is a one stop shop for all Indian Forest Service Examination, CSE, ESE and other competitive exam aspirants.

I would like to acknowledge efforts of entire MADE EASY team who worked day and night to solve previous years' papers in a limited time frame and I hope this book will prove to be an essential tool to succeed in competitive exams and my desire to serve student fraternity by providing best study material and quality guidance will get accomplished.

With Best Wishes

B. Singh
CMD, MADE EASY Group
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SYLLABUS

Part-A
ENGINEERING MECHANICS, STRENGTH OF MATERIALS AND STRUCTURAL ANALYSIS

ENGINEERING MECHANICS:
Units and Dimensions, SI Units, Vectors, Concept of Force, Concept of particle and rigid body. Concurrent, Non Concurrent and parallel forces in a plane, moment of force and Varignon's theorem, free body diagram, conditions of equilibrium, Principle of virtual work, equivalent force system. First and Second Moment of area, Mass moment of Inertia. Static Friction, Inclined Plane and bearings.

Kinematics and Kinetics:

STRENGTH OF MATERIALS:

STRUCTURAL ANALYSIS:
Castiglianio’s theorems I and II, unit load method, method of consistent deformation applied to beams and pin jointed trusses. Slope-deflection, moment distribution, Kani’s method of analysis and column Analogy method applied to indeterminate beams and rigid frames. Rolling loads and Influences lines: Influences lines for Shear Force and Bending moment at a section of beam. Criteria for maximum shear force and bending Moment in beams traversed by a system of moving loads. Influences lines for simply supported plane pin jointed trusses.

Arches:
Three hinged, two hinged and fixed arches, rib shortening and temperature effects, influence lines in arches.

Matrix methods of analysis:
Force method and displacement method of analysis of indeterminate beams and rigid frames.

Plastic Analysis of beams and frames:
Theory of plastic bending, plastic analysis, statitical method, Mechanism method.

Unsymmetrical bending:
Moment of inertia, product of inertia, position of Neutral Axis and Principle axes, calculation of bending stresses.

Part-B
DESIGN OF STRUCTURES: STEEL, CONCRETE AND MASONRY STRUCTURES.

STRUCTURAL STEEL DESIGN:

DESIGN OF CONCRETE AND MASONRY STRUCTURES:
Concept of mix design. Reinforced Concrete: Working Stress and Limit State method of design- Recommendations of I.S. codes design of one way and two way slabs, stair-case slabs, simple and continuous beams of rectangular, T and L sections. Compression members under direct load with or without eccentricity, Isolated and combined footings. Cantilever and Counterfort type retaining walls.

Water tanks:
Design requirements for Rectangular and circular tanks resting on ground.

Prestressed concrete:
Methods and systems of prestressing, anchorages, Analysis and design of sections for flexure based on working stress, loss of prestress.

Design of brick masonry as per I.S. Codes.
Design of masonry retaining walls.
Part-C
FLUID MECHANICS, OPEN CHANNEL FLOW AND HYDRAULIC MACHINES

FLUID MECHANICS

Fluid properties and their role in fluid motion, fluid statics including forces acting on plane and curve surfaces. Kinematics and Dynamics of Fluid flow: Velocity and accelerations, stream lines, equation of continuity, irrotational and rotational flow, velocity potential and stream functions, flownet, methods of drawing flownet, sources and sinks, flow separation, free and forced vortices. Control volume equation, continuity, momentum, energy and moment of momentum equations from control volume equation, Navier-Stokes equation, Euler’s equation of motion, application to fluid flow problems, pipe flow, plane, curved, stationary and moving vanes, sluice gates, weirs, orifice meters and Venturi meters.

Dimensional Analysis and Similitude: Buckingham’s Pi-theorem, dimensionless parameters, similitude theory, model laws, undistorted and distorted models.

Laminar Flow: Laminar flow between parallel, stationary and moving plates, flow through tube.

Boundary layer: Laminar and turbulent boundary layer on a flat plate, laminar sub-layer, smooth and rough boundaries, drag and lift.

Turbulent flow through pipes: Characteristics of turbulent flow, velocity distribution and variation of pipe friction factor, hydraulic grade line and total energy line, siphons, expansion and contractions in pipes, pipe networks, water hammer in pipes and surge tanks.

Open channel flow: Uniform and non-uniform flows, momentum and energy correction factors, specific energy and specific force, critical depth, resistance equations and variation of roughness coefficient, rapidly varied flow, flow in contractions, flow at sudden drop, hydraulic jump and its applications surges and waves, gradually varied flow, classification of surface profiles, control section, step method of integration of varied flow equation, moving surges and hydraulic bore.

HYDRAULIC MACHINES AND HYDROPOWER:

Centrifugal pumps: Types, characteristics, Net Positive Suction Height (NPSH), specific speed. Pumps in parallel.

