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Civil Services Mains Examination

(1995-2016)

Civil Engineering

Paper-I

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Civil Services Mains Examination Previous Solved Papers : Civil Engineering (Paper-I)

First Edition: 2017

Preface

B. Singh (Ex. IES)

Civil Services Examination is considered as most prestigious job in India which is being preferred by engineers now-a-days. There was a need of good book, which contains error free questions with apt solutions that even a begineer student can understand. I am glad to launch the first edition of this book.

MADE EASY team has made deep study of previous exam papers of Civil Services Mains Examination and observed that a good percentage of questions has been asked in Engineering Services Exam, Indian Forest Services as well as State Services Exam. Therefore it is advisable to study this book before one takes the exam. This book is also useful for ESE Mains appearing candidates and other competitive examinations for engineering graduates.

The first edition of this book is prepared with due care to provide complete solutions to all questions with accuracy. I would like to give credit of publishing this book to MADE EASY Team for their hard efforts in solving previous years papers in a limited time frame.

I have true desire to serve student community by providing good source of study and quality guidance. I hope this book will be proved as an important tool to succeed in competitive exams. Any suggestions from the readers for improvement of this book are most welcome.

With Best Wishes

B. Singh

CMD, MADE EASY Group

Previous Years Solved Papers of

Civil Services Mains Examination

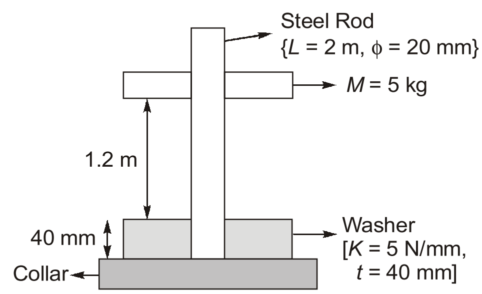
Civil Engineering : Paper-I

CONTENTS

01 CHAPTER Properties of Metals and Basic Concepts

Q.1 A solid steel rod of length 2 m diameter 20 mm hangs from the ceiling and has a collar firmly attached to it. Above the collar an annular rubber washer of 40 mm thickness having a stiffness K = 5 N/mm is placed. Determine maximum stress in the rod caused by a mass of 5 kg falling through a height of 1.2 m. [2004 : 20 Marks]

Solution:



Maximum stress will occur just at time of impact.

Let it be σ_{max}.

Let maximum deflection of Rod be δ_{max}

and Let deformation of washer be δ_{washer}.

$$\text{Load} = M = 5 \text{ kg}$$

Applying conservation of energy.

$$\text{Loss in potential energy} = \text{Gain in strain energy}$$

$$mg[h + \delta_{max} + \delta_{washer}] = \frac{(\sigma_{max})^2}{2E} \times AL + \text{Strain energy in washer} \quad ...(1)$$

For washer, $K = 5$ N/mm

$$\text{Strain energy in washer} = \frac{1}{2} \cdot P \cdot \delta_{washer} = \frac{1}{2} \cdot P \cdot \frac{P}{K} = \frac{P^2}{2K}$$

$$P = mg = 5 \times 10 = 50 \text{ N}$$

Keeping this in equation (1)

$$mg[h + \delta_{max} + \delta_{washer}] = \frac{(\sigma_{max})^2}{2E} \times AL + \frac{1}{2} \times \frac{P^2}{K}$$

$$50\left[1.2 + \frac{\sigma_{max}L}{AE} + \frac{P}{K}\right] = \frac{(\sigma_{max})^2}{2E} \times AL + \frac{P^2}{2K}$$

$$50 \times 1.2 + \frac{50 \times \sigma_{max} \times 2}{AE} + \frac{50 \times 50}{5 \times 10^3} = \frac{(\sigma_{max})^2}{2E} \times \frac{\pi}{4} \times (0.02)^2 \times 2 + \frac{1}{2} \times \frac{50^2}{5 \times 10^3}$$

$$60 + \frac{100 \times \sigma_{max}}{E} + 0.5 = \frac{(\sigma_{max})^2}{E} \times 3.1416 \times 10^{-4} + 0.25$$

For steel, $E = 2 \times 10^5$ N/mm^2

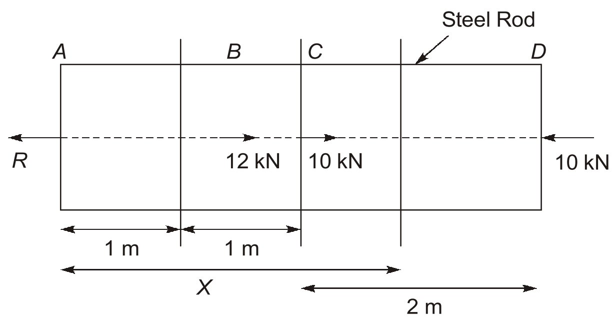
Keeping the value and simplifying the equation

$1.5708 \times 10^{-15} (\sigma^2_{max}) - 5 \times 10^{-10} \sigma_{max} - 60.25 = 0$

$$\Rightarrow \quad \sigma_{max} = 196006740.5 \text{ N/m}^2 = 196.0067 \text{ N/mm}^2$$

Q.2 (i) Through stress-strain diagram of a ductile material explain: Yield stress, Rupture strength and Proof stress. Also state how ductility is estimated.

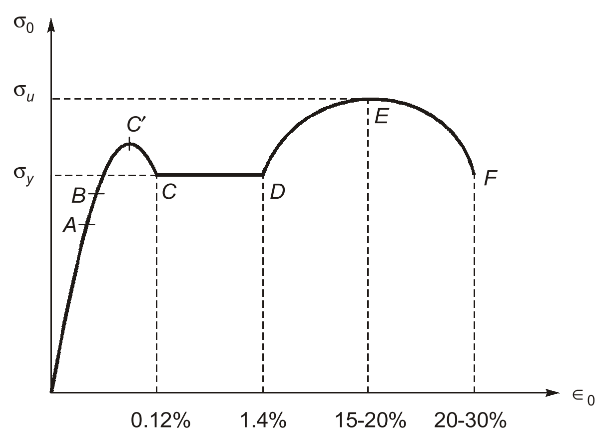
(ii) A steel rod having a diameter of 40 mm is loaded as shown in fig. Assume E = 200 kN/mm^2. Determine the deformation of the free end. Also evaluate the distance 'X' from the left end support to a point at which deformation is zero.



[2006 : 6 + 14 = 20 Marks]

Solution:

(i)



Stress-strain Curve for Mild Steel in Tension

A → Proportional Limit

B → Elastic Limit

C' → Upper Yield Point

C → Lower Yield Point/Actual Yield Point

E → Ultimate point

F → Fracture point

σ_u → Ultimate stress/Tenacity

σ_y → Yield stress

CD → Yield Plateau

DE → Strain hardening

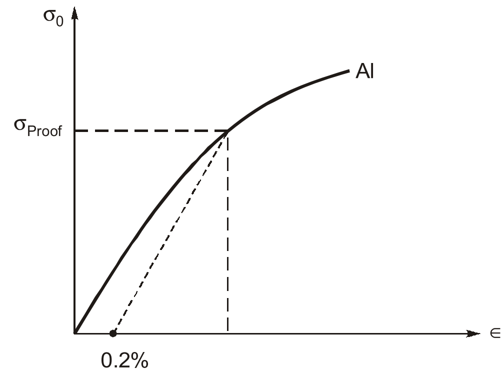
EF → Strain Softening/Necking Region

Yield Stress → The stress at which material begins to deform plastically. Once the yield point is passed, some fraction of the deformation will be permanent.
e.g. σ_y in the above curve.

Rupture Strength → It is defined as the stress in a material just before it yields in a flexure test. In other words it is tensile stress at failure in bending. It is also known as Flexural strength.

Proof Stress $\rightarrow$ It is the design stress which is defined for those ductile metals which do not represent clear yield point in stress-strain curve. e.g. Al, Cu, Ag, Au.

It is determined by offset method. An offset of permissible plastic strain {say 0.2% for Al} is marked on x-axis and straight line parallel to stress-strain curve {Slope = E} is drawn. Intersection of offset line with the stress-strain curve gives proof stress as shown in figure.