Part-D
GEO TECHNICAL ENGINEERING

Types of soil, phase relationships, consistency limits particles size distribution, classifications of soil, structure and clay mineralogy. Capillary water and structural water, effective stress and pore water pressure, Darcy’s Law, factors affecting permeability, determination of permeability, permeability of stratified soil deposits. Seepage pressure, quick sand condition, compressibility and consolidation, Terzaghi’s theory of one dimensional consolidation, consolidation test. Compaction of soil, field control of compaction. Total stress and effective stress parameters, pore pressure coefficients. Shear strength of soils, Mohr Coulomb failure theory, shear tests. Earth pressure at rest, active and passive pressures, Rankine’s theory, Coulomb’s wedge theory, earth pressure on retaining wall, sheetpile walls, Braced excavation. Bearing capacity, Terzaghi and other important theories, net and gross bearing pressure. Immediate and consolidation settlement. Stability of slope, Total Stress and Effective Stress methods, Conventional methods of slices, stability number. Subsurface exploration, methods of boring, sampling, penetration tests, pressure meter tests. Essential features of foundation, types of foundation, design criteria, choice of type of foundation, stress distribution in soils, Boussinessq’s theory, Newmarks’s chart, pressure bulb, contact pressure, applicability of different bearing capacity theories, evaluation of bearing capacity from field tests, allowable bearing capacity, Settlement analysis, allowable settlement. Proportioning of footing, isolated and combined footings, rafts, buoyancy rafts, Pile foundation, types of piles, pile capacity, static and dynamic analysis, design of pile groups, pile load test, settlement of piles, lateral capacity. Foundation for Bridges. Ground improvement techniques-preloading, sand drains, stone column, grouting, soil stabilisation.
Q.1 The bars $AB$, $AC$ and $AD$ shown in indeterminate system (see figure below) are made of steel and have same cross-sectional area of 350 mm$^2$ and they together carry a load of 75 kN, applied at $A$ as shown. There is no initial stress in bars before application of load. $\alpha = 30^\circ$ and $l = 3000$ mm. Find force in each bar and vertical displacement of point $A$ after load is applied. Take $E = 205$ kN/mm$^2$. [15 marks : 2002]

Solution:

\[ \Delta_{AX} = \frac{\Delta_{AD} - \Delta_{AB}}{\cos 30^\circ} \]
\[ \Delta_{AX} = \frac{F_{AD} \times 3000 \times 1000}{350 \times 205 \times 1000} \]
\[ \Delta_{AD} = \frac{F_{AB} \times 1000 \times 3464.1}{350 \times 205 \times 1000} \]
\[ \Delta_{AB} = \frac{F_{AD} \times 1000 \times 3464.1}{350 \times 205 \times 1000} \]

\[ (3000)F_{AC} = \frac{F_{AB} \times 3000}{\sqrt{3}/2} \]
\[ (3000)F_{AC} = \frac{F_{AB} \times 3000}{3/4} \]

Also
\[ 3F_{AC} = 4F_{AB} \quad \ldots \,(i) \]
\[ F_{AC} + F_{AB} \cos 30^\circ + F_{AD} \cos 30^\circ = 75 \quad \ldots \,(ii) \]
\[ F_{AB} \sin 30^\circ = F_{AD} \sin 30^\circ \]
\[ F_{AB} = F_{AD} \quad \ldots \,(iii) \]
\[ F_{AC} + \sqrt{3}F_{AB} = 75 \]
\[ F_{AC} + \frac{3\sqrt{3}}{4}F_{AC} = 75 \]
\[ F_{AC} = \frac{75 \times 4}{(4 + 3\sqrt{3})} = 32.622 \text{ kN} \]
\[ F_{AB} = \frac{3F_{AC}}{4} = 24.47 \text{ kN} \]

If 1 kN load is applied then

\[ k_{AC} = F_{AC} = \frac{4t}{4 + 3\sqrt{3}} = 0.435 \text{ kN} \]

\[ k_{AB} = k_{AD} = 0.3263 \text{ kN} = \frac{24.47}{75} \]

\[ \Delta V = \sum \frac{F_i k_i I_i}{AE} \]

\[ \Delta V = \frac{32.622 \times 0.435 \times 1000 \times 3000 + 2 \times (24.47 \times 1000 \times 3464.1 \times 0.3263)}{350 \times 205 \times 1000} \]

\[ \Delta V = 1.364 \text{ mm} \]

Q.2 Find elongation of a bar of uniform cross-section area ‘A’ and length ‘l’ under action of its own weight. The bar weight ‘w’ per unit length. \( E \) = Modulus of elasticity. See figure below:

Solution:

Take element of cross-sectional area ‘A’ with length ‘dx’ elongate under weight ‘w’

\[ w' = wx \]

\( \Delta l = \text{Total elongation, } dl = \text{Elements elongation} \]

\[ dl = \frac{(wx)dx}{AE} \]

\[ \int dl = \Delta l = \int_0^l \frac{(wx)dx}{AE} \]

\[ \Delta l = \frac{wl^2}{2AE} = \frac{Wl}{2AE} \]

Q.3 A 50 mm cube is subjected to uniform pressure of 200 MPa. When the change in dimension between 2 parallel faces of cube is 0.025 mm. Determine change in volume of cube, \( \mu = 0.25 \).

Solution:
\[ \rho = 200 \text{ MPa} \]

\[ \Delta l = \left[ -\frac{\rho}{E} + \mu \frac{\rho}{E} + \mu \frac{\rho}{E} \right] \times l \]

Given, \( \Delta l = -0.025 \text{ mm} \), \( l = 50 \text{ mm} \), \( \rho = 200 \text{ MPa} \)

\[ \frac{50 \times 200}{E} \times (2 \times 0.25 - 1) = -0.025 \]

\[ E = 2 \times 10^5 \text{ MPa} \]

\[ B = \text{Bulk modulus} = \frac{E}{3(1-2\mu)} \]

\[ B = \frac{2 \times 10^5}{3 \times (1 - 2 \times 0.25)} = 1.33 \times 10^5 \text{ MPa} \]

\[ B = \frac{-\rho}{(\Delta V/V)} \quad (\rho \text{ should be insulated with sign}) \]

\[ 1.33 \times 10^5 = \frac{(-200)}{\Delta V/(50)^3} \]

\[ \frac{\Delta V}{(50)^3} = 150 \times 10^{-5} \]

\[ \Delta V = 187.5 \text{ mm}^3 \]