Ductility $\rightarrow$ Property of a metal due to which a piece of a metal can be drawn into wire of thin section through tensioning process. A metal is called ductile if its *Post Elastic strain* {plastic strain} *is greater than* 5%.

(ii) Reaction at left end = R

Considering Horizontal Equlibrium Condition

$$\Sigma F_x = 0$$
$$R - 12 - 10 + 10 = 0$$
$$R = 12 \text{ kN}$$

Free End is D. Deformation, $\Delta_{AD} = \Delta_D - \Delta_A$

as End A is fixed $\Delta_A = 0$

$\Rightarrow$
$$\Delta_{AD} = \Delta_D$$
$$\Delta_{AD} = \Delta_{AB} + \Delta_{BC} + \Delta_{CD}$$
$$\Delta_D = \frac{R \times L_{AB}}{A \times E} + \frac{(R-12) \times L_{BC}}{AE} + \frac{(-10) \times L_{CD}}{AE}$$
$$= \frac{12 \times 1 \times 10^3}{A \times E} + \frac{(12-12) \times 1 \times 10^3}{AE} + \frac{(-10) \times 2 \times 10^3}{AE}$$
$$= \frac{-8 \times 10^3}{\frac{\pi}{4} \times 40^2 \times 200} = -0.03183 \text{ mm} \qquad \text{Contraction}$$

Note: negative sign here Signifies Contraction.

Let point of zero deformation be at 'x' distance from left end support.

$$\Delta_{AX} = \Delta_x - \Delta_A = 0 - 0 = 0$$
$$\Delta_{AX} = \Delta_{AB} + \Delta_{BC} + \Delta_{CX} = 0$$
$$\frac{R \times L_{AB}}{AE} + \frac{(R-12) \times L_{BC}}{AE} + \frac{(-10)(x-2) \times 10^3}{AE} = 0$$
$$\frac{12 \times 1 \times 10^3}{AE} - \frac{10(x-2) \times 10^3}{AE} = 0$$
$$x = 3.2 \text{ m from Left End Support}$$

Q.3 A steel rod of length 300 mm and diameter 30 mm, is subjected to a pull 'P' while the temperature is 100°C. If the total extension of the rod is 0.40 mm, calculate the magnitude of 'P'. Take 'α' for steel = 12×10^{-6} °C and $E = 0.215 \times 10^{12}$ N/m^2. **[2011 : 12 Marks]**

Solution:

Given: $E = 0.215 \times 10^{12}$ N/m^2 = 2.15×10^5 N/mm^2; $A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 30^2 = 706.8583$ mm^2; $L = 300$ mm;

Pull = 'P' kN

Let us assume room temperature = 20°C

$$\therefore \quad \text{Change in Temp.} = 100 - 20 = 80°C$$

$$\text{Free Expansion of rod} = L \cdot \alpha \cdot (\Delta T) = 300 \times 12 \times 10^{-6} \times (80°C) = 0.288 \text{ mm}$$

$$\text{Total Expansion} = 0.40 \text{ mm}$$

$$\text{New free length of rod at } 100°C = 300 + 0.288 \text{ mm} = 300.288 \text{ mm}$$

$$\text{Change in diameter due to temp.} = d \cdot \alpha \cdot (\Delta T) = 30 \times 12 \times 10^{-6} \times 80°C = 0.0288 \text{ mm}$$

$$\text{New diameter} = 30.0288 \text{ mm}$$

Now pull of P 'kN' is applied

$$\text{Expansion due to pull} = 0.40 - 0.288$$

$$\frac{P.L}{A.E} = 0.112 \text{ mm}$$

$$\frac{P \times 10^3 \times 300.288}{\frac{\pi}{4} \times 30.0288^2 \times 2.15 \times 10^5} = 0.112$$

$$P = 56.7916 \text{ kN}$$

Q.4 A circular copper bar is carrying a tensile load of 200 kN as shown in figure. Calculate the displacement of point B of the copper bar.

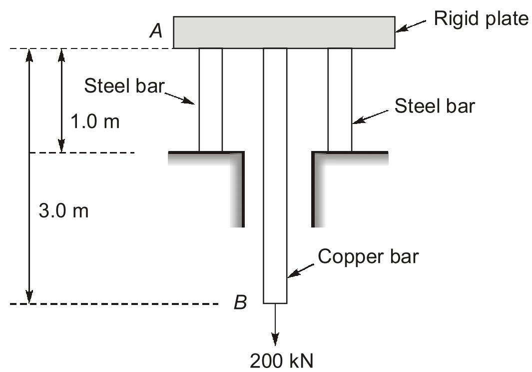
Cross-sectional area of the copper bar = 5000 mm^2

Cross-sectional area of each of the steel bars = 5000 mm^2

Length of the copper bar = 3.0 m.

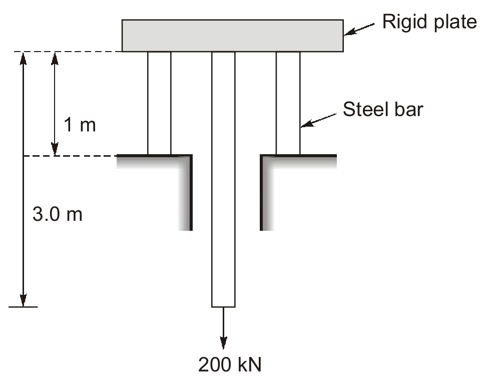
Length of each of the steel bars = 1.0 m

E_{steel} = 210 GPa, E_{copper} = 120 GPa

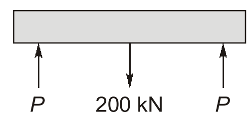


[2016 : 10 Marks]

Solution:



From FBD of rigid plate.



$$200 = 2P$$

$$P = 100 \text{ kN}$$

$$\therefore \quad \text{force in steel bar} = 100 \text{ kN}$$

$$\therefore \quad \text{reduction in length of steel} = \frac{PL}{AE} = \frac{100 \times 10^3 \times 1000}{5000 \times 210 \times 10^3} = 0.095 \text{ mm}$$

$$\text{elongation of copper bar} = \frac{PL}{AE} = \frac{200 \times 10^3 \times 3000}{5000 \times 120 \times 10^3} = 1 \text{ mm}$$

$$\text{Total displacement of } B = 1 + 0.095 = 1.095 \text{ mm}$$

Q.5 Three metal cubes are arranged as shown in figure. The central metal cube is subjected to a uniform compressive stress of 1000 N/mm^2. Calculate the volumetric strain of cube 2, if displacement is restrained about the x-axis.

(i) $A_1 = A_2 = A_3 = A$

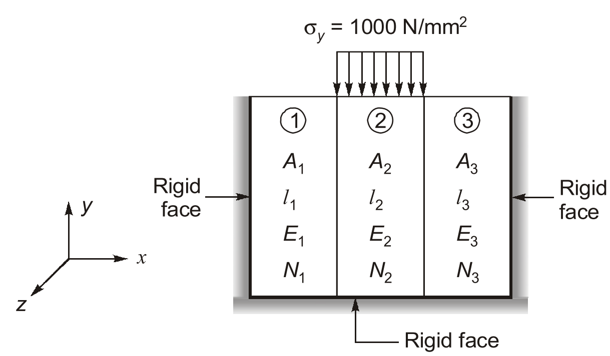
(ii) $l_1 = l_2 = l_3 = l$

(iii) $E_1 = E_3 = 1.2 \times 10^5$ N/mm^2

(iv) $E_2 = 2.1 \times 10^5$ N/mm^2

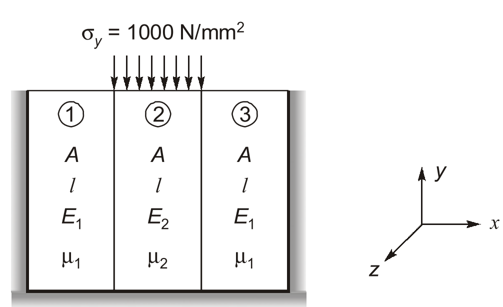
(v) $N_1 = N_3 = 0.20$

(vi) $N_2 = 0.3$



[2016 : 20 Marks]