**Alternative**

\[ \Delta l_1 = 0.025 \text{ mm} \]

\[ \frac{\Delta V}{V} = \varepsilon_v = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \]

As it is cube so

\[ \Delta l_1 = \Delta l_2 = \Delta l_3 \quad \text{(Uniform pressure)} \]

\[ \varepsilon_v = \left( \frac{0.025}{50} \right) \times 3 = 1.5 \times 10^{-3} \]

\[ \frac{\Delta V}{V} = 1.5 \times 10^{-3} \]

\[ \Rightarrow \]

\[ \Delta V = (1.5 \times 10^{-3}) V \]

\[ \Delta V = 187.5 \text{ mm}^3 \]

**Q.4** A rigid bar \( AD \) is pinned at \( A \) and attached to the bars \( BC \) and \( ED \) as shown in figure. The entire system is initially stress free and weights of all bars are negligible. The temperature of bar BC is lowered 25°C and of bar ED is raised 25°C. Neglecting any possibility of lateral buckling, find normal stress in bars BC and ED. For BC which is brass, assume \( E = 90 \text{ GPa} \), \( \alpha = 20 \times 10^{-6}/\text{°C} \) and for ED, which is steel take \( E = 200 \text{ GPa} \) and \( \alpha = 12 \times 10^{-6}/\text{°C} \). Cross-sectional area of BC is 500 mm² and of ED is 250 mm².

[10 marks : 2011]

**Solution:**

Let stress in brass = \( \sigma_b \) (Tensile)

Stress in steel = \( \sigma_s \) (Compressive)  \( \text{(Assume)} \)
\[ (\sum M)_A = 0 \] for static rotational equilibrium

\[ \sigma_b A_b \times 250 + \sigma_s A_s \times 600 = 0 \]
\[ \sigma_b \times 500 \times 250 + \sigma_s \times 250 \times 600 = 0 \]
\[ \sigma_b = (-\sigma_s) \times 1.2 \] \( \ldots (i) \)

Now as ABD bar is rigid so

\[ \frac{\Delta l_1}{250} = \frac{\Delta l_2}{600} \]
\[ \Delta l_1 = \frac{300 \times 20 \times 10^{-6} \times 25 - \sigma_b \times 0.9 \times 10^5 \times 300}{2 \times 10^5} \]
\[ \Delta l_2 = 0.15 - 3.33 \times 10^{-3} \sigma_b \text{ (mm)} \]

All stress values are in N/mm²

\[ E = \text{in N/mm}^2 \text{ or MPa} \]

\[ \Delta l_2 = l_s \alpha_3 \Delta T - \frac{\sigma_s}{E_s} \]
\[ \Delta l_2 = 250 \times 12 \times 10^{-6} \times 25 - \frac{\sigma_s}{2 \times 10^5} \times 250 \]
\[ \Delta l_2 = 0.075 - 1.25 \times 10^{-3} \sigma_s \]

Now,
\[ 0.075 - 1.25 \times 10^{-3} \sigma_s = 2.4 (0.15 - 3.33 \times 10^{-3} \sigma_b) \]
\[ \Rightarrow 8 \times 10^{-3} \sigma_b - 1.25 \times 10^{-3} \sigma_s = 0.285 \]
\[ 8 \sigma_b - 1.25 \sigma_s = 285 \]
\[ \sigma_b = 31.521 \text{ N/mm}^2 \text{ (Tensile)} \]
\[ \sigma_s = -26.27 \text{ N/mm}^2 \]
\[ \sigma_s = 26.27 \text{ N/mm}^2 \text{ (Tensile)} \]

Q.5 A metallic bar 250 mm x 100 mm x 50 mm is loaded as shown in the below figure. Workout change in volume. What should be change that should be made in 4 MN load in order that there should be no change in the volume of the bar? Assume \( E = 2 \times 10^5 \) N/mm² and \( \mu = 0.25 \).

Solution:

Initial volume = \( 250 \times 100 \times 50 = 1250000 \text{ mm}^3 \)
\[ l_1 = 100 \text{ mm} \]
\[ l_2 = 250 \text{ mm} \]
\[ l_3 = 50 \text{ mm} \]

[15 marks : 2012]
\( \varepsilon_y = \frac{2 \times 10^6 \times 100}{250 \times 50 \times 2 \times 10^5} + \frac{0.25 \times 4 \times 10^6 \times 50}{250 \times 100 \times 2 \times 10^5} - \frac{0.25 \times 400 \times 10^3 \times 250}{100 \times 50 \times 2 \times 10^5} = 0.065 \)

\( \varepsilon_x = \frac{400 \times 10^3 \times 250}{100 \times 50 \times 2 \times 10^5} + \frac{0.25 \times 4 \times 10^6 \times 50}{250 \times 100 \times 2 \times 10^5} - \frac{0.25 \times 2 \times 10^9 \times 100}{250 \times 50 \times 2 \times 10^5} = 0.09 \)

\( \varepsilon_z = \frac{-4 \times 10^6 \times 50}{250 \times 100 \times 2 \times 10^5} + \frac{0.25 \times 400 \times 10^3 \times 250}{100 \times 50 \times 2 \times 10^5} - \frac{0.25 \times 2 \times 10^6 \times 100}{250 \times 5 \times 2 \times 10^5} = -0.085 \)

\( l'_y = l_1 (1 + \varepsilon_y) = 100 (1 + 0.065) = 106.5 \text{ mm} \)

\( l'_x = l_2 (1 + \varepsilon_x) = 250 (1 + 0.09) = 272.5 \)

\( l'_z = l_3 (1 + \varepsilon_z) = 50 (1 - 0.085) = 45.75 \)

\( V = l'_y l'_x l'_z = 106.5 \times 272.5 \times 45.75 = 1327022.188 \text{ mm}^3 \)

\( \Delta V = V' - V = 777222.1875 \text{ mm}^3 \)

Let

\[ P_1 = 2000 \text{ kN} \implies \sigma_1 = 160 \text{ MPa} \]

\[ P_2 = 400 \text{ kN} \implies \sigma_2 = 80 \text{ MPa} \]

\[ P_3 = ? \implies \sigma_3 = -\frac{P_3 \times 1000 \times 1000}{250 \times 100} = -40 \text{ MPa} \]

\( \varepsilon_y = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} + \mu \frac{\sigma_3}{E} \)

\[ = \frac{1}{E} \left[ 160 - 0.25 \times 80 + 0.25 \times 40P_3 \right] \]

\[ = \frac{1}{E} \left[ 140 + 10P_3 \right] \]

\( \varepsilon_x = \frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E} + \mu \frac{\sigma_3}{E} \)

\[ = \frac{1}{E} \left[ 80 - 0.25 \times 160 + 0.25 \times 40P_3 \right] \]

\[ = \frac{1}{E} \left[ 40 + 10P_3 \right] \]

\( \varepsilon_z = -\frac{\sigma_3}{E} - \mu \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} \)

\[ = \frac{1}{E} \left[ -40P_3 - 0.25 \times 160 - 0.25 \times 80 \right] \]

\[ = \frac{1}{E} \left[ -40P_3 + 60 \right] \]

\( \Delta V = \varepsilon_y + \varepsilon_x + \varepsilon_z = 0 \)

\[ \Rightarrow \frac{1}{E} (140 + 10P_3 + 40 + 10P_3 - 40P_3 - 60) = 0 \]

\[ \Rightarrow 120 - 20P_3 = 0 \]

\[ \Rightarrow P_3 = 6 \text{ MN} \]

Q.6 \( E = 2 \times 10^5 \text{ N/mm}^2 \)
A member \( ABCD \) is subjected to concentrated loads as shown. Calculate
(i) Force \( P \) necessary for equilibrium
(ii) Total elongation of bar
Solution:

(i) \[ \Sigma F = 0 \]
\[ (P + 300) - (150 + 600) = 0 \]
\[ P = 450 \text{kN} \]

(ii) \[ \Delta_{\text{Total}} = \Delta_{AB} + \Delta_{BC} + \Delta_{CD} \]
\[ = \frac{150 \times 1000 \times 1000}{600 \times 2 \times 10^5} - \frac{300 \times 1000 \times 1000}{2400 \times 2 \times 10^5} + \frac{300 \times 600 \times 1000}{1200 \times 2 \times 10^5} \]
\[ = 1.25 - 0.625 + 0.75 \]
\[ = 1.375 \text{mm (elongation)} \]

Q.7 Three steel bars \( A, B \) and \( C \) having same axial rigidity \( AE \) support a horizontal rigid beam \( A, B, C \) as shown in figure. Determine distance 'a' between bars \( A \) and \( B \). In order that rigid beam will remain horizontal. When a load '\( P \)' is applied at its mid point. The value of length is given within parenthesis.

Solution:

The beam is rigid and as per given condition/situation of beam to be remain horizontal following conditions must be satisfied.
1. Static equation
2. Axial elongation in all steel bars is equal and it takes place gradually.
3. There should be no net moment about any point.

From left, strings are named as 1, 2, and 3 respectively.
\[ F_1 + F_2 + F_3 = F \] ... (i)
\[ \frac{F_1 (4b)}{AE} = \frac{F_2 (3b)}{AE} = \frac{F_3 (2b)}{AE} \] ... (ii)
\[ 4F_1 = 3F_2 = 2F_3 \] ... (iii)

\[ \Rightarrow \quad F_1 + \frac{4F_1}{3} + \frac{4F_1}{2} = F \]
\[ \Rightarrow \quad \frac{13}{3} F_1 = F \]
\[ F_1 = \frac{3}{13} F \]

\[ F_2 = \frac{4}{13} F \]

\[ F_3 = \frac{6F}{13} \]

Now, \((\Sigma M)_A = 0\) to prevent any rigid body rotation.

\[ F_2 a + F_3 l = F \left( \frac{l}{2} \right) \]

\[ \frac{4F}{13} a + \frac{6F}{13} l = \frac{Fl}{2} \]

\[ \frac{4F}{13} a = Fl \times \frac{1}{26} \]

\[ a = \frac{l}{8} \]

Q.8 An aluminium square bar having the cross-section 50 mm x 50 mm and length 3 metres is fixed between two rigid supports as shown in the figure. Two loads, 15 kN and 30 kN are applied concentrically to the rod through collars as shown. Determine the stress developed at the right end of the bar. Young’s modulus of aluminium is \(70 \times 10^9\) N/m².

**Solution:**

Given, Cross-section of aluminium bar = 50 mm x 50 mm

Young’s modulus of aluminium, \(E = 70 \times 10^9\) N/m²

Let the reactions at A and D are \(R_A\) and \(R_D\) respectively.