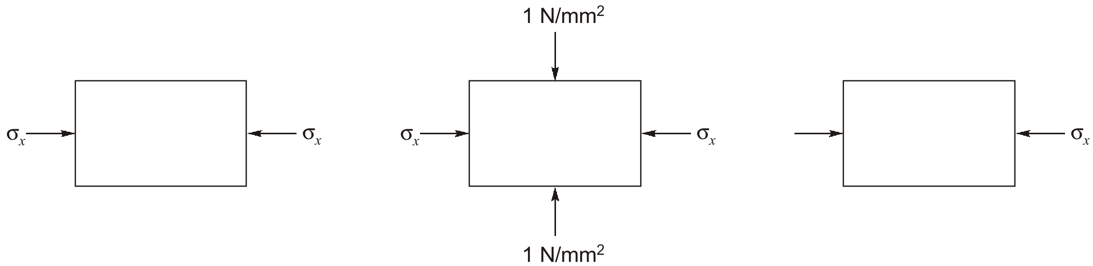
Solution:



Considuring cubes are free in z-direction,

$$\varepsilon_2 + \varepsilon_1 + \varepsilon_3 = 0$$

$$\frac{\Delta_2}{L} + \frac{\Delta_1}{L} + \frac{\Delta_3}{L} = 0$$



$$\varepsilon_1 = -\frac{\sigma_x}{E_1}$$

$$\varepsilon_2 = -\frac{\sigma_x}{E} + u.\frac{\sigma_y}{E}$$

$$\varepsilon_3 = -\frac{\sigma_x}{E_1}$$

Putting values,

$$-\frac{\sigma_x}{E_1} - \frac{\sigma_x}{E_2} + u.\frac{\sigma_y}{E_2} - \frac{\sigma_x}{E_1} = 0$$

$$+\sigma_x\left[\frac{1}{E_1} + \frac{1}{E_1} + \frac{1}{E_2}\right] = \mu_2\frac{\sigma_y}{E_2}$$

$$\sigma_x\left[\frac{1}{1.2} + \frac{1}{1.2} + \frac{1}{2.1}\right]\frac{1}{10^5} = \frac{0.3 \times 1000}{2.1 \times 10^5}$$

$$\sigma_x = 66.67 \text{ N/mm}^2$$

$$e_x = \left[-\frac{66.67}{2.1} + 0.3 \times \frac{1000}{2.1}\right] \times \frac{1}{10^5} = 111.1 \times 10^{-5}$$

$$e_y = \left[-\frac{1000}{2.1} + 0.3 \times \frac{66.67}{2.1}\right] \times \frac{1}{10^5} = -466.67$$

$$e_z = \left[0.3 \times \frac{1000}{2.1} + 0.3 \times \frac{66.67}{2.1}\right] \times \frac{1}{10^5} = 152.38 \times 10^{-5}$$

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = -203.18 \times 10^{-5}$$

■■■■

02 CHAPTER

Shear Force and Bending Moment

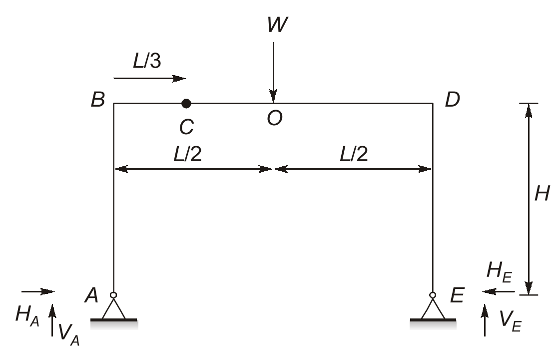
Q.1 A Portal frame *ABCDE* has vertical lengths *AB* and *ED* each of height *H*. Beam *BD* has a span '*L*' and is hinged at *C*. *BC* being equal to *L*/3. The ends *A* and *E* are also hinged. The beam carries a load '*W*' at mid-span. Draw bending moment diagram for the beam only. Take $H = L/3$.

[2000 : 12 Marks]

Solution:

Let H_A, V_A, H_E, V_E be the horizontal and vertical reactions at *A* and *E* respectively.

Using conditions of equilibrium.



$$\Sigma F_x = 0$$

$$H_A - H_E = 0$$

$$H_A = H_E$$

$$\Sigma F_Y = 0$$

$$V_A + V_E = W$$

$$\Sigma M_A = 0$$

$$V_E \times L - \frac{WL}{2} = 0$$

$$V_E = \frac{W}{2}$$

$$\Rightarrow \qquad V_A = \frac{W}{2}$$

Hinge at $C \Rightarrow$ $\qquad M_C = 0$

$$H_A \times H - V_A \times \frac{L}{3} = 0$$

$$H_A \times \frac{L}{3} = \frac{W}{2} \times \frac{L}{3}$$

$$\Rightarrow \qquad H_A = \frac{W}{2}$$

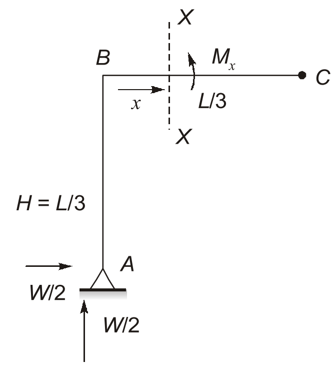
also $\qquad H_E = \frac{W}{2}$

Bending Moment Calculations:

Taking outer face of Beam *BD* as reference face.

Sign convention ⇒ Sagging Bending Moment = Positive B.M.

B.M. in BC:



$$M_x = \frac{W}{2}\times x - \frac{W}{2}\times H$$

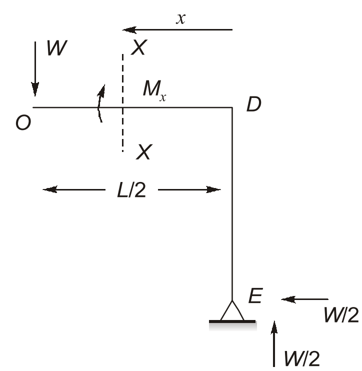
$$= \frac{W}{2}x - \frac{W}{2}\times\frac{L}{3}$$

$$= \frac{Wx}{2} - \frac{WL}{6}$$

B.M. at B, $$M_B = \frac{W\times 0}{2} - \frac{WL}{6} = -\frac{WL}{6}$$

B.M. at C, $$M_C = \frac{W\times L}{2\times 3} - \frac{WL}{6} = 0$$

B.M. in OD:



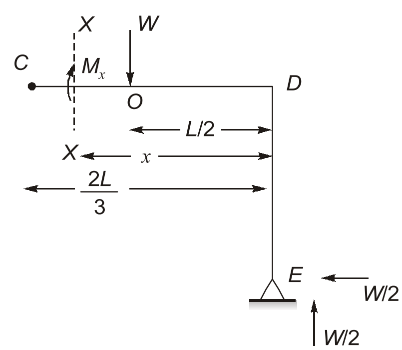
$$M_x = \frac{W}{2}x - \frac{W}{2}\times H$$

$$= \frac{W}{2}x - \frac{W}{2}\times\frac{L}{3} = \frac{W}{2}x - \frac{WL}{6}$$

$$M_D = \frac{W}{2}\times 0 - \frac{WL}{6} = -\frac{WL}{6}$$

$$M_O = \frac{W}{2}\times\frac{L}{2} - \frac{WL}{6} = \frac{WL}{12}$$

B.M. in OC:



$$M_x = \frac{W}{2}x - \frac{W}{2}\times H - W\left(x - \frac{L}{2}\right)$$

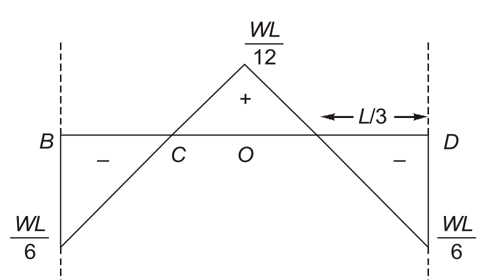
$$= \frac{W}{2}x - \frac{WL}{6} - Wx + \frac{WL}{2}$$

$$= \frac{WL}{3} - \frac{Wx}{2}$$

$$M_C = \frac{WL}{3} - \frac{W}{2}\times\frac{2L}{3} = 0$$

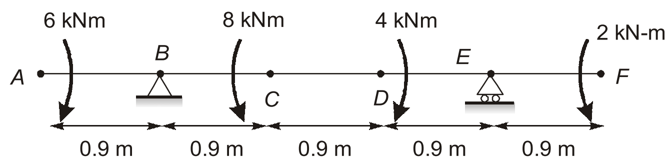
$$M_O = \frac{WL}{3} - \frac{W}{2}\times\frac{L}{2} = \frac{WL}{12}$$

Bending Moment Diagram of Beam BD.