Drawing free body diagrams individually;

Let there is tension in CD, let it be \(x\), then

\[ x = R_A - 15 = 30 - R_D \]

\[ R_A + R_D = 45\ \text{kN} \]

\[ \Delta_{AB} = \frac{R_A \times 1000}{AE} \] (Elongation) \(\because\) Bar is prismatic, \(AE\) is constant
\[
\Delta_{BC} = \frac{x \times 1000}{AE} \quad \text{(Elongation)}
\]
\[
\Delta_{CD} = \frac{R_D \times 1000}{AE} \quad \text{(Compression)}
\]

But \(A\) & \(D\) are fixed,
\[
\frac{R_A \times 1000}{AE} + \frac{x \times 1000}{AE} - \frac{R_D \times 1000}{AE} = 0
\]
\[R_A \times x - R_D = 0\]
\[\Rightarrow R_A + (R_A - 15) - R_D = 0\]
\[\Rightarrow 2R_A - 15 - R_D = 0\]
\[\Rightarrow 2R_A - R_D = 15 \quad \text{...(ii)}\]

Solving equation (i) and (ii), we get
\[R_A = 20 \text{ kN}\]
\[R_D = 25 \text{ kN}\]

\[\therefore \text{ Stress developed on right end of bar = Stress at fixed end D}\]
\[\sigma_0 = \frac{-R_D}{A} = \left(\frac{-25000}{50 \times 50}\right) = -10 \text{ N/mm}^2 \quad \text{(Compression)}\]

**Q.9** If two pieces of materials ‘A’ and ‘B’ have the same bulk modulus, but the value of Modulus of Elasticity for ‘B’ is 1% greater than that for ‘A’, find the value of Modulus of Rigidity for the material ‘B’ in terms of Modulus of Elasticity and Modulus of Rigidity for material ‘A’.

[8 marks : 2017]

**Solution:**

Let \(E_A, K_A, G_A\) and \(E_B, K_B, G_B\) be the modulus of elasticity, bulk modulus and modulus of rigidity of materials \(A\) and \(B\) respectively.

Given,
\[K_A = K_B \text{ and } E_B = 1.01 E_A\]

We know,
\[E = \frac{9KG}{3K + G}\]

or,
\[3KE + EG = 9 KG\]

or,
\[3K(3G - E) = EG\]

from where, we get
\[K = \frac{EG}{3(3G - E)}\]

Hence,
\[
\frac{E_A G_A}{3(3G_A - E_A)} = \frac{E_B G_B}{3(3G_B - E_B)}
\]
\[\therefore E_A G_A(3G_B - E_B) = E_B G_B(3G_A - E_A)\]

or,
\[3E_A G_A G_B - E_A G_B = 3G_A G_B - E_A E_B G_B\]

or,
\[G_B(3E_A G_A - 3G_A E_B + E_A E_B) = E_A G_B E_B\]

\[\therefore G_B = \frac{E_A G_A E_B}{3E_A G_A - 3G_A E_B + E_A E_B}\]

\[\Rightarrow G_B = \frac{1.01E_A G_A E_A}{3E_A G_A - 3 \times 1.01E_A G_A + 1.01E_A E_A}\]

or,
\[G_B = \frac{1.01E_A G_A}{3G_A - 3.03G_A + 1.01E_A} = \frac{101E_A G_A}{10E_A - 3G_A}\]
Q.10  A structural frame is loaded as shown in figure. It has 3 hinges at C, D and E. C and E being at same level. Calculate reaction at A and B.

Solution:

\[ r_e = 3 + 3 = 6 \]
\[ m = 4 \]
\[ r' = 3 \text{ (3 internal hinge)} \]
\[ j = 5 \]
\[ D_{pe} = 3m + r_e - 3j - r' = 3 \times 4 + 6 - 3 \times 5 - 3 = 0 \text{ (Statically determinate)} \]

\[ V_A + V_B = 20 \] \quad ...(i)

\[ H_A + H_B = 0 \] \quad ...(ii)

\[ (\Sigma M)_A = 0 \]

\[ M_{A} + 13V_B - M_B - H_B \times 1 - 25 - 10 \times 7 = 0 \]

\[ M_{A} + 13V_B - M_B - H_B = 95 \] \quad ...(iii)

\[ M_C = 0 \text{ (Left)} \]

\[ M_D = 0 \text{ (Right)} \]

\[ M_E = 0 \text{ (Right)} \]

\[ 5V_A - M_A = 25 \] \quad ...(iv)

\[ 4V_B = 4H_B + M_B \] \quad ...(v)

\[ V_B = H_B + M_B \] \quad ...(vi)

From eq. (v) and (vi)

\[ M_B = 0 \]

\[ V_B = 4H_B \]

\[ V_A = 20 - H_B \]

\[ 100 - 5H_B - 25 = M_A \]

\[ M_A = 75 - 5H_B \]

\[ 75 - 5H_B + 13H_B - 0 - H_B = 95 \]

\[ 7H_B = 20 \]

\[ \Rightarrow H_B = \frac{20}{7} = 2.86 \text{ kN} \]

\[ \Rightarrow \]

\[ H_A = -2.86 \text{ kN} \]

\[ V_B = 2.86 \text{ kN} \]

\[ V_A = 17.14 \text{ kN} \]

\[ 5 \times 16.8 - M_A = 25 \]

\[ \Rightarrow M_A = 60.7 \text{ kNm} \]
Q.11 (i) Find support reactions at A and B of structure shown in figure. The structure has an internal hinge at C.

(ii) Draw the free body diagram for spans AB, BC and CD and show thrust acting on support A, B, C and D.

Solution:

(i) 
\[ H_A + H_B = 6 \] 
\[ V_A + V_B = 0 \] 
\[ \sum M_C = 0 \] 
\[ 8V_B + 3 = 6 \times 5 \] 
\[ V_B = 3.375 \text{ kN} \uparrow \] 
\[ V_A = 3.375 \text{ kN} \downarrow \] 
\[ H_C = 0 \] 

Taking moment about C from right side
\[ 8H_B = 4V_B \] 
\[ H_B = 1.6875 \text{ kN} \leftarrow \] 
\[ H_A = 6 - H_B \] 
\[ H_A = 4.3125 \text{ kN} \rightarrow \]

(ii)
\[ V_A + V_{BA} = W_1 \]  
\[ V_{BA} + V_{BC} = V_B \]  
\[ V_{CB} + V_{CD} = V_C \]  
\[ M_{BA} + M_{BC} = 0 \]  
\[ M_{CB} + M_{CD} = 0 \]  
\[ V_{BC} + V_{CB} = W_2 \]  
\[ V_{CD} + V_D = W_3 \]

\[ A \text{ and } D \text{ are simple pinned discontinuous supports hence } M_A, M_D = 0 \] 
\[ B \text{ and } C \text{ are continuous supports } M_B, M_C \neq 0 \]

Equation of equilibrium of joint resists accompanied by static equilibrium.

Q.12 Draw the bending moment and shear force diagrams of the following beam as shown in the figure. The beam has an internal hinge at \( D \).

**Solution:**

Shear force diagram:
Let \( V_A, V_B \) and \( V_F \) be the support reactions
\[ \therefore M_O = 0 \]
Taking moment from RHS,
\[ V_F \times 10 = 10 \times 5 \]
\[ \Rightarrow V_F = 5 \text{kN (\( \uparrow \))} \]
Similarly taking moment from LHS,
\[ V_A \times 20 + V_B \times 10 = 3 \times 10 \times 15 + 10 \times 5 \]
\[ 20 V_A + 10 V_B = 500 \text{kN} \]
\[ 2V_A + V_B = 50 \] ...(i)

For equilibrium of beam, \( \Sigma F_y = 0 \)
\[ V_A + V_B + V_F - 30 - 10 - 10 = 0 \]
\[ V_A + V_B + V_F = 50 \]
\[ V_A + V_B = 45 \text{kN} \] ...(ii)

Solving eq. (i) and (ii), we get
\[ V_A = 5 \text{kN} (\uparrow), V_B = 40 \text{kN} (\uparrow) \]

For portion AB;
\[ SF_x = V_A - 3x = 5 - 3x \]
At \( x = 0 \), \( (SF)_A = 5 \text{kN} (\uparrow) \)
At \( x = 10 \text{ m} \), \( (SF)_B = 5 - 3 \times 10 = -25 \text{kN} \)

For \((SF)_x = 0 \)
\[ 5 = 3x \]
\[ x = 1.67 \text{ m} \]

For portion BD;
\[ (SF)_D = V_A - 3 \times 10 + V_B - 10 = 45 - 40 = 5 \text{kN} \]
\[ (SF)_B = -25 \text{kN} + 40 = 15 \text{kN} \]

For portion DF;
\[ (SF)_E = (SF)_D - 10 = (5 - 10) = -5 \text{kN} \]
\[ (SF)_F = -5 \text{kN} \]

For portion AB; \( 0 < x \leq 10 \text{ m} \)
\[ M_x = V_A x - \frac{3x^2}{2} = 5x - 1.5x^2 \]
At \( x = 0 \), \( M_A = 0 \)
and
\[ x = B = 10 \text{ m} = 50 - 1.5 \times 100 = -100 \text{kNm} \]

For maximum value,
\[ \frac{dM_x}{dx} = -5 - 3x \]
\[ x = \frac{5}{3} = 1.67 \text{ m} \]
\[ M_x = 0 = x (5 - 1.5 x) \]
\[ x = \frac{5}{1.5} = 3.33 \text{ m} \]
\[ M_{\text{max}} = 5 \times 1.67 - 1.5 \times (1.67 \times 1.67) = 4.167 \text{kNm} \]

For portion BC, \( 0 < x < 5 \text{ m} \)
\[ M_x = 5(x + 10) + 40x - 30 (5 + x) \]
At \( x = 0 \)
\[ M_B = 50 - 30 \times 5 = -100 \text{kNm} \]
At \( x = 5 \)
\[ M_C = 5 \times 15 + 40 \times 5 - 30 \times 10 = 75 + 200 - 300 = -25 \text{kNm} \]

For portion CD; \( 0 < x < 5 \)
\[ M_x = 5(x + 15) + 40(x + 5) - 30(x + 10) - 10 \times x \]
At \( x = 0 \)
\[ M_C = -25 \text{kNm} \]
At \( x = 5 \)
\[ M_D = 100 + 400 - 450 - 10 \times 5 = 0 \]
From RHS of the beam:
Portion $FD; \ 0 < x < 5$
\[ M_x = 5x \]
At $x = 0$
\[ M_F = 0 \]
At $x = 5 \text{ m}$
\[ M_F = 25 \text{ kNm} \]
Portion $ED; \ 0 < x < 5$
\[ M_x = 5(x + 5) - 10x \]
\[ M_D = 50 - 50 = 0 \]

Q.13 A right-angled rigid pipe is fixed to the wall at $A$ and is additionally supported through the cable $CD$ as shown in the figure. Determine the magnitudes of the moments about the $x, y$ and $z$ axes, if the tensile force applied to the cable is $3 \text{ kN}$.