Point of Contraflexure: $$M_x = 0 \Rightarrow \frac{W}{2}x - \frac{WL}{6} = 0 \Rightarrow x = \frac{L}{3}$$

Q.2 Draw SF and BM diagrams for the beam shown in the figure. The beam is simply supported at *B* and *E* having overhangs *BA* and *EF*



[2001 : 12 Marks]

Solution:

Let H_B and V_B be Horizontal and upward vertical reaction at *B* respectively.
Let V_E be upward vertical reaction at *E*.
Using equations of equilibrium

$$\Sigma F_x = H_A = 0$$

$$\Sigma F_y = 0$$

$$\Rightarrow \quad V_B + V_E = 0$$

$$\Sigma M_E = 0$$

$$+6 + V_B \times (0.9 + 0.9 + 0.9) - 8 + 4 - 2 = 0$$

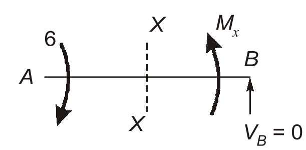
$$V_B \times 2.7 = 0$$

$$\Rightarrow \quad V_E = 0$$

SF $\Rightarrow$ As there is no reaction force, SF = 0 for full beam

Bending Moment between *A* and *B*

$$M_x = 6 \text{ kN-m}$$

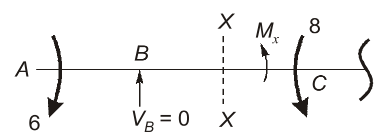


Bending Moment between *B* and *C*

$$M_x = +6$$

$$M_B = 6 \text{ kN-m}$$

$$M_{C^-} = 6 \text{ kN-m}$$



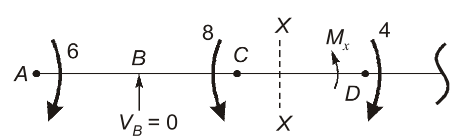
Bending Moment between *C* and *D*

$$M_x = 6 - 8$$

$$= -2 \text{ kN-m}$$

$$M_{C^+} = -2 \text{ kN-m}$$

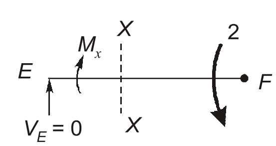
$$M_{D^-} = -2 \text{ kN-m}$$



Bending Moment between *E* and *F*

$$M_x = 2 \text{ kN-m}$$

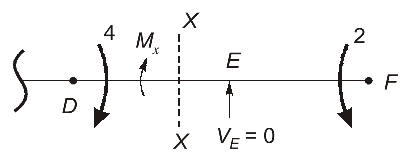
$$M_{E^+} = 2 \text{ kN-m}$$



Bending Moment between *E* and *D*

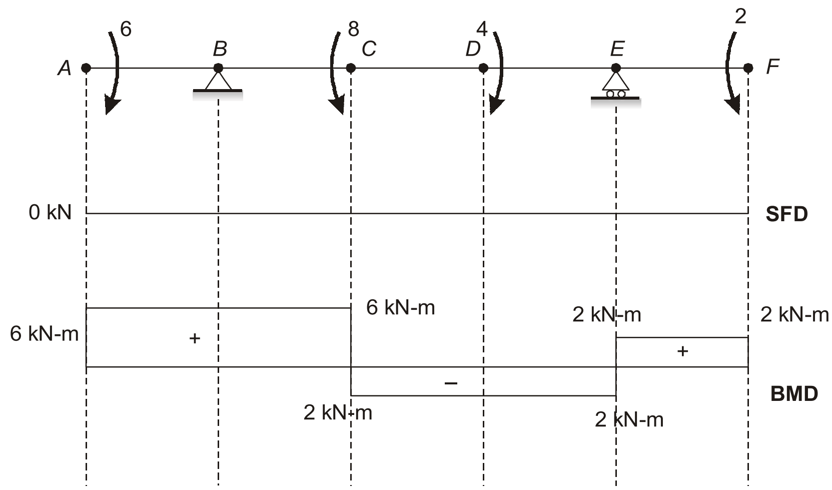
$$M_x = 2 \text{ kN-m}$$

$$M_{D^+} = 2 \text{ kN-m}$$

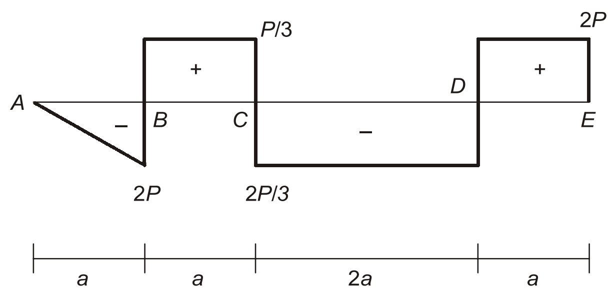


SFD and BMD : Sign Convention $\Rightarrow$ Shear force = $\uparrow \big| \downarrow$ = Positive SF.

Sagging Bending Moment = Positive Bending Moment



Q.3 The shear force diagram of a beam is shown in the figure. Draw bending moment and load diagrams.



[2002 : 12 Marks]

Solution:

Shear force sign convention ⇒ = Positive shear force

Between A and B : Let w kN/m be the *udl*.

Shear force just left of B = S.F.$_{B^-}$ = $-2P$

$$\text{S.F.}_{B^-} = -w \times a = -2P$$

$$w = \frac{2P}{a}$$

Let V_B be the upward force at B.

⇒ $$\text{S.F.}_{B^+} = -2P + V_B = P/3 \Rightarrow V_B = \frac{7P}{3}$$

Let P_1 be the downward load at C.

⇒ $$\text{S.F.}_{C^+} = -\frac{2P}{3} = \frac{P}{3} - P_1 \Rightarrow P_1 = \frac{3P}{3} = P$$

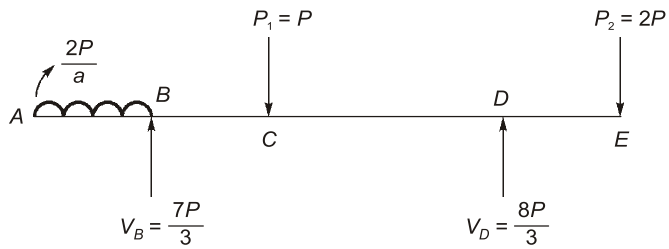
Let P_2 be the downward load at E

⇒ $$\text{S.F.}_{E^-} = 2P = P_2 \Rightarrow P_2 = 2P$$

Let V_D be the upward force at D.