Solution:

[8 marks : 2016]
Let \( F_x \) and \( F_y \) be the resolved components of forces in \( x \) and \( y \) directions of cable force 3 kN at free end \( C \).

\[
\begin{align*}
F_x &= 3 \sin \theta \\
F_y &= 3 \cos \theta
\end{align*}
\]

Also,

\[
\tan \theta = \frac{0.8}{1.8}
\]

\[
\Rightarrow \quad \theta = 23^\circ 57' 45'' \approx 23.962^\circ
\]

At joint \( B \),

\[
M_{xy} = \text{Moment about } xy \text{-axes} = F_x \times 0.70 = 3 \cos 23.962^\circ \times 0.70 = 1.28 \text{ kN}
\]

\[
M_x = \text{Moment about } x \text{-axis} = F_y \times 0.7 = 3 \sin 23.962^\circ \times 0.70 = 0.853 \text{ kN}
\]

\[
M_z = 0
\]

For joint \( A \),

\[
M_x = F_y \times 0.7 = 3 \sin 23.962^\circ \times 0.70 = 0.853 \text{ kN}
\]

\[
M_y = F_x \times 0.70 = 3 \cos 23.962^\circ \times 0.7 = 1.28 \text{ kN}
\]

\[
M_z = F_y \times 1.8 = 3 \sin 23.962^\circ \times 1.8 = 2.193 \text{ kN}
\]

Q.14 Find the support reactions of the beam shown in the figure. The beam has an internal hinge at \( C \).

\[8 \text{ marks : 2016}\]

Solution:

Let \( V_A \) and \( V_D \) be the support reactions at \( A \) & \( D \) respectively and \( M_A \) be the bending moment at \( A \).

We know,

\[
\Sigma F_y = 0
\]

\[
\Rightarrow \quad V_A + V_D = 5 \text{ kN}
\]

Taking moment about hinge

\[
M_C = 0
\]

\[
\Rightarrow \quad V_D \times 3 = 6
\]

\[
\Rightarrow \quad V_D = 2 \text{ kN (↑)}
\]

\[
\Rightarrow \quad V_A = (5 - 2) = 3 \text{ kN (↑)}
\]

Taking moment about \( C \) from LHS;

\[
V_A \times 6 + M_A = 5 \times 3
\]

\[
3 \times 6 + M_A = 15
\]

\[
\Rightarrow \quad M_A = -3 \text{ kNm}
\]

3. Bending Stress and Shear Stress

Q.15 A simply supported hollow rectangular beam of outside width 200 mm, outside depth 160 mm and material thickness 20 mm is subjected to UDL of 10 kN/m for entire span of 10 m. Find maximum shear stress induced in beam.

\[10 \text{ marks : 2000}\]
Solution:

\[ q = \frac{V(\bar{y})}{Ib} \]

Shear stress will be maximum at Neutral Axis because \( V(\bar{y}) \) will be maximum at NA coupled with minimum value of web thickness.

Also

\[ V_{\text{max}} = \frac{10 \times 10}{2} = 50 \text{ kN} \]

\[ I = \frac{200 \times 160^3}{12} - \frac{(200 - 2 \times 20) \times (160 - 2 \times 20)^3}{12} \]

\[ I = \frac{13568 \times 10^4}{3} \text{ mm}^4 \]

\[ b \text{ at Neutral axis} = 2 \times 20 = 40 \text{ mm} \]

\[ A\bar{y} = 2 \times [60 \times 20 \times 30] + 200 \times 20 \times 70 = 352000 \text{ mm}^3 \]

\[ b = 40 \text{ mm} \]

\[ q = \frac{V(\bar{y})}{Ib} = \frac{50 \times 1000 \times 352000}{13568 \times 10^4 \times 40} = 9.73 \text{ N/mm}^2 \]

Q.16 A T-beam (x-section shown in figure) is simply supported at ends over span of 7 m. It is subjected to UDL of 600 kgf/m (6 kN/m). 1°C its own weight.

Calculate maximum shear stress in flange and at ends.

\[ I_{\text{xx}} = 235.42 \text{ cm}^4 \]

centroid at 3.25 cm from Top.

[10 marks : 2006]

Solution:

\[ \text{UDL} = 6 \text{ kN/m} \]

Shear force at ends \( = \frac{6 \times 7}{2} = 21 \text{ kN} \)

Shear stress \( (\tau) = \frac{V(\bar{y})}{Ib} \)

\( I = \) Moment of area

\( A = \) Area above the section where \( \tau \) is to be determined

\( \bar{y} = \) distance of centroid of \( A \) from N.A.

\( b = \) width of section
\[ I = 2354200 \text{ mm}^4 \]

Maximum shear stress will be at Neutral axis because \( A \bar{y} \) is maximum at NA while ‘b’ is minimum

\[ A \bar{y} = 100 \times 10 \times 27.5 + 22.5 \times 10 \times \frac{22.5}{2} \]
\[ A \bar{y} = 30031.25 \text{ mm}^3 \]
\[ b = 10 \text{ mm} \]

\[ \tau_{\text{max}} = \frac{VA \bar{y}}{1b} = \frac{21 \times 1000 \times 30031.25}{2354200 \times 10} = 26.79 \text{ N/mm}^2 \]

Shear stress will be zero on whole of section at mid span.

**Q.17** Compare bending strength of three beams one having a square cross-section, a rectangular section (depth is twice the width) and a circular cross-section; all the three beams having same weight and having a cross-sectional area of 95000 sq. mm each.