⇒ $$\text{S.F.}_{D^-} = 2P - V_D = \frac{-2P}{3} \Rightarrow V_D = 2P + \frac{2P}{3} = \frac{8P}{3}$$

Loading Diagram:



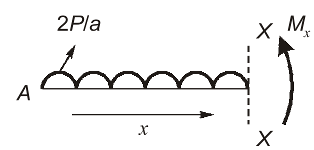
Checking for equilibrium condition

$$\Sigma F_Y = \frac{2P}{a}\times a + P + 2P - \frac{7P}{3} - \frac{8P}{3} = 0$$

$$\Sigma M_B = \frac{2P}{a}\times a \times \frac{a}{2} - P\times a + \frac{8P}{3}\times 3a - 2P\times 4a$$

$$= P.a - P.a + 8Pa - 8Pa = 0$$

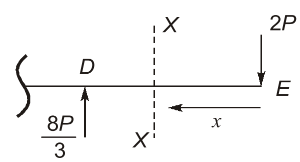
Bending moment equation between A and B



$$M_x = -\frac{2P}{a}\times x \times \frac{x}{2} = -\frac{P.x^2}{a}$$

$$M_{B^-} = -\frac{P\times a^2}{a} = -P.a$$

Bending Moment between E and D

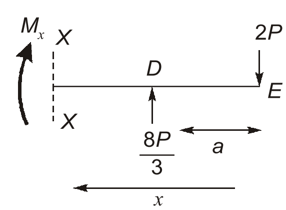


$$M_x = -2P\times x = -2Px$$

$$M_E = 0$$

$$M_{D^+} = -2P\times a = -2Pa$$

Bending Moment equation betwwen D and C



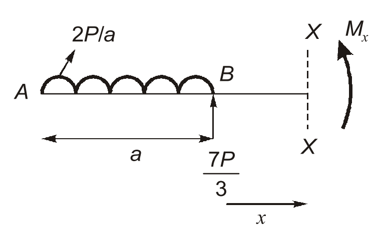
$$M_x = -2P\times x + \frac{8P}{3}(x-a)$$

$$= -2Px + \frac{8P}{3}(x-a) = \frac{2}{3}Px - \frac{8P}{3}a$$

$$M_{D^-} = -2Pa$$

$$M_{C^+} = \frac{2}{3}\times P\times 3a - \frac{8P}{3}a = -\frac{2}{3}Pa$$

Bending Moment between B and C



$$M_x = -\frac{2P}{a}\times a\times\left(\frac{a}{2}+x\right) + \frac{7P}{3}x$$

$$= -Pa - 2Px + \frac{7P}{3}x$$

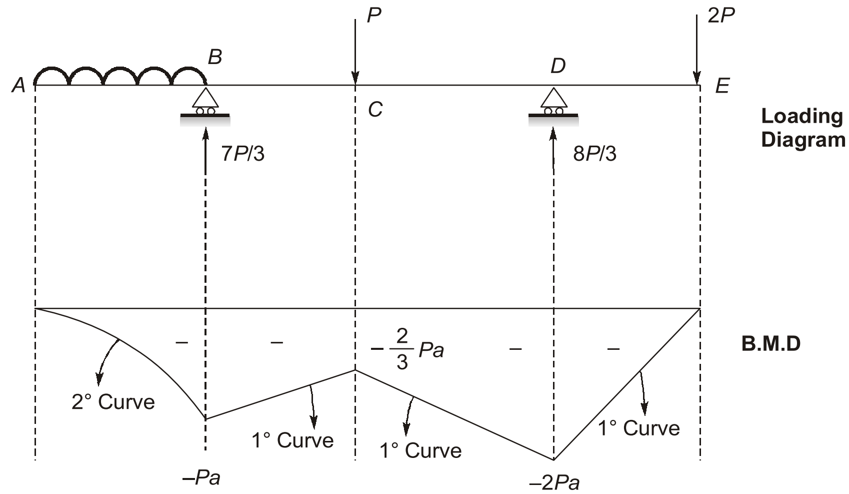
$$= \frac{P}{3}x - Pa$$

$$M_{B^+} = -Pa$$

$$M_{C^-} = \frac{P}{3}\times a - Pa = -\frac{2}{3}Pa$$

Bending Moment Diagram

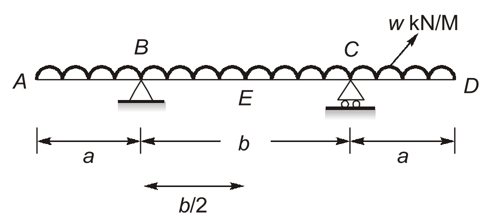
Sign Convention ⇒ Sagging Bending Moment = Positive B.M.



Q.4 A beam of uniform cross-section resting on two supports '*b*' m apart has an equal overhang of '*a*' m on either end. Determine the ratio of *b*/*a* when the magnitude of mid span bending moment is equal to the magnitude of support bending moment due to its own weight. For this condition draw the Bending moment diagram, locate contraflexure points.

[2004 : 20 Marks]

Solution:



Let beam be as shown in figure.

Let w kN/m be the self weight of beam

Let Vertical Reaction at B & C be V_B & V_C respectively.

Let Horizontal Reaction at B be H_B.

Using Equations of equilibrium

$$\Sigma F_H = 0$$

$$H_B = 0$$

$$\Sigma F_Y = 0$$

$$V_B + V_C = w \times (b + 2a)$$

From symmetry

$$V_B = V_C$$

⇒

$$V_B = V_C = \frac{w}{2}(b+2a)$$

Bending Moment at support B & C

$$M_B = -w \times a \times \frac{a}{2} = -wa^2/2$$

Bending moment at mid point of beam i.e. E

$$M_E = -w\left(a+\frac{b}{2}\right)\left(\frac{a+\frac{b}{2}}{2}\right)+\frac{w}{2}(b+2a)\times\frac{b}{2}$$

$$= -\frac{w}{2}\left[a^2+\frac{b^2}{4}+ab\right]+\frac{wb^2}{4}+\frac{wab}{2}$$

$$= -\frac{wa^2}{2}-\frac{wb^2}{8}-\frac{wab}{2}+\frac{wb^2}{4}+\frac{wab}{2} = \frac{wb^2}{8}-\frac{wa^2}{2}$$

A/c to the condition in the equation,

$$|M_B| = |M_E|$$

$$\pm\left[-\frac{wa^2}{2}\right] = \pm\left(\frac{wb^2}{8}-\frac{wa^2}{2}\right)$$

Case-I:

$$-\frac{wa^2}{2} = \frac{wb^2}{8}-\frac{wa^2}{2}$$

$$b = 0 \Rightarrow \text{not possible}$$

Case II:

$$-\frac{wa^2}{2} = -\frac{wb^2}{8}+\frac{wa^2}{2}$$

$$wa^2 = \frac{wb^2}{8}$$

$$b = 2\sqrt{2}\,a$$

Case III:

$$+\frac{wa^2}{2} = \frac{wb^2}{8}-\frac{wa^2}{2}$$

$$wa^2 = \frac{wb^2}{8}$$

$$b = 2\sqrt{2}\,a$$

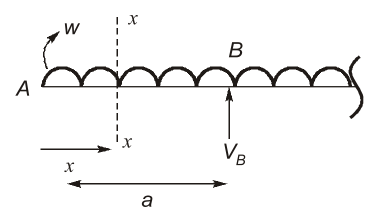
Case IV:

$$+\frac{wa^2}{2} = -\frac{wb^2}{8}+\frac{wa^2}{2}$$

$$b = 0 \Rightarrow \text{not possible}$$

$$\therefore \quad b = 2\sqrt{2}\,a$$

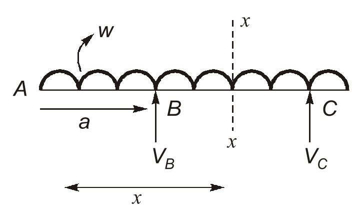
Bending Moment between A and B



$$M_x = -w\cdot x\cdot\frac{x}{2} = -\frac{wx^2}{2}$$

$$M_{B^-} = -\frac{wa^2}{2}$$

Bending Moment between B and C



$$M_x = -w\cdot x\cdot\frac{x}{2}+V_B(x-a)$$

$$= -\frac{wx^2}{2}+\frac{w}{2}(b+2a)(x-a)$$

$$M_x = -\frac{wx^2}{2}+\frac{wbx}{2}-\frac{wba}{2}+wax-wa^2$$

$$= -\frac{w}{2}x^2+\left(\frac{wb}{2}+wa\right)x-\frac{wba}{2}-wa^2$$

Bending Moment at Mid point of beam i.e. E

$$M_E = -\frac{w}{2}\left\{a+\frac{b}{2}\right\}^2+\left(\frac{wb}{2}+wa\right)\left(\frac{b}{2}+a\right)-\frac{wba}{2}-wa^2$$

$$M_E = -\frac{w}{2}\left\{a^2+\frac{b^2}{4}+ab\right\}+\frac{wb^2}{4}+\frac{wab}{2}+\frac{wab}{2}+wa^2-\frac{wba}{2}-wa^2$$

$$M_E = -\frac{wa^2}{2} - \frac{wb^2}{8} - \frac{wab}{2} + \frac{wb^2}{4} + \frac{wab}{2} = -\frac{wa^2}{2} + \frac{wb^2}{8}$$

We know $b = 2\sqrt{2}a \Rightarrow$ $$M_E = -\frac{wa^2}{2} + \frac{w}{8}(2\sqrt{2}a)^2 = \frac{wa^2}{2}$$

Points of Contraflexure $M_x = 0$

$$-\frac{wx^2}{2} + \left(\frac{wb}{2} + wa\right)x - \frac{wba}{2} - wa^2 = 0$$

$$x^2 - (2a + b)x + a(2a + b) = 0$$

$$x = \frac{(2a+b) \pm \sqrt{(2a+b)^2 - 4a(2a+b)}}{2}$$

$$= \frac{(2a+b) \pm \sqrt{4a^2 + b^2 + 4ab - 8a^2 - 4ab}}{2} = \frac{(2a+b) \pm \sqrt{b^2 - 4a^2}}{2}$$

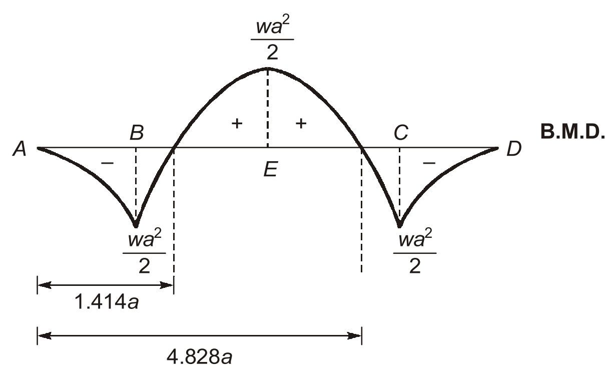
We know, $b = 2\sqrt{2}a$ $$x = \frac{(2a+b) \pm \sqrt{8a^2 - 4a^2}}{2} = \frac{(2a+b) \pm 2a}{2} \Rightarrow x = \frac{4a+b}{2}, \frac{b}{2}$$

$$= (2a + b), \frac{b}{2} \qquad \{\text{keeping } b = 2\sqrt{2}a\}$$

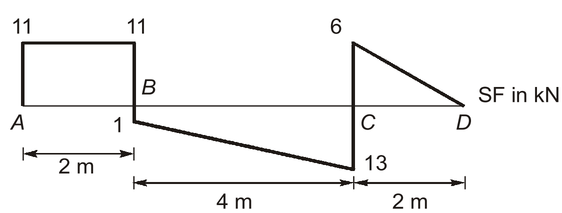
$$= 4.828a, 1.414a$$

Bending Moment Diagram : Using Symmetry

Sign Convention : Sagging Moment = Positive B.M. (+)



Q.5 The shear force diagram of a beam is shown in the figure. Draw its load and moment diagrams.



[2005 : 12 Marks]

Solution:

Sign convention for shear force ⇒ = Positive shear force

Considering section *AB*

Shear force just right of A, $SF_{A^+} = 11$ kN

Let force acting at A be V_A

⇒ $$SF_{A^+} = V_A = 11 \text{ kN}$$

At *B* : Let us suppose point load *P* acts

Shear force just right of $B = SF_{B+} = -1$ kN

$$SF_{B^+} = V_A - P = -1$$

$$11 - P = -1$$

$$P = 12 \text{ kN}$$

Between *B* & *C* : Let *W* kN/m *udl* be acting

Shear force just left of $C = SF_{C^-} = -13$ kN

$$SF_{C^-} = V_A - P - W \times 4 = -13$$

$$11 - 12 - 4W = -13$$

$$W = 3 \text{ kN/m}$$

Between *C* & *D* : Let W_2 kN/m downward *udl* be acting, shear force just right of $C = SF_{C^+} = 6$ kN

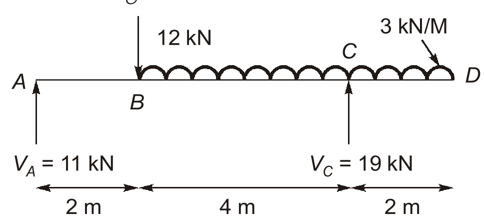
$$SF_{C^+} = W_2 \times 2 = 6 \text{ kN}$$

$$W_2 = 3 \text{ kN/m}$$

At *C* : Let upward force be V_C

$$\text{S.F. just left of } C = SF_{C^-} = -13 = 6 - V_C$$

$$V_C = 19 \text{ kN}$$



Checking equilibrium conditions

$$\Sigma F_Y = V_A - P + V_C - 3 \times 6$$

$$= 11 - 12 + 19 - 18 = 0$$

$$\Sigma M_A = 12 \times 2 - 19 \times 6 + 3 \times 6 \times (2 + 3)$$

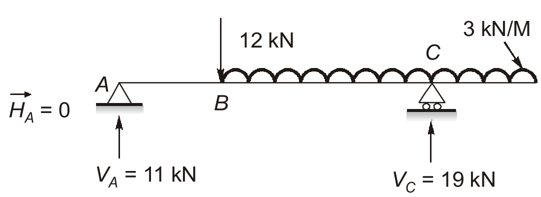
$$= 24 - 114 + 90 = 0$$

$$\Sigma M_C = 11 \times 6 - 12 \times 4 - 3 \times 4 \times 2 + 3 \times 2 \times 1$$

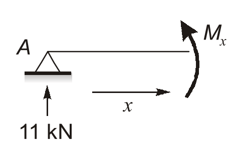
$$= 66 - 48 - 24 + 6 = 0$$

$$\Sigma F_H = 0 \Rightarrow H_A = 0$$

Loading Diagram:



Bending Moment Equation between *A* & *B*

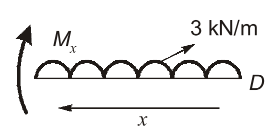


$$M_x = 11 \times x$$

$$M_A = 0$$

$$M_{B^-} = 11 \times 2 = 22 \text{ kN-m}$$

Bending Moment Equation between *C* & *D*

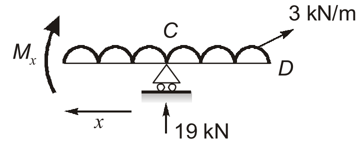


$$M_x = -(3 \times x)\left(\frac{x}{2}\right)$$

$$= -1.5\,x^2$$

$$M_{C^+} = -1.5 \times 2^2 = -6 \text{ kN-m}$$

Bending Moment Equation between C & B



$$M_x = -(3 \times 2 \times (x+1)) + 19x - 3 \times x \times \frac{x}{2}$$
$$= 19x - 6x - 6 - 1.5x^2 = -1.5x^2 + 13x - 6$$
$$M_{B^+} = -1.5 \times 4^2 + 13 \times 4 - 6 = 22 \text{ kN-m}$$

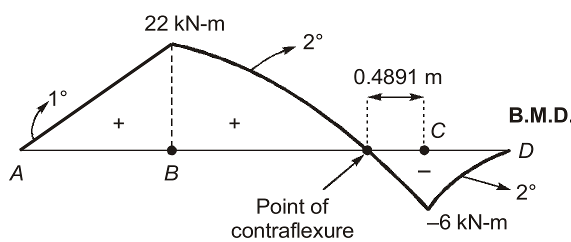
Point of Contraflexure: $M_x = 0$

$$-1.5x^2 + 13x - 6 = 0$$

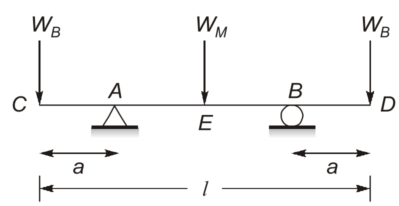
$\Rightarrow$ $x = 0.4891$ m from C towards B.

Bending Moment Diagram : Sign Convention $\Rightarrow$ Sagging Moment

= Positive B.M. $\boxed{+}$



Q.6 A section of a scaffold consists of a plank laid across two supports & extending a distance 'a' on either side of the supports. A mason working at the centre of the plank thinks that he should stack his supply of bricks on the ends of the plank in order to minimise the bending moment in the plank. Is he correct? If equal number of bricks are stacked at each end of the plank, for what weight of bricks W_B is the maximum bending moment in the plank a minimum? The man weighs W_M



[2007 : 20 Marks]

Solution:

Let Vertical Reactions at A and B be V_A and V_B respectively & Horizontal Reaction at A be H_A.