* [10 marks : 2015]

**Solution:**

Same weight having same cross-sectional area implies that they have same material and equal value of maximum bending stress.

\[ A = 95000 \text{ mm}^2 \]

**Case-1: Square**

\[ d^2 = 95000 \]
\[ d = 308.22 \text{ mm} \]

\[ \text{Bending strength} = \sigma_{\text{max}} \times \text{Section modulus (z)} \]

\[ z = \frac{bd^2}{6} = \frac{d^3}{6} \text{square} \]

\[ z = \left( \frac{308.22}{6} \right)^3 \]
\[ (z)_1 = 4.88 \times 10^6 \text{ mm}^3 \]

**Case-2: Rectangle**

\[ bd = 95000 \]
\[ 2b^2 = 95000 \]

\[ b = 217.94 \text{ mm} \]
\[ d = 435.9 \text{ mm} \]

\[ (z)_2 = \frac{bd^2}{6} = \frac{2}{3} b^3 = \frac{2}{3} \times 217.94^3 \]
\[ (z)_2 = 6.90 \times 10^6 \text{ mm}^3 \]

**Case-3: Circular**

\[ \frac{\pi D^2}{4} = 95000 \]

\[ D = 347.72 \text{ mm} \]

\[ (z)_3 = \frac{\pi D^4}{64 (D/2)^2} = \frac{\pi D^3}{32} = \frac{22}{7} \times \frac{317.72^3}{32} \]
\[ (z)_3 = 4.13 \times 10^6 \text{ mm}^3 \]

\[ (z)_2 > (z)_1 > (z)_3 \]

Bending strength = \( M \)

\[ M_2 > M_1 > M_3 \]
Q.18 A steel beam having cross-section of an 'I' with overall depth 300 mm and flange width 150 mm is simply supported at both ends. The thickness of the flange, as well as the web is 20 mm for each. The beam needs to carry a concentrated load of 50 kN at its mid span. If the permissible bending stress is to be limited to 120 N/mm², determine
(i) the maximum possible length of the beam,
(ii) the depth of an equivalent rectangular section, with the width fixed to be 100 mm. Also, determine the percentage increase in weight of the beam as compared to the beam with 'I' section.

Solution:
Given, Maximum concentrated load,

\[ W = 50 \text{ kN} \]

Permissible bending stress,

\[ \sigma_{B, \text{max}} = 120 \text{ N/mm}^2 \]

(i) Let \( l_{\text{max}} \) be maximum possible length of beam.

\[ M_{\text{max}} = \frac{WL_{\text{max}}}{4} = \frac{50l_{\text{max}}}{4} \]

\[ = 12.5 l_{\text{max}} \text{ kNm} \]

We know,

\[ \frac{M_{\text{max}}}{I} = \frac{\sigma_{B, \text{max}}}{y_{\text{max}}} \]

\[ I = \frac{20 \times (260)^3}{12} + 150 \times 20 (130 + 10)^2 + 150 \times 20 \times (140)^2 \]

\[ = 146.893 \times 10^6 \text{ mm}^4 \]

\[ \Rightarrow \frac{12.5 l_{\text{max}} \times 10^6}{146.893 \times 10^6} = \frac{120}{150} \]

\[ \Rightarrow l_{\text{max}} = 9.4 \text{ metre} \]

(ii)

\[ I = \frac{bd^3}{12} = \frac{100d^3}{12} \]

Let \( d \) be depth of equivalent section (rectangular)

\[ \therefore \frac{M}{I} = \frac{\sigma_{B, \text{max}}}{y} \]

\[ \Rightarrow \frac{12.5 \times 9.4 \times 10^6 \times 12}{100 \times d^3} = \frac{120 \times 2}{d} \]

\[ \Rightarrow \quad d = 242.4 \text{ mm} \]

We know, weight of beam \( W \approx A \)
Let weight of equivalent rectangular beam, \( W_e \propto A_e \),
\( \Rightarrow \quad A_e = bd = 242.384 \times 100 = 24238.4 \text{ mm}^2 \)
Weight of I-section beam \( \propto A_I \)
\( A_I = 150 \times 20 \times 2 + 260 \times 20 \)
\( \Rightarrow \quad A_I = 11200 \text{ mm}^2 \)
Hence,
\[ \% \text{ increase} = \left( \frac{24238.4 - 11200}{11200} \right) \times 100 = 116.41\% \]

4. Torsion

Q.19  A shaft is made up of partly solid section and partly hollow section as shown as figure.

What is maximum torque that can be transmitted when maximum shear stress is 80 MPa, with modulus of rigidity = 80 GPa? What is maximum free end rotation?

[12 marks : 2010]

Solution:
Let

Maximum torque = \( T_{\text{max}} \) (in kNm)

Maximum shear stress in hollow shaft

\[ \tau = \frac{T_{\text{max}} \times \left( \frac{150}{2} \right) \times 10^6}{22 \times \left( \frac{150^4 - 100^4}{32} \right)} \]

\[ \tau_{\text{max}} = 1.879721 \times 10^6 \text{ N/mm}^2 \quad \ldots(\text{i}) \]

Maximum shear stress for solid shaft

\[ \tau_{\text{max}} = \frac{T_{\text{max}} \times 16 \times 10^6}{7 \times (100)^3} = 5.091 T_{\text{max}} \quad \ldots(\text{ii}) \]

\( \tau_{\text{max}} \leq 80 \text{ MPa} \)

5.091 \( T_{\text{max}} \) = 80 MPa

\( T_{\text{max}} = 15.714 \text{ kNm} \)

\( \theta_{\text{max}} = ? \) (at free end)

So,

\[ \theta_{\text{max}} = \frac{T_{\text{max}} L_1}{GJ_1} + \frac{T_{\text{max}} L_2}{GJ_2} \]

\[ = \frac{15.714 \times 10^6}{80 \times 1000} \times \left[ \frac{1200 \times 32 \times 7}{22 \times (150^4 - 100^4)} + \frac{1200 \times 7 \times 32}{22 \times 100^4} \right] \]

\[ = 0.02991 \text{ radian} = 1.713^\circ \]