From equilibrium equations

$$\Sigma F_H = 0$$
$$H_A = 0$$
$$\Sigma F_Y = 0$$
$$V_A + V_B = W_M + 2W_B$$

From symmetry $V_A = V_B$

$$\Rightarrow \quad V_A = V_B = \frac{1}{2}(W_M + 2W_B) = \frac{W_m}{2} + W_B$$

Maximum Hogging moment will be at supports i.e. A & B

$$M_A = -W_B \times a$$

Maximum Sagging B.M. will occur at E.

$$M_E = V_A \times \left(\frac{l}{2} - a\right) - W_B \times \frac{l}{2}$$

$$= \left[W_B + \frac{W_M}{2}\right]\left[\frac{l}{2} - a\right] - W_B \times \frac{l}{2} = \frac{W_M l}{4} - \frac{W_M a}{2} - W_B \cdot a$$

Condition for maximum Bending Moment to be minimum is

Maximum sagging B.M. = Maximum Hogging B.M.

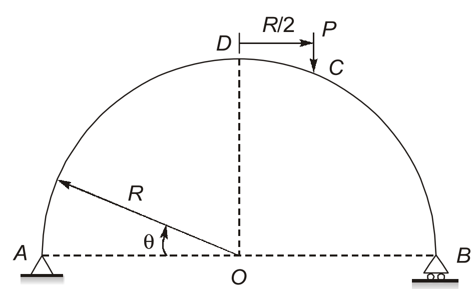
$$\frac{W_M \cdot l}{4} - \frac{W_M \cdot a}{2} - W_B \cdot a = W_B \cdot a$$

$$\Rightarrow \quad 2W_B \cdot a = W_M\left[\frac{l}{4} - \frac{a}{2}\right]$$

$$W_B = \frac{W_M}{2a \times 2}\left[\frac{l}{2} - a\right] = \frac{W_M}{4a}\left[\frac{l}{2} - a\right]$$

Clearly we can see from the equation of B.M. that if the Mason keeps the stack not on end but between end of plank and support, then Hogging moment will reduce but sagging moment will increase as the components with negative sign in maximum sagging B.M. equation will decrease. Therefore leading to increase in sagging B.M.

Q.7 For the semicircular simply supported member shown in the figure, find the value of bending moment as a function of θ, and also draw its bending moment diagram.

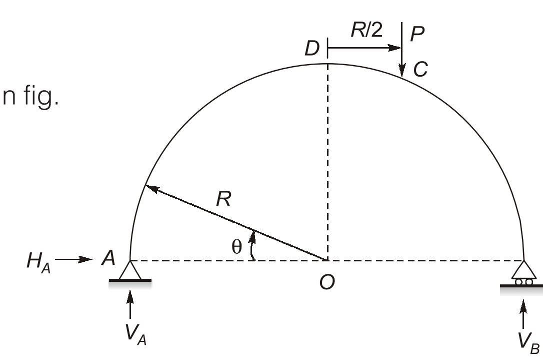


[2013 : 15 Marks]

Solution:

Let Horizontal Reaction at A be H_A.

Let Vertical Reaction at A and B be V_A and V_B as shown in fig.



Applying equations of equilibrium

$$\Sigma F_H = 0$$

$$H_A = 0$$

$$\Sigma F_Y = 0$$

$$V_A + V_B = P \quad \text{...(1)}$$

$$\Sigma M_B = 0$$

$$V_A \times 2R - P \times R/2 = 0$$

$$V_A = P/4$$

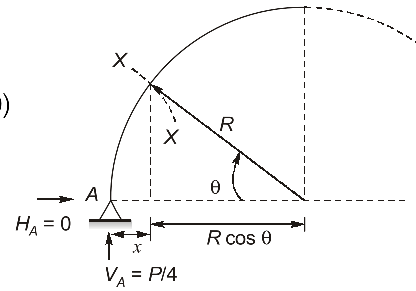
Keeping this in Equation (1)

$$\frac{P}{4} + V_B = P$$

$$V_B = \frac{3P}{4}$$

BM equation between *AD*.

Let us consider a section x-x at an angle θ.



$$M_x = V_A \times x = \frac{P}{4} \times (R - R\cos\theta)$$

$$= \frac{PR}{4}(1 - \cos\theta)$$

Here θ varies from 0° to 90°

$$M_{0°} = 0$$

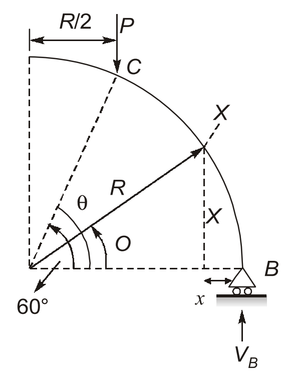
$$M_{30°} = \frac{PR}{4}\left(1 - \frac{\sqrt{3}}{2}\right) = 0.0335\,PR$$

$$M_{45°} = \frac{PR}{4}\left(1 - \frac{1}{\sqrt{2}}\right) = 0.0732\,PR$$

$$M_{60°} = \frac{PR}{4}\left(1 - \frac{1}{2}\right) = 0.125\,PR$$

$$M_{90°} = \frac{PR}{4}(1 - 0) = 0.25\,R$$

BM equation between BC



$$\angle COB = \cos^{-1}\left(\frac{R/2}{R}\right) = \cos^{-1}\left(\frac{1}{2}\right) = 60°$$

Let us consider section x-x at an angle θ.

$$M_x = V_B \times x$$

$$= \frac{3P}{4} \times (R - R\cos\theta) = \frac{3PR}{4}(1 - \cos\theta)$$

Here θ varies between 0° to 60°

$$M_{0°} = 0$$

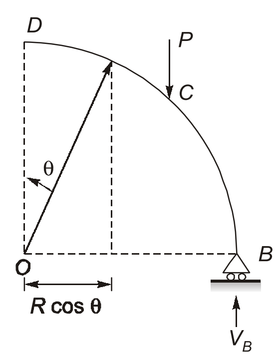
$$M_{30°} = \frac{3PR}{4}\left(1 - \frac{\sqrt{3}}{2}\right) = 0.1005\,PR$$

$$M_{45°} = \frac{3PR}{4}\left(1 - \frac{1}{\sqrt{2}}\right) = 0.2205\,PR$$

$$M_{60°} = \frac{3PR}{4}\left(1 - \frac{1}{2}\right) = 0.375\,PR$$

BM Equation between CD.

Let section be x-x at angle θ such that θ varies between 60° to 90°.



$$M_x = V_B \times x - P_X(x - R/2)$$

$$= \frac{3P}{4} \times (R - R\cos\theta) - P_X\left(R - R\cos\theta - \frac{R}{2}\right)$$

$$= \frac{3PR}{4}(1 - \cos\theta) - PR\left(\frac{1}{2} - \cos\theta\right)$$

$$= PR\left\{\frac{3}{4}-\frac{3}{4}\cos\theta-\frac{1}{2}+\cos\theta\right\} = PR\left\{\frac{1}{4}+\frac{1}{4}\cos\theta\right\} = \frac{PR}{4}(1+\cos\theta)$$

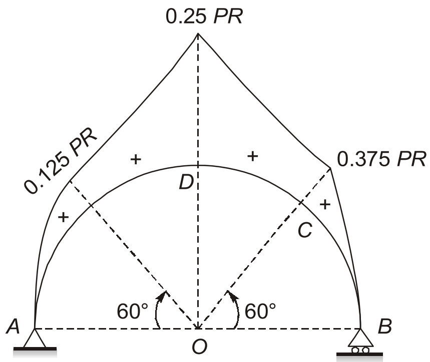
$$M_{90^\circ} = \frac{PR}{4}(1+0) = 0.25\ PR$$

Bending Moment Diagram:

Sign Convention ⇒ Sagging = Positive B.M.

Hogging = Negative B.M.

Considering loading face as Reference face



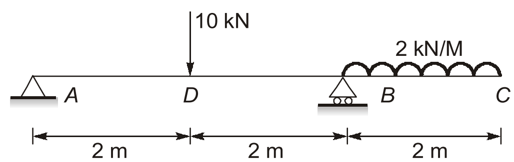
Slope of Curve between AD $= \dfrac{dM_x}{d\theta} = \dfrac{d}{d\theta}\left\{\dfrac{PR}{4}\{1-\cos\theta\}\right\}$

$$= \frac{PR}{4}\sin\theta$$

Slope of Curve between BC $= \dfrac{3PR}{4}\sin\theta$

Slope of Curve between CD $= \dfrac{PR}{4}(-\sin\theta) = -\dfrac{PR}{4}\sin\theta$

Q.8 Draw the bending moment diagram and shear force for the beam as shown in the figure.



[2013 : 10 Marks]

Solution:

Let Horizontal Reaction at A be H_A and Vertical Reaction of A and B be V_A and V_B respectively.

Applying Equations of Equilibrium

$$\Sigma F_H = 0$$
$$H_A = 0$$
$$\Sigma F_y = 0$$
$$V_A + V_B = 10 + 2 \times 2 = 14 \text{ kN}$$
$$\Sigma M_B = 0$$
$$V_A \times 4 - 10 \times 2 + (2 \times 2) \times 1 = 0$$
$$V_A = 4 \text{ kN}$$
$$\Rightarrow \quad V_B = 10 \text{ kN}$$

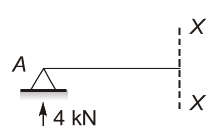
Sign Conventions: Shear force ⇒ $\uparrow\,|\,|\downarrow$ = Positive shear force

Bending Moment ⇒ Sagging = Positive BM

Considering Loading face as Reference face

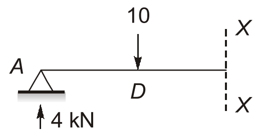
Shear force between A & D

$$S_x = +V_A = 4 \text{ kN}$$



Shear force between D & B

$$S_x = V_A - 10$$
$$= 4 - 10$$
$$= -6 \text{ kN}$$



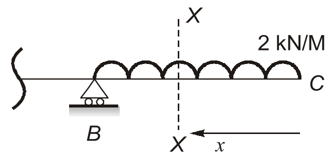
S.F. just left of D i.e. $S_{D^-} = 4$ kN

S.F. just right of D i.e. $S_{D^+} = -6$ kN

Shear force between B & C

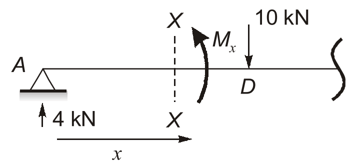
$$S_x = +2 \times x$$
$$S_x = 2x$$

S.F. just right of B i.e. $S_{B^+} = 2 \times 2$
$$= 4 \text{ kN}$$



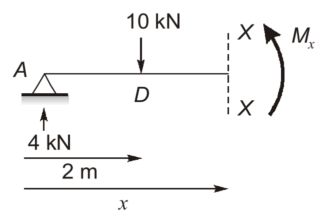
Bending Moment between A & D

$$M_x = 4 \times x$$
$$M_A = 0$$
$$M_D = 4 \times 2 = 8 \text{ kN-m}$$



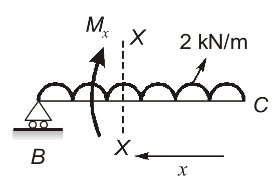
Bending Moment between D & B

$$M_x = 4 \times x - 10 \times (x - 2)$$
$$= 20 - 6x$$
$$M_{D^+} = 20 - 6 \times 2$$
$$= 8 \text{ kN-m}$$
$$M_{B^-} = 20 - 6 \times 4$$
$$= -4 \text{ kN-m}$$



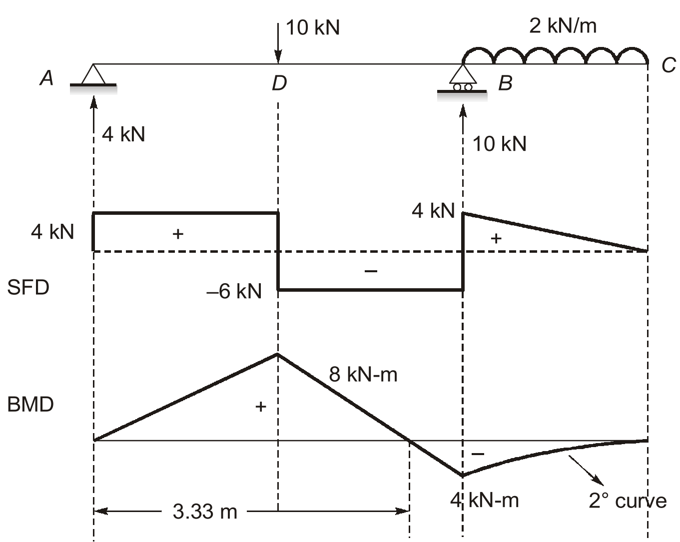
Bending Moment between B & C

$$M_x = -2 \times x \times \frac{x}{2}$$
$$= -x^2$$
$$M_{B^+} = -2^2$$
$$= -4 \text{ kN-m}$$

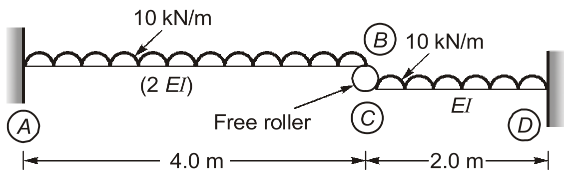


Location of zero B.M. between D & B

$$M_x = 0$$
$$20 - 6x = 0$$
$$x = 3.33 \text{ m from } A$$

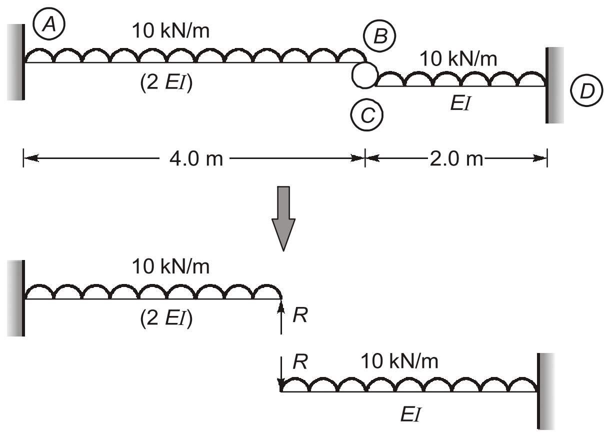


Q.9 Draw the Bending Moment diagram for the CD portion only for the beam shown in figure.



[2016 : 10 Marks]

Solution:



Because of campatibility of structure deflection of both the parts of beams will be equal.

$$\frac{w_1 l_1^4}{8EI_1} - \frac{Rl^3}{3EI_1} = \frac{w_2 l_2^4}{8EI_2} + \frac{Rl_2^3}{3EI_2}$$

$$\frac{10\times 4^4}{8(2EI)} - \frac{R\times 4^3}{3(2EI)} = \frac{10\times 2^4}{8EI} + \frac{R2^3}{3EI}$$

$$140 = \frac{40\times R}{3}$$

$$R = 10.5\text{ kN}$$

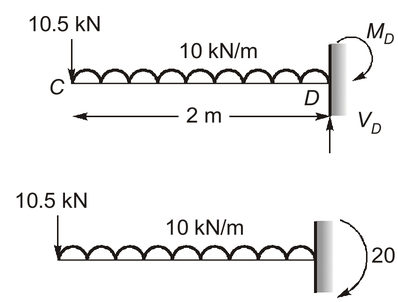
$$\Sigma F_y = 0$$

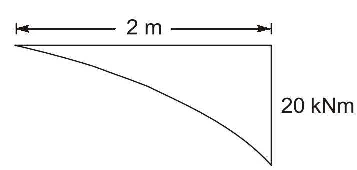
$$V_D = 10.5 + 10\times 2 = 30.5$$

$$\Sigma M_c = 0$$

$$10\times 2\times 1 - M_D = 0$$

$$M_D = 20\text{ kNm}$$





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